ISOPLANARITY IN VISION SYSTEMS

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A criterion is suggested for estimating the dimensions of the central isozone for the scheme of vertical viewing and results of these estimations are presented. Their dependence on the optical-geometric parameters of three types of vision systems is investigated. The calculations are performed in the single scattering approximation and (taking multiple scattering into account) using the Monte Carlo method.

It is known that the brightness image of any object can be obtained by different methods. The most characteristic of these are the following[1,2]:

a) the radiation brightness is recorded in the same fixed direction at different points of the plane in which the receiver is located (spatial distribution);

b) the angular brightness distribution is recorded at one fixed point in space;

c) the image is formed by a raster-type system in which angular scanning is carried out along one coordinate and the receiver is displaced along the other.

In general, when there is a scattering medium between the object and the receiver, all three received images appear different. To restore the actual form of the object, it is necessary to eliminate distortions caused by the medium. This can be done within the framework of the linear systems approach if the point spread function (PSF) of the system or its optical transfer function are known. As is well-known, the image of the object in the theory of linear systems is written as

$$g(x,y) = \int_{-\infty}^{\infty}\int G(x',y')h(x,y;\ x',y')dx'dy', \tag{1}$$

where $G(x', y')$ is the function that defines the object and $h(x, y; x', y')$ is the PSF of the vision system.

In the case of an isoplanar system,[3,4] the infinite set of functions $h(x, y; x', y')$ needed to retrieve the actual shape of the object can be restricted to those functions $h(x, y; x', y')$ of the form

$$h(x,y;\ x',y') = h(x-x';\ y-y'). \tag{2}$$

This makes it considerably easier to restore the image. Strictly speaking, only the system that forms the image according to procedure "a" under conditions of a horizontally homogeneous medium has the property of isoplanarity. Nevertheless, in most applied problems of vision theory one can find ranges of the values of x and y for procedures "b" and "c" within which relation (2) is satisfied with some prescribed accuracy. These ranges are conventionally called zones of isoplanatism. In this paper we suggest a criterion for estimating the dimensions of the central isozone for a scheme of vertical observation and describe the results of such estimations.

CRITERION SELECTION

It is clear that that part of the frame in which the brightness images obtained by procedures "a," "b," and "c' coincide with a specified degree of accuracy should be taken as the zone of isoplanatism.

Let us consider the image of the simplest object, i.e., a point source. From Eq. (1) and (2) it follows that

$$g(x,y) = \int_{-\infty}^{\infty}\int \delta(x'-\xi)\delta(y'-\eta)h(x-x',y-y')dx'dy' = h(x-\xi,y-\eta).$$

This means that in isoplanar system "a" the image of the point is its own spread function. Therefore, to determine the zone of isoplanatism, it is sufficient to determine the region where the PSF coincides with the angular distribution of the radiation brightness of the point source with the prescribed degree of accuracy.

Let us consider the vision system. A diagram of it is shown in Fig. 1. The diffuse point source (the object of observation) is assumed to be located at the origin of the XYZ coordinate system. The receiver is located at point A (0, 0, L). The plane-parallel, horizontally homogeneous, scattering medium is characterized by the profiles of the scattering $\sigma_s(z)$ and extinction $\sigma_t(z)$ coefficients and also by the scattering phase function g. Let us assume that the observation is vertical; therefore, the system has circular symmetry and $h(x, y) = h(r)$, where $r = \sqrt{x^2 + y^2}$. Let Θ be the angle between the direction of observation (in our case the $0Z$ axis) and the direction of the point source

$$B\left[\Theta = \mathrm{arctg}\frac{r}{L}\right].$$

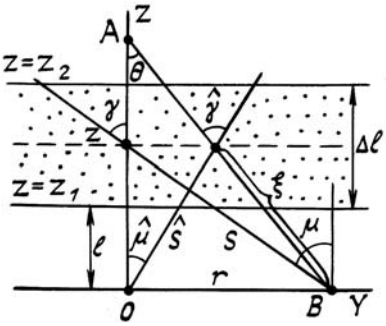

FIG. 1. Geometric diagram of image formation in vision systems.

We denote by $\hat{h}(r)$ the angular brightness distribution at the point A. We define the degree of coincidence of $h(r)$ and $\hat{h}(r)$ as follows:

$$\varepsilon(r) = \frac{|h(r)-\hat{h}(r)|}{h(r)}. \quad (3)$$

The isoplanatism zone dimension R_1 can be found from the condition

$$\varepsilon(R_1) = \varepsilon_0. \quad (4)$$

In practice it is more convenient and pictorial to use the angular isoplanatism zone dimension $R_\varphi = \mathrm{arctg}\frac{R_1}{L}$.

The goal of our efforts was to try to define the dependence of R_φ on the optical-geometric parameters of the vision system.

CALCULATIONAL METHODS

The functions $h(r)$ and $\hat{h}(r)$ were calculated in the single-scattering approximation (SSA) and by the Monte-Carlo method, which takes account of multiple scattering. To check the obtained results, a number of laboratory experiments were carried out.[5]

Within the framework of SSA we obtain the following expressions for the unknown functions:

$$h_1(r) = \frac{1}{2\pi^2}\int_0^L \frac{\sigma_s(z)}{s^2(z)}\,\mu(r,z)e^{-[\tau_1(z)+\tau_2(z)]} \times$$
$$\times\, g(\mu(r,z))dz; \quad (5)$$

$$\hat{h}_1(r) = \frac{1}{2\pi^2}\int_0^{\sqrt{r^2+L^2}} \frac{\sigma_s(\xi)\hat{\mu}(\xi)}{\hat{s}^2(\xi)}\, e^{-[\tau_1(\xi)+\tau_2(\xi)]} g(\hat{\gamma})d\xi, \quad (6)$$

where (Fig. 1) $\arccos\mu(\hat{\mu})$ is the angle between the radiation direction and the axis OZ; $\arccos\hat{\gamma}$ is the angle between the beam direction from the source and the direction from the scattering point to the point A; τ_1 and τ_2 are the optical distances from the source to the scattering point and from this point to the receiver; ξ is the coordinate along the line segment BA; $s(\hat{s})$ is the distance from the source to the scattering point. From among the various Monte-Carlo method algorithms one can use to calculate $h(r)$ and $\hat{h}(r)$, we chose the algorithm of local counting on conjugate trajectories.[6] The local estimate has the form[7]

$$\xi_{i,k} = 2\kappa^k\exp(-\tau_{i,k})g(\gamma_{i,k})\mu_{i,k}/(2\pi s^2_{i,k}),$$

where i is the path number; k is the number of the scattering event; κ is the probability of photon survival (we assume that $\kappa(s) = \kappa = \mathrm{const}$).

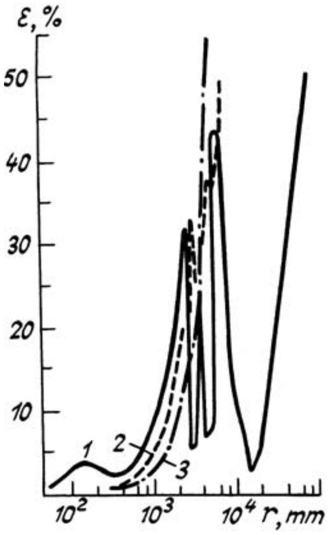

FIG. 2. General for the functions ε(r). The medium is haze H; Δl = 500 mm. τ = 3, t = 0.1 (curve 1); τ = 1, t = 0.5 (curve 2); τ = 1, t = 0.9 (curve 3); ε_0 = 20%.

In the first stage of the investigation we carried out a number of numerical experiments which simulated the observation conditions in the laboratory setup.[5] The receiver is located at a distance $L = 10^4$ mm from the source. A planar scattering layer of geometric thickness Δl = 30, 500 mm is located between the source and the receiver. We specify the position of the layer along the observation path by the parameter $t = \left(l + \frac{\Delta l}{2}\right)\Big/L$ (l is the distance from the source to the lower boundary of the layer): $0.025 \le t \le 0.9$. As the scattering medium we chose the haze-H model and the C.1 cloud model at the wavelength $\lambda = 0.53$ µm. The characteristic parameters of these two models are $\gamma_H = g_H(0)/g_H(\pi) \approx 143$, $\eta_H \approx 27.7$ (the asymmetry factor), $\gamma_{C.1} \approx 4480$, and $\eta_{C.1} \approx 28.4$. The optical depth varied within the range $0.1 \le \tau \le 12$. The scattering and extinction coefficients σ_s and σ_t were taken to be constant. Figure 2 shows some examples of the functions $\varepsilon(r)$. The characteristic feature of these functions is a pronounced nonmonotonicity in the majority of cases. Analysis of the mean values of the integrals h and $\hat{h}$ showed that this feature of the behavior of the functions $\varepsilon(r)$ is due to the variation of contributions of different magnitudes entering into h and $\hat{h}_1$, as functions of r. Figure 3, which presents examples of the following functions:

$$\varepsilon_h(r) = \frac{|h_1(r)-\hat{h}_1(r)|}{h_1(r)}; \quad \varepsilon_f(r) = \frac{|f(r)-\hat{f}(r)|}{f(r)};$$

$$\varepsilon_g(r) = \frac{|g(r)-\hat{g}(r)|}{g(r)}; \quad \varepsilon_{cos}(r) = \frac{|\mu/s^2-\hat{\mu}/\hat{s}^2|}{\mu/s^2};$$

$$\varepsilon_{exp}(r) = \frac{|\exp(-\tau)p-\exp(-\hat{\tau})\hat{p}|}{\exp(-\tau)p},$$

where

$$z = \frac{z_2-z_1}{2}; \quad g = g(\gamma(z,r)); \quad \hat{g} = g(\hat{\gamma}(\xi,r));$$

$$f = g(\gamma)\frac{\mu}{s^2}\exp(-\tau)p; \quad \hat{f} = g(\hat{\gamma})\frac{\hat{\mu}}{\hat{s}^2}\exp(-\hat{\tau})\hat{p};$$

$$p = \Delta l; \quad \hat{p} = \Delta l/\sin\Theta;$$

$$\tau = \sigma_t\frac{\Delta l}{2}\left[1 + \frac{1}{\sin\gamma}\right]; \quad \hat{\tau} = \sigma_t\frac{\Delta l}{2}\left[\frac{1}{\sin\hat{\mu}} + \frac{1}{\sin\Theta}\right]$$

(Fig. 1 illustrates this result), nonmonotonicity of the function $\varepsilon(r)$ can result a nonuniqueness in the determination of magnitude of R_φ. This depends on the choice the error level ε_0. The minimum error level which R_φ can be calculated uniquely is determined by the parameters of the optical-geometric observation scheme. Using the criterion (3), (4) to estimate the size of the isoplanatism zone, among all the possible values of R_φ corresponding to the prescribed error level ε_0, it is natural to choose the smallest.

Figure 4 depicts the dependence $R_\varphi(t)$ for different values of τ. The difference in the qualitative form of the curves $R_\varphi(t)$ for $\tau \leq 1.5$ and $\tau > 1.5$ is of particular interest. The presence of a pronounced maximum is a typical feature of $R_\varphi(t)$ for $\tau < 1.5$. A physical explanation of the position of the maximum has not yet been found. However, it follows from an analysis of Eqs. (5) and (6) that its existence and coordinates for $\tau \leq 1.5$ can be due to the combined action of the geometric factor (μ/s^2, $\hat{\mu}/\hat{s}^2$), and the scattering phase function. It makes sense, however, that the maximum of the dependence $R_\varphi(t)$ lies in the range of small values of t. When τ changes within the limits $1.5 < \tau \leq 12$, variations in the parameter f do not lead to substantial changes in the dimension R_φ. A small increase in R_φ is observed as $t \to 0$ and $t \to 1$. Moreover, $\lim_{t\to 1} R_\varphi(t, \tau) = \text{const}$.

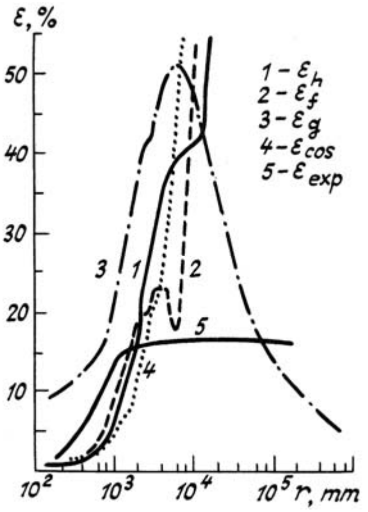

FIG. 3. Variation of the character of the interaction of various quantities that enter into h_1 and $\hat{h}_1$. The medium is haze H; Δl = 500 mm; τ = 1; t = 0.05; ε_0 = 20%.

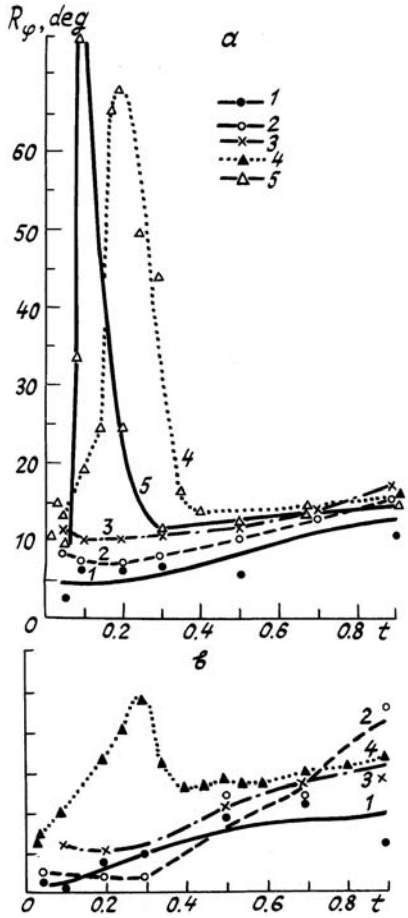

FIG. 4. The dependence $R_\varphi(t)$ for different values of τ. The medium, is: haze H (a); cloud C.1 (b); Δl = 500 mm; ε_0 = 20%; τ = 12 (curve 1); τ = 6 (curve 2); τ = 3 (curve 3); τ = 1 (curve 4); τ = 1.5 (curve 5).

The functions $R_\varphi(\tau)$ (Fig. 5) for different values of t are nonmonotonic in the range $\tau \le 1.5$, where the dependence of the functions $h(r)$ and $\hat{h}(r)$ on the scattering phase function is the strongest. As τ increases, the tendency of the value of R_φ to decrease becomes evident.

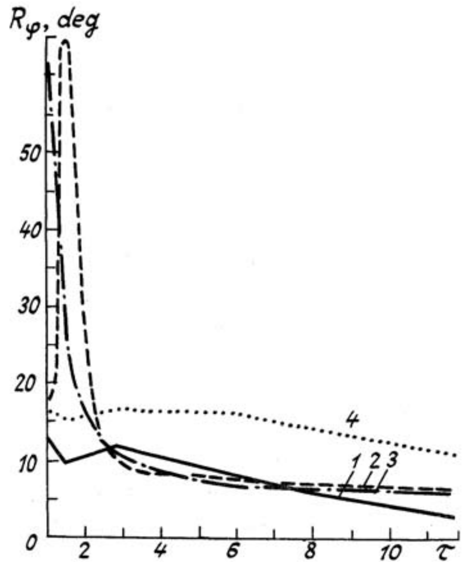

FIG. 5. The dependence $R_\varphi(\tau)$ for different values of t. The medium is haze H; Δl = 500 mm; ε_0 = 20%; τ = 0.05 (curve 1); τ = 0.1 (curve 2); τ = 0.2 (curve 3); τ = 0.9 (curve 4).

One of the factors substantially affecting the dimension of the isoplanatism zone is the scattering phase function. The dependence of R_φ on g can be' estimated from Figs. 4a and b. As the elongateness of the scattering phase function is varied, the above-noted regularities in the behavior of $R_\varphi(t, \tau)$, as a whole, remain unchanged. An increase in the phase function parameters η and γ results, as could be expected, in a decrease in R_φ. It is most evident in the region of those values of t where the maximum of $R_\varphi(t)$ is attained.

Both the growth in the optical depth and increase in t ($t \to 1$) lead to a convergence of the dimensions of the isoplanatism zones for media with different elongateness of the scattering phase function.

In Refs. 8 and 9 it is noted that a variation in the geometric depth of the scattering layer Δl does not substantially affect the functions $h(r)$ and $\hat{h}(r)$ if $\lg(\Delta l_1/\Delta l_2) \le 2$. A similar conclusion can be also drawn in relation to the influence of Δl on the isoplanatism zone dimension.

Finally, an increase in the prescribed error level ε_0 leads to an obvious increase in the value of R_φ with the character of the above regularities remaining practically unchanged.

As obvious as it may be, it is nevertheless important to note that the considered criterion (3), (4) is, as the analysis indicates, very sensitive to variations of the optical-geometric parameters. An integral criterion appears to be slightly more stable in this respect. According to such a criterion the isoplanatism zone dimension $\tilde{R}_i(\tilde{R}_\varphi)$ is determined from the condition

$$\delta = \frac{|\eta(R) - \hat{\eta}(R)|}{\eta(R)} = \delta_0, \qquad (7)$$

where

$$\eta(R) = 2\pi\int_0^R rh(r)dr \text{ and } \hat{\eta}(R) = 2\pi\int_0^R r\hat{h}(r)dr.$$

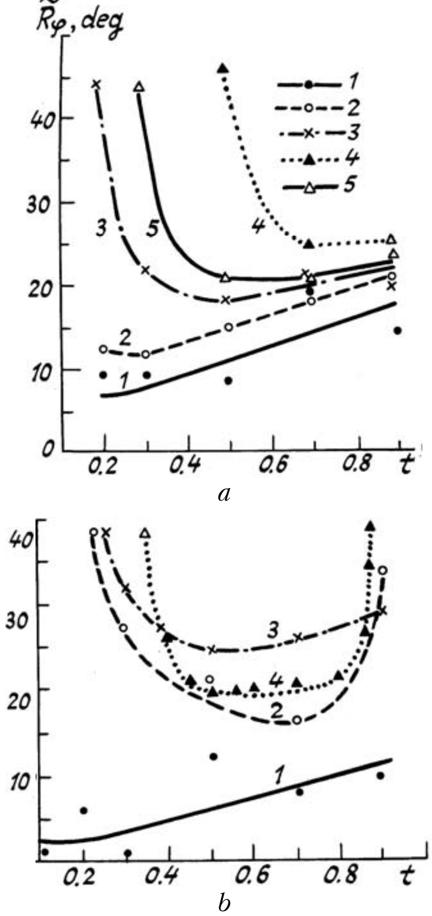

FIG. 6. The dependence $\tilde{R}_\varphi(t)$ for different values of τ. The medium is: a) haze H; b) C.1 cloud; Δl = 500 mm. τ = 12 (curve 1); τ = 6 (curve 2); τ = 3 (curve 3); τ = 1 (curve 4); τ = 1.5 (curve 5); δ_0 = 20%. In the case of cloud C.1 and τ = 1 δ = 5%.

An example of the dependences $\tilde{R}_\varphi(t, \tau)$ is shown in Fig. 6. In general, they exhibit the same regularities as were noted in the behavior of $R_\varphi(t, \tau)$. The differences between $R_\varphi(t, \tau)$ and $\tilde{R}_\varphi(t, \tau)$ have to do with the influence of the scattering phase function. That is, the increase in the elongateness of the scattering phase function results (in this case) in an increase in the dimension $\tilde{R}_\varphi$ at least for $\tau \le 6$. This is probably due to the fact that in the case of a more elongate scattering phase function up to certain values of τ the contributions of the integrands $h(r)$ and $\hat{h}(r)$ to $\eta(R)$ and $\hat{\eta}(R)$ are most significant when these functions differ only slightly from one another. For $\tau > 6$, strong multiple scattering effects considerably extend the range $[0, R]$ over which the integrals are summed. Therefore, in the case of a less elongate scattering phase function the isoplanatism zone dimension turns out to be larger.

The second stage of our investigations included numerical experiments for models of the actual atmosphere. A mean-cyclic model of the continental aerosol at the wavelength $\lambda = 0.53$ μm was taken as the starting point.[10] Calculations were also carried out for the case of the presence in the atmosphere of a solid cloud cover $\Delta l = 300$ m with the extinction coefficient $\sigma_t \approx 17$ km^{-1}. Its height above the Earth's surface was taken to be $l = 0.25$, 12, and 20 km. Table I displays calculated results for the angular dimension of the central isozone R_φ in the atmosphere for the case of vertical observation.

TABLE I.

ε_0, %	Cloudless atm.	l=0.25 km	l=12 km	l=20 km
20	5.84	0.11	2.05	10.03
50	9.78	0.52	3.24	11.95

It follows from the data in Table I that for observation conditions in the model that are close to actual ones, the dimension of the isoplanatism zone decreases (compared with those considered above) even for a cloudless atmosphere. The presence of a cloud layer can lead to different results: for small values of l, the value of R_φ becomes smaller than the value that it has in the case of a cloudless atmosphere. As l increases, the value of R_φ grows. But in every one of the above-mentioned cases R_φ does not exceed 10°-12° (for criterion (7) it is 20°) even for an error level of 50%.

In conclusion, we emphasize that, the isoplanarity property is not the same as that of "foreshortened invariance,"[1,2] for, as shown in Ref. 2, when the "foreshortened invariance" condition is violated, isoplanarity remains.

Thus, we can draw the following conclusions:

1. According to the criteria considered above, the isoplanatism zone dimension is a complex multivalued function of the optical-geometric parameters of the particular vision system in question (in particular, optical depth, the vertical profiles of the coefficients σ_s and σ_t, and the scattering phase function g).2. Using the Monte Carlo method, quantitative estimates of the central isozone dimension were obtained for concrete cases. However, the complex character of the dependences $R_\varphi(t, \tau, g)$ does not allow us to extrapolate the obtained estimates with any degree of confidence to other observation conditions for small optical depths ($\tau \leq 1.5$) and cases in which the medium contains layers with higher turbidity characterized by the parameter t in the limit $t \to 0$.

3. These facts should be taken into consideration when using one point spread function to restore the images obtained with the help of procedures "b" and "c".

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