ON ESTIMATION OF THE BRIGHTNESS PATTERN OF A SCATTERING VOLUME

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We consider specific features in the formation of light field scattered by space-limited-objects depending on optical characteristics of the medium within the objects. To determine the asymmetry of the brightness pattern, we propose to use an integral parameter introduced in the paper. It is shown that the behavior of the brightness asymmetry factors as functions of the medium parameters can be used to estimate applicability of the Bouguer law, as well as the onset of deep-in regime, and the optical depth of the medium starting from which it can be considered infinite.

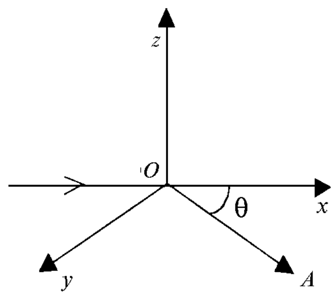
Calculations of the brightness fields of space-limited objects, that scatter radiation, make up a part of investigations of the radiation balance in the atmosphere.[1,2] Detailed information on the fields of scattered light is difficult to be obtained, both experimentally and theoretically. So, it is worth introducing an integral parameter to enable estimations of the configurations of the scattered radiation fields. The scattering theory usually uses brightness patterns to describe configuration of the radiation field of a large scattering volume, while using the scattering phase functions in the case of elementary volumes. The scattering phase function may be used when detailed information on the angular structure of scattering is required, and the asymmetry parameter can be employed when only the anisotropy of scattering is of interest. Among the characteristics of scattering, that are also widely used, is the mean cosine of scattering angle $\langle\cos\theta\rangle$. These parameters are appropriate when operative information on the angular distribution of scattered radiation is needed, or when a classification must be done of the scattering objects. Usually, such characteristics are used when assuming the scattering to be symmetric.

The irregular shaped particles scatter the radiation asymmetrically, what makes it necessary to use integral parameters that are, in the general case, defined as follows:

$$\mu = \int_{\Omega} \chi(\theta) \cos\theta \, d\Omega , \tag{1}$$

where $\chi(\theta)$ is the scattering phase function, and Ω is the solid angle.

Let us denote as μ_{-x}, μ_{+x}, μ_{-y}, μ_{+y}, μ_{-z}, μ_{+z} the integral parameters referring to the corresponding axes of a Cartesian coordinate system. The diagram below shows the geometry of light scattering on a single particle or a unit elementary volume.



Scattering diagram

We assume that light is incident along the positive x direction. The viewing direction OA is in the plane XY and makes the angle θ with the incidence direction of the beam. Integral parameters are being determined depending on which viewing direction is chosen. Thus, for instance, for the direction along the X axis

$$\mu_{+x} = 2\pi \int_{0}^{\pi/2} \chi(\theta) \sin\theta \cos\theta \, d\theta . \tag{2}$$

Define the coefficients of asymmetry along the x, y, and z directions as, correspondingly, a_x, a_y, and a_z, which are as follows:

$$a_x = \frac{\mu_{+x} + 1/2(\mu_{+y} + \mu_{-y} + \mu_{+z} + \mu_{-z})}{\mu_{-x} + 1/2(\mu_{+y} + \mu_{-y} + \mu_{+z} + \mu_{-z})} ;$$
$$a_y = \frac{\mu_{+y} + 1/2(\mu_{+x} + \mu_{-x} + \mu_{+z} + \mu_{-z})}{\mu_{-y} + 1/2(\mu_{+x} + \mu_{-x} + \mu_{+z} + \mu_{-z})} ; \tag{3}$$
$$a_z = \frac{\mu_{+z} + 1/2(\mu_{+x} + \mu_{-x} + \mu_{+y} + \mu_{-y})}{\mu_{-z} + 1/2(\mu_{+x} + \mu_{-x} + \mu_{+y} + \mu_{-y})} .$$

The coefficients of scattering asymmetry are introduced because the mean cosine of the scattering angle, as well as other such characteristics, inadequately describes the scattering properties of a medium (e.g., Rayleigh and isotropic scattering phase functions have the same $\langle\cos\theta\rangle$).

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Using the above proposed coefficients one can allow for the asymmetry of scattering along the coordinate axes. This, in its turn, removes the ambiguity of the ratio between the parameters of a medium and radiation characteristics. This is most important in the case of irregular shaped particles that may take a preferred orientation in space under the action of different physical factors like, for instance, convective flows, electric fields, and so on.

The brightness pattern as well as the scattering phase function describes the angular distribution of radiation inside the scattering macrovolume. However, in many cases it is very difficult to determine these characteristics inside the medium. For this reason, the angular distribution of radiation is mainly considered in the space outside the scattering volume. By analogy with the asymmetry factor of the scattering phase function of an elementary volume, it is worth introducing the asymmetry factor for the radiation coming from a macrovolume. Again, as in formulas (3), we shall describe the anisotropy of scattering by a macrovolume using the coefficient

$$A_s = (I_{+x} + B)/(I_{-x} + B) , \quad (4)$$

where

$$B = (I_{+y} + I_{-y} + I_{+z} + I_{-z})/2 . \quad (5)$$

Here I_{+x} is the radiative flux leaving the medium along the direction of incident flux, that is along the OX-axis, I_{-x} is the flux reflected from the medium; and $I_{\pm y}$ and $I_{\pm z}$ are the fluxes of scattered radiation leaving the medium along the directions $\pm y$ and $\pm z$.

In the particular case of a medium with a square-shaped optical cross-section, and a symmetric, about OX-axis, scattering phase function, the fluxes $I_{+y}=I_{-y}=I_{+z}=I_{-z}=I$, so that formula (4) takes the following form:

$$A_s = (I_{+x} + 2I)/(I_{-x} + 2I) . \quad (6)$$

Figures 1–4 show the examples of results calculated, by formulas (2)–(6). These expressions allow the number of the brightness pattern measurements required to be minimized and its deformations to be followed up using the above defined parameters.

Let us now consider some results of investigations of the radiation fields from a space limited scattering volume as functions of the optical properties and sizes of the medium. The shape of the brightness pattern of the scattering volume was estimated from the mean cosine of the scattering angle $\langle\cos\varphi\rangle$, which was determined in the same way as $\langle\cos\theta\rangle$ for an elementary volume.

Deformations of the brightness pattern of a volume are determined by the volume size, shape, density of the medium as well as by the particle size distribution. Typical dependences of $\langle\cos\varphi\rangle$ on these parameters are shown in Fig. 1. These dependences enable one to determine the optical sizes of the medium starting from which no further changes in the radiation budget characteristics occur. In particular, curves *3–6* in the figure reach their asymptotic at $\tau_y \geq 8$, while for shorter wavelengths λ (curves *1* and *2*), the saturation is reached at larger τ values. The mean cosine $\langle\cos\varphi\rangle$ is quite sensitive to the radiative field variations so it can be used as a criterion, when estimating the limiting size of the medium at which it may be considered spatially infinite.

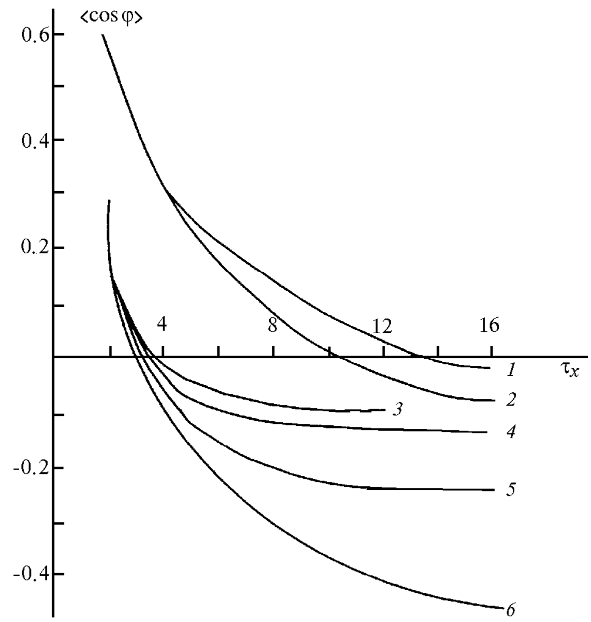

FIG. 1. Dependence of $\langle\cos\varphi\rangle$ *on optical depth* τ: $\Lambda = 0.9$ *(curves 1–3 and 5) and* 0.98 *(curves 1, 3, and 4);* $\tau_y = \tau_z = 1$ *(curves 1, 3, and 4) and* $\tau_y = \tau_z = 50$ *(curves, 2, 5, and 6); radiation wavelength* $\lambda = 0.5$ μm *(curves 1 and 2) and* 50 μm *(curves 3–6).*

Consider now the dependence of the asymmetry factor given by expression (6) on the optical depth of the scattering medium (Fig. 2).

Calculations were made for two scattering phase functions with the asymmetry parameters $a_{x1} = 1$ and $a_{x2} = 12.09$, transverse optical depths $\tau_y = \tau_z = 10$, and the single scattering albedos $\Lambda = 1$ and 0.95, respectively. It is obvious, that the radiation field is being formed by the multiply-scattered radiation and direct radiation attenuated according to the Bouguer law. The former one increasingly dominates over the latter one as the optical sizes of the medium increase. For this reason, the coefficient A_s stronger depends on τ_x in a scattering medium whose scattering phase function has higher anisotropy.

The absorption of radiation by the medium, if any, increases the A_s value in the region of small optical sizes (curves *2* and *3*), because of a decrease in the multiple scattering contribution to the total radiation budget. The results presented can also be used to establish the applicability limits of the Bouguer law.

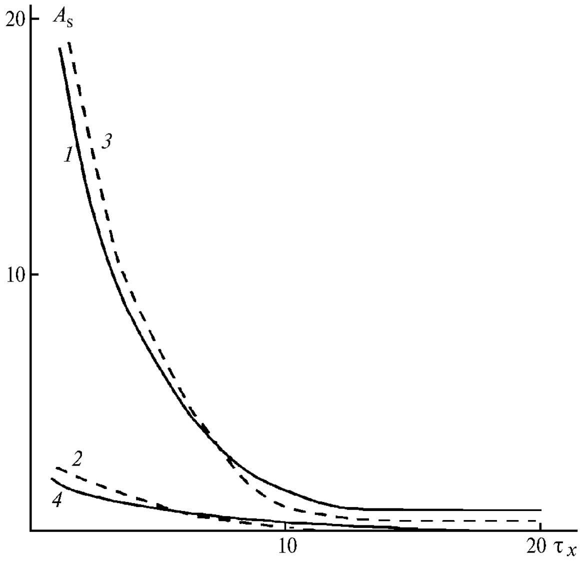

FIG. 2. Dependence of the asymmetry parameter A_s *on the optical depth* τ *of the medium for* $\tau_y = \tau_z = 10$: $\Lambda = 1$ *and* $a_{x2} = 12.09$ *(curve 1);* $\Lambda = 0.95$ *and* $a_{x1} = 1$ *(curve 2);* $\Lambda = 0.95$ *and* $a_{x2} = 12.09$ *(curve 3); and* $\Lambda = 1$ *and* $a_{x1} = 1$ *(curve 4).*

In Fig. 3 the asymmetry parameter is shown as a function of transverse optical sizes of the medium, in situations modeled so that the longitudinal size τ_z and one of the transverse sizes, a_y, are fixed. Such situations may occur in the atmospheric and ocean optics studies. The calculations have been made using same scattering phase functions as above and assuming the scattering medium to be conservative. The data obtained exhibit quite a weak dependence of the A_s coefficient on the transverse optical size of the medium when two its sizes are fixed while the third one is being varied. At the same time, this characteristic keeps to be strongly dependent on the scattering phase function. From analysis of the A_s dependence on the transverse optical size, one can establish the limiting size of the medium at which it can be considered spatially infinite (in transverse direction). Besides, independence of the coefficient A_s of the optical sizes of the medium suggests that the brightness pattern is already formed, what is equivalent to the onset of the deep-in regime.

Figure 4 presents the relation of the medium macroparameters to its microparameters expressed in terms of the dependence of the coefficient A_s on the asymmetry parameter of scattering phase function of particles that make up a cubic-shaped medium with fixed optical sizes ($\tau_y = \tau_z = \tau_x = \tau$).

As seen, these parameters are uniquely related. Evidently, as the scattering phase function becomes more asymmetric, the asymmetry of the brightness pattern of the scattering volume also increases. This tendency is most pronounced at small optical sizes of the medium because of a smaller contribution coming from multiple scattering to the radiation budget.

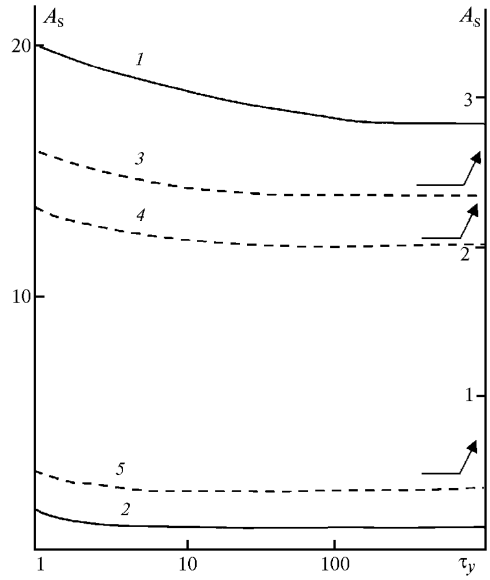

FIG. 3. Dependence of the asymmetry parameter A_s *on the transverse optical depth* τ_y *of the medium for* $\Lambda = 1$: $\tau_x = 1$, $\tau_z = 1$, *and* $a_{x2} = 12.09$ *(curve 1);* $\tau_x = 10$, $\tau_z = 1$, *and* $a_{x2} = 12.09$ *(curve 2);* $\tau_x = 1$, $\tau_z = 1$, *and* $a_{x1} = 1$ *(curve 3);* $\tau_x = 10$, $\tau_z = 10$, *and* $a_{x1} = 1$ *(curve 4); and* $\tau_x = 10$, $\tau_z = 1$, *and* $a_{x1} = 1$ *(curve 5).*

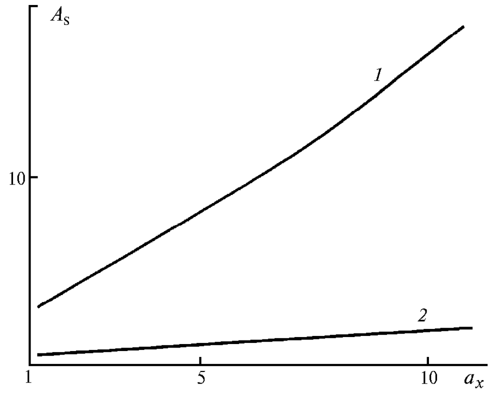

FIG. 4. Relation of the asymmetry parameter A_s *and the asymmetry factor of the scattering phase function of an elementary volume for* $\Lambda = 1$: $\tau = 1$ *(curve 1) and* $\tau = 10$ *(curve 2).*

Thus, it can be concluded, that the asymmetry parameters, proposed here, prove to be an informative and sensitive characteristics, with which the spatial distribution of radiation, scattered by the volume, can be estimated quite accurately. From analysis of how these parameters vary as functions of medium characteristics, it is possible to estimate the applicability limits of the Bouguer law, the onset of deep-in regime, and the limiting optical sizes of the medium at which it can be considered infinite.

REFERENCES

1. E.M. Feigelson, ed., *Radiation in the Cloudy Atmosphere* (Gidrometeoizdat, Leningrad, 1981), 280 pp.
2. J. Lenoble, *Radiative Transfer in the Scattering and Absorbing Atmospheres* (Gidrometeoizdat, Leningrad, 1990), 264 pp.