UNIFORM APPROXIMATION IN APPLICATION TO SMALL PERTURBATION METHODS IN PROBLEMS OF STATISTICAL ANALYSIS OF DOPPLER MEASUREMENTS

A.P. Shelekhov

Institute of Atmospheric Optics, Siberian Branch of the Russian Academy of Sciences, Tomsk
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We study two approaches to statistical analysis of the frequency measurements of non-Gaussian signal of the Doppler lidar when using the perturbation method. The first approach uses the perturbation method proposed for analysis of a random Gaussian signal. The second approach is based on renormalization of the average spectrum parameters. When applying the first method, one faces the difficulties associated with nonuniform approximation and physical interpretation of the results. It is shown that renormalization of the average spectrum parameters gives rise to a perturbation theory series for estimating Doppler frequency. This series meets the requirement of the uniform approximation and allows the interpretation of obtained results.

1. INTRODUCTION

It is known that application of the small perturbation method without meeting the requirement of uniform approximation results in significant difficulties. In the theory of oscillations, hydrodynamics, the theory of laser generation, and other branches of physics,[1,2] the presence of secular terms in serial expansions makes the physical interpretation of the observed phenomena significantly much more complicated. Nonlinear properties of oscillations, generation, etc. are the cause of nonuniform approximation and difficulties in physical approximation of the results that can be overcome using the renormalization method.

This paper considers the methodical problems of the perturbation methods application to statistical analysis of the Doppler lidar signal frequency measurements. Under study are two approaches to statistical analysis of frequency measurements of non-Gaussian signal of the Doppler lidar when applying the perturbation methods. The first approach uses the perturbation theory proposed in Refs. 6 and 7 for analysis of random Gaussian signal. The second approach is based on renormalization of the average spectrum parameters. The characteristic feature of the problem considered is the fact that Doppler lidar signal is non-Gaussian random process and only when the conditions of the central limit theorem hold true the statistical properties become Gaussian.[3–5] It is shown that the nonuniformity in approximation and difficulties in physical interpretation of the results that can be overcome using renormalization method are caused by non-Gaussian properties of a Doppler lidar signal.

2. USE OF PERTURBATION METHODS IN STATISTICAL ANALYSIS OF DOPPLER MEASUREMENTS

The estimate of the Doppler frequency for the method of spectral function[4] has the form

$$\hat{\Omega}_d = \frac{2\pi}{T_s} \sum_{m=-(N_s-1)}^{N_s-1} \omega_m \, \hat{\mathbf{S}}(\omega_m) / \hat{\mathbf{S}}_0 \, , \tag{1}$$

where

$$\hat{\mathbf{S}}(\omega_m) = \frac{1}{M N_s \Delta t} \sum_{i=1}^{M} \left| \sum_{n=0}^{N_s-1} j(t_i + n\Delta t) \exp\left(-i\, \omega_m \, n\Delta t\right) \right|^2$$

is the spectrum estimate; $\omega_m = \dfrac{2\pi m}{N_s \, \Delta t} = \dfrac{2\pi m}{T_s}$, $m = 0, \pm 1, \ldots, \pm(N_s - 1)$; $\hat{\mathbf{S}}_0 = \dfrac{2\pi}{T_s} \sum\limits_{m=-(N_s-1)}^{N_s-1} \hat{\mathbf{S}}(\omega_m)$;

M is the number of cases of the length T_s; Δt is the discreteness range; $N_s = T_s/\Delta t$, $j(t)$ is the Doppler lidar signal. It is assumed that for the Doppler lidar signal the assumption is true that only single scattering of optical radiation by atmospheric particles moving in turbulent flow[3,4] occurs. As to the medium, it is assumed that particles are distributed in the scattering volume homogeneously and independently. The particle number follows the Poisson law, and the atmospheric turbulence flow motion velocity is distributed following the

Gaussian law. The noise fluctuations are the Gaussian white noise.

When studying the uniform approximation, under consideration are the case of Gaussian statistics and two cases corresponding to non-Gaussian statistics of the Doppler lidar signal. Gaussian statistics of the Doppler lidar signal is observed under the following conditions: $d = \infty$ (where $2d$ is the characteristic size of the scattering volume), that means that the influence of correlation of the local Doppler frequency fluctuations is totally neglected.

The first case associated with the non-Gaussian statistics corresponds to the case of a strong correlation of the local Doppler frequency fluctuations, $d \to 0$. The second case associated with the non-Gaussian statistics is the case of weak correlation of the local Doppler frequency fluctuations, $d \to \infty$. In this case, the influence of correlation properties of local Doppler frequency fluctuations is taken into account as a small perturbation.[4,5] Thus, it is characteristic of the first and second cases, that non-Gaussian properties of the Doppler lidar signal manifest themselves in maximal and minimal form, respectively.

2.1. Analysis of a random Gaussian signal by perturbation methods

In spite of different behavior of the signal fluctuations, the case of Gaussian statistics ($d = \infty$) and the case of weak correlation ($d \to \infty$) have common regularities. Therefore let us consider separately the approach to statistical analysis of Doppler measurements of a random Gaussian signal for the method of spectral function. The approach to statistical analysis of Doppler measurements of a random Gaussian signal was proposed in Refs. 6 and 7 for the method of autocorrelation function and the maximum likelihood method. In this statistical analysis two assumptions are used by turn. First, it is assumed that as a zero order of the perturbation theory the average characteristics of the Doppler lidar signal, for example the autocorrelation function, can be taken. Then, having written the perturbation series, the assumption of Gaussian properties of the signal is used. Therefore let us present the spectrum estimate as the average spectrum and small perturbation

$$\hat{\mathbf{S}}(\omega_m) = \mathbf{s}(\omega_m) + \Delta\mathbf{s}(\omega_m) + \dots ,$$

$$\hat{\mathbf{S}}_0 = \mathbf{s}_0 + \Delta\mathbf{s}_0 + \dots , \tag{2}$$

where $\mathbf{s}(\omega_m)$ is average spectrum; $\mathbf{s}_0 = \dfrac{2\pi}{T_s} \sum\limits_{m=-(N_s-1)}^{N_s-1} \mathbf{s}(\omega_m)$; $\Delta\mathbf{s}(\omega_m)$ and $\Delta\mathbf{s}_0$ are the first-order terms of the perturbation theory. It follows from Eqs. (1) and (2) that in the first order of perturbation theory the estimate of the Doppler frequency takes the following form:

$$\hat{\Omega}_d = \omega_d - \frac{2\pi}{T_s} \sum_{m=-(N_s-1)}^{N_s-1} (\omega_m - \omega_d)\, \Delta\mathbf{s}(\omega_m)/\mathbf{s}_0 , \tag{3}$$

where ω_d is the average Doppler frequency.

Then, following Refs. 6 and 7, let us assume that the parameter $\Delta\mathbf{s}(\omega_m)$, entering in the Eq. (3), is defined by Gaussian properties of the Doppler lidar signal. In the case of Gaussian statistics, the expression for measurement error in the average Doppler frequency $\mathrm{var}\,\hat{\Omega}_d = \langle(\hat{\Omega}_d - \omega_d)^2\rangle$, calculated based on Eq. (3) under the condition $d = \infty$ as a result of averaging over random position and number of particles in the scattering volume, over fluctuations of the velocity of the atmospheric turbulent flux motion, as well as noise fluctuations, takes the form

$$\mathrm{var}\,\hat{\Omega}_d = \frac{\sqrt{\pi}}{2MT_s} \sqrt{\langle\Delta\omega'^{\,2}_{d,g}\rangle} + \frac{2}{M\, S/N} + \\ + \langle\Delta\omega'^{\,2}_{d,g}\rangle + \frac{\pi^2}{3MT_s^2\, S^2/N^2} , \tag{4}$$

where $\langle\Delta\omega'^{\,2}_{d,g}\rangle$ is the average spectrum halfwidth $\mathbf{s}(\omega_m)$; S/N is the signal-to-noise ratio. Equation (4) has simple physical meaning. As seen from the equation, the first two terms are functions of the average spectrum halfwidth $\mathbf{s}(\omega_m)$. Consequently, they characterize the value of measurement error of the average Doppler frequency, resulting from the spectrum broadening. The last term differs from zero at $\langle\Delta\omega'^{\,2}_{d,g}\rangle = 0$, therefore it is the measure of statistical uncertainty in measurements of harmonic oscillation frequency against the background noise. Thus, for a random Gaussian signal the measurement error of average Doppler frequency is governed by two factors: spectrum broadening and measure of statistical uncertainty of the harmonic oscillation measurements against the noise. It should be noted that Eq. (4) coincides with similar expression for the method of autocorrelation function obtained in Refs. 6 and 7. This coincidence follows from the fact that the methods of autocorrelation and spectral functions fall, in their essence, in the category of nonoptimal approaches, and therefore should have the same accuracy.

2.2. Statistical analysis of Doppler measurements of non-Gaussian signal when using the perturbation method[6, 7]

Let us consider the first approach to statistical analysis of the frequency measurements of non-Gaussian signal of a Doppler lidar when using the perturbation methods. This approach is based on the use of perturbation method presented in Refs. 6 and 7.

The expansion (2), as well as the perturbation series for estimation of the Doppler frequency (3) will be considered valid for a random non-Gaussian signals. However, in contrast to derivation of Eq. (4), let us assume that parameter $\Delta\mathbf{s}(\omega_m)$, entering in Eq. (3), is defined by non-Gaussian properties of a Doppler lidar. The value of measurement error of average Doppler frequency $\mathrm{var}\,\hat{\Omega}_d = \langle(\hat{\Omega}_d - \omega_d)^2\rangle$, calculated based on Eq. (3), takes the form

$$\mathrm{var}\,\hat{\Omega}_d = \sigma^2_{\omega_d}\int |p(\mathbf{r}_m)|^2\, |p(\mathbf{r}_n)|^2\, R_{\omega_d}(\mathbf{r}_m, \mathbf{r}_n)\mathrm{d}\mathbf{r}_m \mathrm{d}\mathbf{r}_n +$$

$$+\frac{\sqrt{\pi}}{2MT_s}\sigma_{\omega_d}\int |p(\mathbf{r}_m)|^2 |p(\mathbf{r}_n)|^2 \frac{1+R_{\omega_d}(\mathbf{r}_m, \mathbf{r}_n)}{\sqrt{1-R_{\omega_d}(\mathbf{r}_m, \mathbf{r}_n)}}\mathrm{d}\mathbf{r}_m\mathrm{d}\mathbf{r}_n +$$

$$+\frac{2}{MS/N}\sigma^2_{\omega_d} + \frac{\pi^2}{3MT_s^2 S^2/N^2}, \quad (5)$$

where $\sigma^2_{\omega_d} R_{\omega_d}(\mathbf{r}_m, \mathbf{r}_n) = \langle\omega'_d(\mathbf{r}_m)\, \omega'_d(\mathbf{r}_n)\rangle$; $\sigma^2_{\omega_d}$ is the variance; $R_{\omega_d}(\mathbf{r}_m, \mathbf{r}_n)$ is the normalized correlation function; $\omega'_d(\mathbf{r}_m)$ are fluctuations of the local Doppler frequency; $p(\mathbf{r}_m)$ is the cross-section of the directional patterns.[4,5]

Let us analyze Eq. (5) for the case of Gaussian statistics and for two cases corresponding to non-Gaussian statistics of the Doppler lidar signal. In the case of Gaussian statistics of Doppler lidar signal ($d = \infty$), the contribution of correlation properties of the local Doppler frequency fluctuations is totally neglected, i.e. $R_{\omega_d}(\mathbf{r}_m, \mathbf{r}_n) = 0$. In this case, the expression for measurement error (5) coincides with the Eq. (4), taking into account the fact that $\langle\Delta\omega'^2_{d,g}\rangle = \sigma^2_{\omega_d}$. This coincidence is natural, since initially the expansion (2), as well as the perturbation series (3) is basic assumption in the statistical analysis of Doppler measurements of a random Gaussian signal, and asymptotic behavior $d = \infty$ of Eq. (5) corresponds to the case of Gaussian statistics.

If for the Gaussian statistics the behavior of Eq. (5) agrees with Eq. (4) and, consequently, with the results of Refs. 6 and 7, then analysis of this equation for two cases, corresponding to a non-Gaussian signal of a Doppler lidar, leads to contradictory results. For strong correlation, i.e. in the first case, the expression $1 - R_{\omega_d}(\mathbf{r}_m, \mathbf{r}_n) \to 0$ and corresponding integrand in Eq. (5) increase unlimitedly, so the value $\mathrm{var}\,\hat{\Omega}_d \to \infty$ at $d \to 0$.

From the physical viewpoint, the unlimited behavior of the measurement error at $d \to 0$ is a contradictory result. Really, on the one hand, $\mathrm{var}\,\hat{\Omega}_d \to \infty$ means that the average Doppler frequency cannot be measured at strong correlation. On the other hand, at $d \to 0$ measured is the frequency of a random harmonic oscillation against the noise. Consequently, the uncertainty in measurements of the average Doppler frequency due to the spectrum broadening disappears. Therefore the value of the measurement error in the average Doppler frequency must be defined by a limited parameter, related to fluctuations of the frequency of harmonic oscillations and the measure of statistical uncertainty in measurements of the average frequency of the harmonic oscillation against the noise.

Thus, the behavior of measurement error at $d \to 0$ is similar to the behavior of solution, for example, in the theory of oscillations[1,2] before applying the renormalization methods. The measurement error in the average Doppler frequency and oscillation energy grow unlimitedly, that contradicts to physical meaning. The case $d \to 0$ is characterized by the fact that non-Gaussian properties of the Doppler lidar signal manifest themselves in a maximum degree. Therefore we can conclude that the cause of nonuniform approximation and difficulties in physical interpretation of the results is in non-Gaussian properties of the Doppler lidar signal.

In the case of a weak correlation ($d \to \infty$) the result is also difficult for physical interpretation. In this case the expression for measurement error of the average Doppler frequency takes the following form:

$$\mathrm{var}\,\hat{\Omega}_d = \sigma^2_{\omega_d}\int |p(\mathbf{r}_m)|^2\, |p(\mathbf{r}_n)|^2\, R_{\omega_d}(\mathbf{r}_m, \mathbf{r}_n)\mathrm{d}\mathbf{r}_m \mathrm{d}\mathbf{r}_n +$$

$$+\frac{\sqrt{\pi}}{2MT_s}\sigma_{\omega_d}\int |p(\mathbf{r}_m)|^2 |p(\mathbf{r}_n)|^2\left(1+\frac{3}{2}R_{\omega_d}(\mathbf{r}_m, \mathbf{r}_n)\right)\mathrm{d}\mathbf{r}_m\mathrm{d}\mathbf{r}_n +$$

$$+\frac{2}{MS/N}\sigma^2_{\omega_d} + \frac{\pi^2}{3MT_s^2 S^2/N^2}. \quad (6)$$

For a weak correlation, in behavior of the measurement error in the average Doppler frequency, regularities must be observed, which are common for the behavior of the same parameter but for Gaussian statistics. Therefore one should expect here that the second and third terms in the right-hand side of Eq. (6) should have the form similar to Eq. (4). However, the parameter $\mathrm{var}\,\hat{\Omega}_d$ cannot be presented as a function of the average spectrum halfwidth, i.e.

$$\mathrm{var}\,\hat{\Omega}_d \neq \sigma^2_{\omega_d}\int |p(\mathbf{r}_m)|^2\, |p(\mathbf{r}_n)|^2\, R_{\omega_d}(\mathbf{r}_m, \mathbf{r}_n)\mathrm{d}\mathbf{r}_m \mathrm{d}\mathbf{r}_n +$$

$$+\frac{\sqrt{\pi}}{2MT_s}\sqrt{\langle\Delta\omega'^2_{d,g}\rangle} + \frac{2}{MS/N}\langle\Delta\omega'^2_{d,g}\rangle + \frac{\pi^2}{3MT_s^2 S^2/N^2}, \quad (7)$$

and, consequently, have the form similar to Eq. (4). Therefore, in my opinion, it is difficult to give simple physical interpretation to Eq. (6) in the case of a weak correlation.

Thus, the application of perturbation methods[6,7] results in difficulties at statistical analysis of measurements of the Doppler frequency of a non-Gaussian signal. They are nonuniform approximation and difficulties in physical interpretation of the results obtained at weak and strong correlation.

Equation (5) coincides with similar equation from Refs. 8 and 9 at $S/N \to \infty$ and $p(\mathbf{r}_m) = p(z_m)$, where z_m is the projection of vector $\mathbf{r}_m$ onto the measurement direction. Coincidence of the results is indicative of the fact that the approach to statistical analysis of Doppler measurements proposed in Refs. 8 and 9 leads to the same difficulties: nonuniform approximation and difficulties in physical interpretation. It should be noted that in this paper, in contrast to Refs. 8 and 9, we do not assume $\Delta\mathbf{s}_0 = 0$. Therefore the derivation of Eq. (5) is, from the methodical point of view, more correct and to a greater degree corresponds to the basic assumptions.[6,7]

2.3. Statistical analysis of Doppler measurements of a non-Gaussian signal when using the perturbation method based on renormalization of the average spectrum parameters

The second approach to applying of the perturbation method allows us to overcome the above difficulties, related to the divergence of perturbation series and difficulties in physical interpretation of the results. This approach is based on renormalization of the average spectrum parameters. It is known[1,2] that in the theory of nonlinear oscillations, for example, to obtain uniform approximation, perturbation is added not only to solution itself, but to the parameters of solution as well, i.e. the oscillation frequency. It results in the situation when the solution already in zero order depends on nonlinearity parameters. In statistical analysis of Doppler frequency measurements, to obtain uniform approximation, we will add perturbation to both spectrum and parameters of the average spectrum. As spectrum parameters, we take average spectrum moments. For example, the first moment ω_d and the second moment $<\Delta\omega'^2_{d,g}>$ of the average spectrum after renormalization takes the following form:

$$\omega_d \Rightarrow \omega_d + \omega'_{ng} = \omega_d + \int |p(\mathbf{r}_m)|^2\, \omega'_d(\mathbf{r}_m)\, \mathrm{d}\mathbf{r}_m\,, \tag{8}$$

$$<\Delta\omega'^2_{d,g}> \Rightarrow \Delta\omega'^2_{d,ng} = \frac{1}{2}\int |p(\mathbf{r}_m)|^2\, |p(\mathbf{r}_n)|^2 \left(\omega'_d(\mathbf{r}_m) - \omega'_d(\mathbf{r}_n)\right)^2 \mathrm{d}\mathbf{r}_m\, \mathrm{d}\boldsymbol{r}_n\,. \tag{9}$$

It is seen from Eqs. (8) and (9) that, in the zeroth order, the frequency, at which the maximum of spectrum estimate after renormalization is reached, and spectrum halfwidth depend on random parameters. In place of expansion (2) and perturbation series for estimation of Doppler frequency (3), we have respectively the following expressions:

$$\hat{\mathbf{S}}(\omega_m) = \mathbf{S}(\omega_m) + \Delta\mathbf{S}(\omega_m) + \dots\,,$$

$$\hat{\mathbf{S}}_0 = \mathbf{S}_0 + \Delta\mathbf{S}_0 + \dots\,, \tag{10}$$

$$\hat{\Omega}_d = \omega_d + \omega'_{ng} - \frac{2\pi}{T_s}\sum_{m=-(N_s-1)}^{N_s-1} (\omega_m - (\omega_d + \omega'_{ng})\, \Delta\mathbf{S}(\omega_m)/\ \mathbf{S}_0\,, \tag{11}$$

where $\mathbf{S}(\omega_m)$, $\mathbf{S}_0 = \frac{2\pi}{T_s}\sum_{m=-(N_s-1)}^{N_s-1}\mathbf{S}(\omega_m)$ are terms of the zeroth order of smallness; $\Delta\mathbf{S}(\omega_m)$ and $\Delta\mathbf{S}_0$ are terms of the first order of smallness of the perturbation theory.

The value of measurement error in the average Doppler frequency var $\hat{\Omega}_d = <(\hat{\Omega}_d - \omega_d)^2>$, calculated using Eq. (11) as a result of averaging over random positions and number of particles in the scattering volume, over fluctuations of the velocity of atmospheric turbulent flux, as well as over the noise fluctuations, takes the following form:

$$\begin{aligned}\mathrm{var}\,\hat{\Omega}_d = {}& \sigma^2_{\omega_d}\int |p(\mathbf{r}_m)|^2\, |p(\mathbf{r}_n)|^2\, R_{\omega_d}(\mathbf{r}_m, \mathbf{r}_n)\mathrm{d}\mathbf{r}_m\, \mathrm{d}\mathbf{r}_n + \frac{\sqrt{\pi}}{MT_s}\int |p(\mathbf{r}_m)|^2 |p(\mathbf{r}_n)|^2 \left\{ \frac{<(\omega'_d(\mathbf{r}_n) - \omega'_{ng})\,(\omega'_d(\mathbf{r}_m) - \omega'_{ng})>}{\left[\frac{1}{2}<(\omega'_d(\mathbf{r}_n) - \omega'_d(\mathbf{r}_m))^2>\right]^{1/2}} + \right. \\ & \left. + \frac{<(\omega'_d(\mathbf{r}_n) - \omega'_d(\mathbf{r}_m))\,(\omega'_d(\mathbf{r}_n) - \omega'_{ng})> <(\omega'_d(\mathbf{r}_m) - \omega'_d(\mathbf{r}_n))\,(\omega'_d(\mathbf{r}_m) - \omega'_{ng})>}{2\left[\frac{1}{2}<(\omega'_d(\mathbf{r}_n) - \omega'_d(\mathbf{r}_m))^2>\right]^{3/2}} \right\} \mathrm{d}\mathbf{r}_m\, \mathrm{d}\mathbf{r}_n + \\ & \frac{2}{MS/N}\int |p(\mathbf{r}_m)|^2\, <(\omega'_d(\mathbf{r}_m) - \omega'_{ng})^2>\, \mathrm{d}\mathbf{r}_m + \frac{\pi^2}{3MT_s^2\, S^2/N^2}\,. \end{aligned} \tag{12}$$

Let us study the behavior of Eq. (12) in the case of Gaussian statistics of the Doppler lidar signal, as well as in the case of weak and strong correlation. At $d = \infty$, i.e. in the case of Gaussian statistics, Eq.(12) coincides with Eq. (4). This fact indicates that conditions of renormalization have been selected correctly.

In the case of a strong correlation ($d \to 0$), the integrand of the second and third terms in the right-hand side of Eq. (12) tend to zero, therefore the measurement error takes the form

$$\mathrm{var}\,\hat{\Omega}_d = \sigma^2_{\omega_d} + \frac{\pi^2}{3MT_s^2\, S^2/N^2} \neq \infty \ . \qquad (13)$$

As seen from Eq. (13), in contrast to the result (5) this approximation is uniform. From the physical point of view, in the case $d \to 0$ the behavior of the measurement error described by Eq. (12) is not a contradictory result. As was already noted, measured in this case is the frequency of random harmonic oscillation against the noise. The first term in the right-hand side of Eq. (13) describes the fluctuations of the harmonic oscillation frequency. The second term describes the measure of statistical uncertainty in the measured harmonic oscillation frequency against the noise. Thus, use of application of renormalization methods in the statistical analysis of frequency measurements of non-Gaussian signal of a Doppler lidar, as, for example, in the theory of oscillations,[1,2] allows the results to be obtained that do not contradict the physical sense.

Let us consider the physical meaning of Eq. (12) at weak correlation ($d \to \infty$). In this case, the measurement error in the average Doppler frequency has the form

$$\mathrm{var}\,\hat{\Omega}_d = \sigma^2_{\omega_d} \int |p(\mathbf{r}_m)|^2\, |p(\mathbf{r}_n)|^2\, R_{\omega_d}(\mathbf{r}_m, \mathbf{r}_n)\mathrm{d}\mathbf{r}_m\, \mathrm{d}\mathbf{r}_n +$$

$$+\frac{\sqrt{\pi}}{MT_s}\sqrt{<\Delta\omega'^{\,2}_{d,ng}>} + \frac{2}{M\ S/N} + <\Delta\omega'^{\,2}_{d,ng}> + \frac{\pi^2}{3MT_s^2\ S^2/N^2}\ , \qquad (14)$$

where

$$<\Delta\omega'^{\,2}_{d,ng}> = \frac{1}{2}\int |p(\mathbf{r}_m)|^2 |p(\mathbf{r}_n)|^2 <(\omega'_d(\mathbf{r}_m) - \omega'_d(\mathbf{r}_n))^2> \mathbf{r}_m \mathrm{d}\mathbf{r}_n$$

is the average halfwidth of the spectrum $\mathbf{S}(\omega)$.

The terms in the right-hand side of Eq. (14) have the following physical meaning. The first term corresponds to that part of measurement error, which is determined by the frequency fluctuations. At this frequency the maximum in spectrum $\mathbf{S}(\omega)$ is reached. The second and third terms are functions of halfwidth of the spectrum $\mathbf{S}(\omega)$ and have the form similar to Eq.(4). This means that statistical uncertainty in measurements of the average Doppler frequency, determined by these terms is the result of the spectrum $\mathbf{S}(\omega)$ broadening. The forth term is the measure of statistical uncertainty in the frequency measurements of harmonic oscillations against the noise. Thus, in the case of a weak correlation ($d \to \infty$), the behavior of measurement error in the average Doppler frequency has the same regularities as the behavior of the same parameter for the Gaussian statistics ($d = \infty$), that do not contradict the physical sense.

3. CONCLUSION

Thus, considered are two approaches to statistical analysis of measured frequency of a non-Gaussian signal of a Doppler lidar using the perturbation methods. The first approach uses the perturbation method proposed in Refs. 6 and 7 for analysis of a random Gaussian signal. The second approach is based on renormalization of the average spectrum parameters. Obtained are the expressions for estimation of the Doppler frequency, (3) and (11), as well as expressions for measurement error in the average Doppler frequency, (5) and (12).

When using the perturbation theory proposed in Refs. 6 and 7, one faces two main problems. They are nonuniform approximation and difficulties in physical interpretation of the obtained results. Nonuniformity of the approximation manifests itself in the fact that measurement error in the average Doppler frequency increases unlimitedly in the case of a strong correlation. Main difficulties in physical interpretation occur for both strong and weak correlation. In the case of strong correlation, the difficulties with interpretation follow from unlimited growth of the measurement error of average Doppler frequency, which contradicts the physical sense. As was shown, in the case of weak correlation, when the contribution from non-Gaussian properties into the parameters under study is taken into account as a small perturbation, behavior of the second and third terms in the right-hand side of Eq.(5) has no form similar to Eq. (4), that hampers explanation of this result.

Within the framework of the second approach, it is possible to construct the perturbation theory and to obtain the expressions for estimation of the Doppler frequency and measurement error in the average Doppler frequency, which meet the requirement of uniform approximation and allow interpretation of the results. In the case of a strong correlation, the measurement error in the average Doppler frequency is determined by the parameter which is limited and corresponds to the given case from the physical point of view. The measurement error is determined by fluctuations of a harmonic oscillation frequency and the measure of statistical uncertainty in the measurement of the harmonic oscillation frequency against the noise. In the case of a weak correlation, the measurement error in the average Doppler frequency is determined by fluctuations of the frequency, at which the maximum of spectrum $\mathbf{S}(\omega)$ is reached, average halfwidth of spectrum $\mathbf{S}(\omega)$, and the measure of statistical uncertainty in the measurement of a harmonic oscillation frequency against the noise.

By comparing two approaches to application of the perturbation theory, one can conclude that they are the same if ω'_{ng} is considered as an infinitely small parameter at limited values of other parameters in the problem considered. Therefore, when using the expansion (2), as well as the perturbation series (3) for analysis of non-

Gaussian signals, it is implicitly assumed that parameters $\omega'_{ng} = 0$, $<\omega'^2_{ng}> = 0$, etc. The fact that these parameters are zero means that the influence of correlations between fluctuations of local Doppler frequency can be neglected: $R_{\omega_d}(\mathbf{r}_m, \mathbf{r}_n) = 0$. Consequently, such phenomena, as non-Gaussian properties of a signal, spatial averaging of $\omega'_d(\mathbf{r}_m)$ over the scattered volume, are not considered correctly or in full measure. Strictly speaking, incorrect consideration of the correlation results, for example, in the situation when Eq. (5) can be considered reliable only in the case of Gaussian statistics ($d = \infty$). Thus, the approach to statistical analysis of measurements of the Doppler frequency based on perturbation methods[6,7] does not result in a higher accuracy when taking into account non-Gaussian properties of a signal.

REFERENCES

1. L.D. Landau and E.M. Lifshits, *Mechanics* (Nauka, Moscow, 1973), 208 pp.
2. J. Uizem, *Linear and Nonlinear Waves* [Russian translation] (Mir, Moscow, 1977), 624 pp.
3. A.P. Shelekhov, Atmos. Oceanic Opt. **6**, No. 9, 623–627 (1993).
4. A.P. Shelekhov, in: *Proc. SPIE on Lidar Techniques for Remote Sensing* **2310** (1994).
5. A.L. Afanas'ev and A.P. Shelekhov, Coherent Radar, Vol. 19 (Optical Society of America, Washington DC, 1995), pp. 124–127.
6. D.S. Zrnic, IEEE Trans. Aerosp. Electron. Syst. **AES-13**, No. 4, 344–354 (1977).
7. D.S. Zrnic, IEEE Trans. Aerosp. Electron. **GE-17**, No. 4, 113–129 (1979).
8. V.A.Banakh, I.N. Smalikho, et al., Appl. Opt. **34**, No. 12, 2055–2067 (1995).
9. V.A. Banakh, R. Haring, et al., *Laser Remote Sensing of the Mean Wind in the Atmospheric Boundary Layer* (DLR-Forschungsbericht 95–13, 1995), 83 pp.