THE MEAN VALUE OF ALBEDO OF SYSTEM OF BROKEN CLOUD-UNDERLYING SURFACE

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The problem of solar radiation transfer in the system cumulus clouds-underlying surface is studied.

An algorithm based on the Monte Carlo method for calculating the mean fluxes and their partial derivatives with respect to the parameters of the problem has been developed.

The effect of the albedo of the underlying surface, the geometric-optics parameters of the cloud field, and the zenith angle of the Sun on the mean value of albedo of the system and its partial derivative with respect to the cloud amount has been examined.

At present the need for a statistical description of the transfer of optical radiation in cloud fields with random geometry (broken clouds) has been generally recognized. The best-suited methods for such a description with respect to the mathematics and physics of the problem are based on averaging (analytically oh numerically) the stochastic radiation transfer equation over an ensemble of realizations of the cloud field simulated with the help of the Poisson point processes. In the case of a statistically uniform cloud field located above a nonreflective underlying surface (US), the equations for the mathematical expectation, variance, and spatial-angular correlation function of the radiation intensity have been obtained and algorithms for their solution by the Monte Carlo method developed (see Refs. 1 and 2 and the references cited therein). The variability of the mean fluxes of visible solar radiation attendant to changes in the geometric-optics parameters of the clouds and the illumination conditions change was studied in Refs. 1, 3, and 4, where the differences between the mean radiation fluxes for the cumulus clouds and equivalent stratus clouds are also discussed at some length. Equivalence here is taken to mean that the indicated types of clouds differ only in their horizontal size, while the optical and other geometric parameters are identical.

In this paper, we present a statistical simulation algorithm which we developed for calculating the linear functionals of the mean intensity of the solar radiation along with results of our investigation of the radiative characteristics of systems of cumulus clouds-underlying surface.

Cloud simulation and the procedure of problem solution. In the Cartesian coordinate system, the cloud field $OXYZ$ occupies the layer Λ: $0 \le z \le H$, at the top of which a unitary flux of solar radiation is incident upon the plane XOZ in the direction $\omega_{\odot}$. The optical characteristics of the clouds are assigned in the form of random scalar fields of .the extinction coefficient $\sigma\kappa(r)$, photon survival probability $\lambda\kappa(r)$, and the scattering phase function $g(\omega, \omega')\kappa(r)$, $r = (x, y, z)$, and $\omega = (a, b, c)$ is a unit direction vector, random indicator field

$$\kappa(r) = \begin{cases} 1, & r \in G, \\ 0, & r \notin G, \end{cases}$$

where $G \subset \Lambda$ is the random set of points at which the cloud material occurs. The field $\kappa(r)$ is simulated based on the Poisson point processes along each OX and OY coordinate axis.[1] It is statistically uniform and anisotropic; and it has the mean indicator field $\langle\kappa(r)\rangle p$ and the exponential function $B(r_1 - r_2) = \exp\,(-A(\omega_{\odot})\,|r_1 - r_2|\,)$, where p is the probability of cloud occurence; $A(\omega) = A(|a| + |b|)$, $\omega = (r_1 - r_2)\,/\,|r_1 - r_2|$, and A is the average number of points per unit length and is calculated based on the formula

$$A = (1.65\ (p - 0.5)^2 + 1.04)/\mathrm{D},$$

where D is the mean horizontal cloud size.

A closed system of equations for the mean intensity $\langle I(z, \omega)\rangle$ and the function $U(z, \omega) = \langle\kappa(r)\ I(r, \omega)\rangle/p$ (see Ref. 1) 'has been obtained from the stochastic transfer equation without taking into account reflection from the US. This system of equations for $\langle I(z. \omega)\rangle$ and $f(z, \omega) = \langle\kappa(r)\ I(r, \omega)\rangle = \langle\sigma p U(z, \omega)$ can be written down in the form

$$\langle I(z,\ \omega)\rangle = \frac{\lambda}{|c|} \int_{E_z} d\xi \int_{4\pi} \sum_{i=1}^{2} D_i \times$$

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$$\times \exp\left[-\lambda_1 \frac{|z-\xi|}{|c|}\right] g(\omega, \omega') f(\xi, \omega') d\omega' +$$

$$+ \langle j(z)\rangle \delta(\omega - \omega_\odot),$$

$$\langle j(z)\rangle = \sum_{1=1}^{2} C_1 \exp\left[-\lambda_1 \frac{|z-H|}{|c_\odot|}\right], \quad (1)$$

where the angular brackets denote the mean value over the ensemble of realizations of the cloud field and $\langle j(z)\rangle$ is the mean intensity of unscattered light.

$$\lambda_{1,2} = \frac{\sigma + A(\omega)}{2} \pm \frac{\sqrt{[\sigma + A(\omega)]^2 - 4A(\omega)\sigma p}}{2},$$

$$D_1 = \frac{\lambda_2 - \xi}{\lambda - \lambda}, \quad D_2 = 1 - D_1,$$

$$C_1 = \frac{\lambda_2 - \sigma p}{\lambda_2 - \lambda_1}, \quad C_2 = 1 - C_1, \quad (2)$$

$E_z = (0, z)$ at $c > 0$ and $E_z = (z, H)$ at $c < 0$. The function $f(z, \omega)$ has the meaning of the mean density of collisions and is the solution of the integral equation

$$f(x) = \int_x k(x, x) f(x') dx' + \Psi(x); \quad (3)$$

$$k(x', x) = \frac{\lambda g(\mu) \sum_{1=1}^{2} D_1 \lambda_1 \exp[-\lambda_1 |r - r'|]}{2\pi |r - r'|^2} \times$$

$$\times \delta\left[\frac{r - r'}{|r - r'|} - \omega\right], \quad (4)$$

$$\Psi(x) = \sum_{1=1}^{2} C_1 \lambda_1 \exp[-\lambda_1 |r - r_H|] \delta(\omega - \omega_\odot), \quad (5)$$

where X is the phase space of the coordinates and directions, $x = (r, \omega)$, $g(\omega, \omega') = g(\mu)/2\pi$, $\mu = \omega' \cdot (r - r')/|r - r'|$, $r_H \in z = H$. The function $\Psi(x)$ is the mean density of the initial collisions.

Let us consider the problem of calculating . the linear functionals by the Monte Carlo method:

$$J_h = (f, h) = \int_x f(x) h(x)\, dx, \quad (6)$$

In particular, $\langle I(z, \omega)\rangle$ is such a functional. We shall simulate the trajectories of the particles with initial density $\Psi(x)$ and transition density $k(x, x)/\lambda$. To calculate J_h, we have[5]

$$J_h = (f, h) = M\xi, \quad (7)$$

where M is the symbol of mathematical expectation over the ensemble of realizations of the trajectories of the particles;

$$\xi = \sum_{n=0}^{N} Q_n h(x_n), \quad (8)$$

N is the random number of the last state of the trajectory, while the auxiliary weight Q_n is calculated according to the formulas

$$Q = 1, \quad Q_n = \lambda Q_{n-1}. \quad (9)$$

It is pertinent to note that in the algorithm constructed based on the solution of the integral equation for the function $U(x)$ by the Monte Carlo method the weight Q_0 is random.[1]

In accordance with Eq. (1), the mean intensity of the radiation that passes through the plane $z = z_*$ in a direction $\omega_* \neq \omega_\odot$ can be calculated if we set

$$h_{z_*}(x_n) = \begin{cases} \frac{g(\mu_*)}{2\pi|c_*|} \sum_{1=1}^{2} D_1 \exp\left[-\lambda_1 \frac{|z_* - z_n|}{|c_*|}\right], & (z_* - z_n)c_* > 0 \\ 0, & (z_* - z_n)c_* < 0, \end{cases} \quad (10)$$

where $\mu_* = (\omega_* \cdot \omega_n)$, and to calculate the mean radiation flux

$$h_{z_*}(x_n) = \begin{cases} \sum_{1=1}^{2} D_1 \exp\left[-\lambda_1 \frac{|r_* - r_n|}{|c_{n+1}|}\right], & (z_* - z_n)c_{n+1} > 0, \\ 0, & (z_* - z_n)c_{n+1} < 0. \end{cases} \quad (11)$$

Based on the method of dependent tests[5] it is not difficult to construct a statistical simulation algorithm to calculate the partial derivatives of the linear functionals with respect to the parameters of the problem, for instance, relative to the cloud amount N, the horizontal cloud size, etc. Knowledge of the partial derivatives makes it possible to get a more detailed insight into the dependence of the statistical characteristics of the radiation field on the geometric-optics parameters of the clouds, identify the parameters that are mainly responsible for the variability of the radiation field, and in the framework of the linearization method formulate the inverse problems of retrieval of the parameters of the cloud field based on the statistical characteristics of the radiation measured.

It follows from the definition of $f(x)$ as the mean density over the ensemble of realizations of the collisions that the average number of particles that cross the plane $z = z_*$, provides an estimate of the mean radiation flux passing through this plane. Calculations show that the algorithm for calculating the mean fluxes of the solar radiation constructed on the basis of a direct simulation of Markov chains is more efficient than the local estimate (11) by a factor of ~ 2.

The use of the mean collision density $f(x)$ instead of the function $U(x)$ provides the possibility of taking account in a simple way of reflection from the US, which can be regarded as a diffuse transmitter. If the path intersects the US, then, according to the assigned reflectance, a new direction of motion of the particle is played out, while the mean free path up to the point of the first collision is simulated based on the probability density $\Psi(x)$. Comparison with the calculations based on the numerical simulation of the cloud and radiation fields[1] shows that the statistical simulation algorithm developed to calculate the radiative characteristics of the cumulus clouds-underlying surface has a high accuracy.

The results of our calculations. Let a horizontally uniform US have albedo A_s and reflect the radiation incident on it according to Lambert's law, and the scattering phase function be calculated on the basis of the Mie theory for the cloud model C1 (see Ref. 6) for the wavelength 0.69 μm. In order to simplify the problem, we neglect the interaction of visible solar radiation with the aerosol-gaseous atmosphere. The calculations are performed for $0 \le A_s \le 0.8$, but because of limitations on the length of the article, we present only the results that were obtained for $A_s = 0$ and $A_s = 0.315$. These values of A_s correspond approximately to the albedo of the ocean and of some types of dry land.[7] It is obvious that the mean albedo R_{st} of stratus clouds that partially cover the sky and its partial derivative $\delta\bar{R}_{st} / \delta N$ can be written in the form

$$\bar{R}_{st} = (1 - N)A_s + NR_s, \quad \frac{\partial \bar{R}_{st}}{\partial N} = R_s - A_s, \qquad (12)$$

where R_s is the albedo of a continuous horizontally uniform cloud layer which is located above the Lambertian surface. In the case in which $\gamma = \frac{H}{D} \to 0$ it is not difficult to deriveEqs. (12) from Eqs. (1)–(5) for the mean intensity of the radiation modulated by broken clouds. These formulas, without any strict foundation or any estimates of their accuracy, are used to calculate the mean radiation characteristics not only of stratus clouds ($\gamma \ll 1$), but also of cumulus clouds ($\gamma \sim 1$). It is pertinent to note that the derivative $\delta\bar{R}_{st} / \delta N$ is used when estimating the parameter of the sensitivity of the climate to variations of the cloud amount.[8]

Analysis of the obtained results shows that when the cloud amounts are small ($N \sim 0.1$–0.3) a significant portion of the radiation reflected from the US propagates in the gaps between the clouds, the role of multiple reflections between the US and the cloud field being, on the average, not great and the dependence of the mean albedo on A_s being close to linear. For larger cloud amounts ($N \gtrsim 0.6$), the contribution of multiple reflections, on the average, increases, which is responsible for a relatively weak nonlinear dependence of the mean albedo on A_s. The contribution of the US to the mean albedo of the system prevails in the case in which the cloud amounts are small. For instance, for $N = 0.1$, $\bar{R}(A_s = 0.8) / \bar{R}(A_s = 0) \sim 20$ and decreases as N increases since the latter ratio does not exceed 2 for $N = 0.9$.

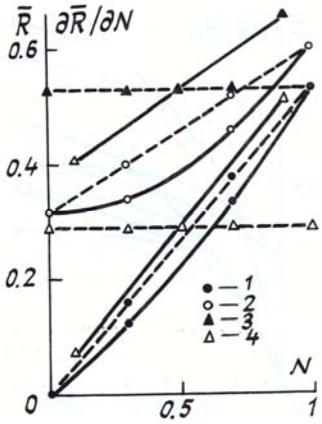

FIG. 1. The effect of the cloud amount N on the mean albedo $\bar{R}$ and the derivative $\delta\bar{R} / \delta N$ for $D = 0.25$ km, $\xi_\odot = 0$, and $\sigma = 30$ km^{-1}. Here and in the following figures $H = 0.5$ km, $\bar{R}$ ($A_s = 0$) (1); $\bar{R}$ ($A_s = 0.315$) (2); $\delta\bar{R} / \delta N$ ($A_s = 0$) (3); and $\delta\bar{R} / \delta N$ ($A_s = 0.315$) (4). The solid curves denote cumulus clouds and the broken curves denote stratus.

As might be expected, regardless of the cloud amounts, $\bar{R}$ and $\bar{R}_{st}$ are monotonically increasing functions of A_s (Fig. 1). It can be seen that the mean albedo is most sensitive to variations of the cloud amount when A_s is small, for example, the case of cumulus cloud fields above the ocean. In the case in which the clouds are located above a good reflecting surface, which snow cover is, the values of the mean albedo of both cumulus and stratus clouds depend weakly on N and are close to A . In the event of small solar zenith angles, the difference ($\bar{R}_{st} - \bar{R}$) increases as A_s grows, reaches its maximum magnitude for some value $\tilde{A}_s > 0$, and then falls to zero as $A_s \to 1$. This can be explained by the fact that in cumulus clouds the diffuse radiation reflected from the US can be attenuated not only by the bottoms, but also by the sides of a great number of individual cumulus clouds. Consequently, the fraction of radiation which has passed through the gaps between the clouds, on the average, is smaller, while the fraction of radiation backscattered towards the US is on average greater in the case of cumulus clouds, and the contribution of the radiation reflected from the surface is smaller than for the equivalent stratus clouds. As regards the derivative $\delta\bar{R}\,/\,\delta N$, if A_s increases, its value substantially decreases, especially for small cloud amounts. In accordance with the foregoing, as A increases from 0 to $\tilde{A}_s$ the difference between the ratios $\delta\bar{R}\,/\,\delta N$ and $\delta\bar{R}_{st}\,/\,\delta N$ becomes greater, for example, $\dfrac{\delta\bar{R}_{st}}{\delta N}\Big/\dfrac{\delta\bar{R}}{\delta N}\sim$ 1.3 is valid for $N = 0.1$ and $A_s = 0$, while for $A_s = 0.315$ this ratio has increased to ~ 4. For the given parameters of the problem $\delta\bar{R}\,/\,\delta N$ depend almost linearly on N, for which reason the dependence of the mean albedo on the cloud amount is close to quadratic.

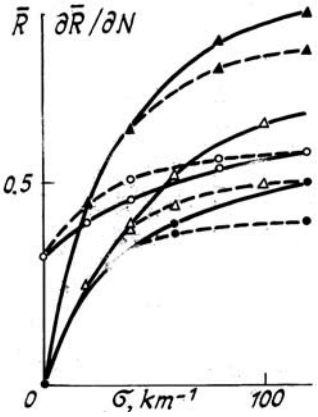

FIG. 2. The mean albedo $\bar{R}$ and the derivative $\delta\bar{R}\,/\,\delta N$ as functions of the extinction coefficient σ at $D = 0.25$ km, $\xi_\odot = 30$, and $N = 0.5$.

The mean albedo and its partial derivative with respect to N grow as the optical thickness $\tau = \sigma H$ of the clouds increases (Fig. 2). If the cloud field is above a surface with a small or intermediate value of A_s, the mean albedo is most sensitive to variations of the optical thickness in the interval $0 \le \tau \le 20$–30. As A_s increases, the sensitivity of the mean fluxes relative to τ decreases, for example, when $A_s \gtrsim 0.8$ and $5 \le \tau \le 60$, $\bar{R}\simeq\bar{R}_{st}\simeq 0.8$ and $\delta\bar{R}\,/\,\delta N\simeq\delta\bar{R}_{st}\,/\,\delta N\simeq 0$. Effects which are random in nature and are associated with the finite horizontal size of the clouds are responsible for the fact that $\bar{R}$ is more sensitive to variations of the optical thickness of the clouds at $\tau \gtrsim 20$–30 than $\bar{R}_{st}$. An explanation of the small differences between $\bar{R}$ and $\bar{R}_{st}$ at $A_s = 0$ and $\tau \le 20$–30 was given in Ref. 4. It is advisable to turn our attention to the strong variability (especially in the case of a cloud field above the ocean) of $\delta\bar{R}\,/\,\delta N$ and $\delta\bar{R}_{st}\,/\,\delta N$ when τ varies greatly.

As long as the underlying surface is a Lambertian source, which emits isotropically regardless of the angular distribution of the transmitted solar radiation, the dependence of the mean albedo on $\xi_\odot$ becomes weaker as A_s increases. For this reason, the mean albedo of the system is most sensitive to variations of the zenith angle of the Sun in the region of small A_s. When $A_s = 0$, the growth of $\xi_\odot$ from 0° to 60° results in an increase of $\bar{R}$ by almost a factor of two, while for $A_s = 0.8$ the mean albedo increases by only ~ 5%. It is obvious that the extinction and scattering of the incident solar radiation by the sides of cumulus clouds increase, on the average, as the ratio $\gamma = H/D$ grows (the other parameters of the problem are kept fixed) and result in an increase of the fraction of the scattered radiation. Therefore, for large solar zenith angles the mean albedo of a field of cumulus clouds can be greater than that of a field of stratus clouds (Figs. 3 and 4). The value of $\xi_\odot$ at which the inequality $\bar{R} < R_{st}$ is reversed depends on A_s and decreases as γ increases.

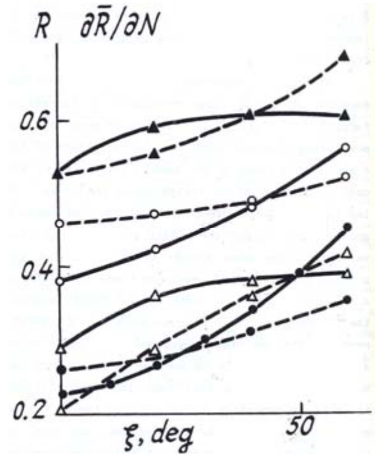

FIG. 3. The effect of solar zenith angle $\xi_\odot$ on the mean albedo $\bar{R}$ and the derivative $\delta\bar{R}\,/\,\delta N$ for $D = 0.25$ km, $N = 0.5$, and $\sigma = 30$ km^{-1}.

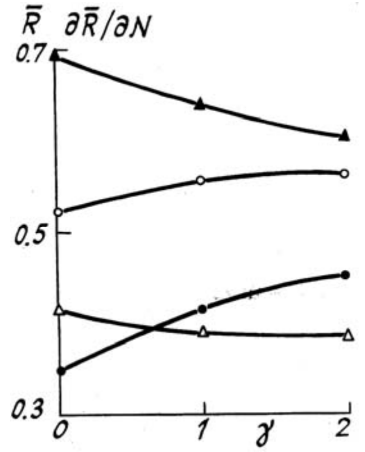

FIG. 4. The mean albedo $\bar{R}$ and the derivative $\delta\bar{R}\,/\,\delta N$ as functions of the parameter $\gamma = H/D$ for $N = 0.5$, $\sigma = 30$ km^{-1} and $\xi_\odot = 60$.

Conclusion. Based on the foregoing numerical experiments it has been determined that in the case of small and average values of A_s the mean albedo of a field of cumulus clouds depends nonlinearly on the cloud amount and differes substantially from the mean albedo of the equivalent stratus clouds. The interaction of the incident radiation with the sides of the cumulus clouds is responsible for the fact that at large solar zenith angles the mean albedo of cumulus clouds can be greater than that of stratus clouds. With an increase in A_s, the derivatives $\delta\bar{R} / \delta N$ and $\delta\bar{R}_{st} / \delta N$ decrease, $\delta\bar{R} / \delta N$ being especially sensitive to variations of A_s when the cloud amounts are small. These peculiarities of the radiative regime of cumulus clouds must be taken into account in numerical models of weather forecasting based on the mesometeorology equations.

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