Characteristics of laser beams under conditions of intermittent small-scale atmospheric turbulence

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Multiparametric analysis of fluctuations of laser radiation propagating along the near-ground paths in a big city allowed us to state that the drastic changes in radiation characteristics are due to the intermittence of the small-scale turbulence. The alternation of quasi-regular and stochastic states of a beam under such conditions can be clearly seen when working with narrow collimated beams having the Fresnel number close to unity. Even slight increase of this parameter leads to the prevalence of the stochastic state. It has been found that the fractal structure of vortical inhomogeneities in the ground layer manifests itself in the fractal structure of radiation fluctuations. The observed phenomenon is theoretically interpreted.

The study of peculiarities in the behavior of laser beams propagated through the near-ground atmosphere under conditions of various hydrodynamic instabilities is an urgent problem nowadays from both the general, or theoretical, and practical points of view. The sporadic dynamics of processes occurring in the troposphere is characteristic of a complex terrain of the underlying surface, in particular, for the urban landscape. Developing instabilities lead to formation of small-scale turbulence with a complex spatiotemporal structure in the surface atmospheric layer,[1,2] and this turbulence shows signs of a fractal structure.[3,4]

In this paper, we consider multiparametric analysis of the behavior of laser beams of different diameters along surface paths under conditions of intermittent small-scale atmospheric turbulence.

The experimental basis includes horizontal and slant atmospheric paths among the buildings of the Moscow State University.[5,6] Paths of this type are typical of the conditions of a big city. The transceiving equipment was placed at the altitude of about 25 m above the surface. The main horizontal path was arranged at this altitude as well. The reflecting mirror of the slant path was at the altitude of 165 m above the surface. The length of the horizontal path in one direction was 280 m, and the length of the slant path was 320 m.

A single-mode He–Ne laser served as a source of cw radiation at the wavelength $\lambda = 0.63\ \mu$m. The optical arrangement used allowed us to work with the horizontal and slant paths simultaneously. To determine the characteristics of the radiation having passed along the path, we used a TV video digitizer to input beam images and shear interferograms into a computer memory. The signals from photodiode characterizing local intensity fluctuations were also entered into to the computer through an analog-to-digital converter. Specially developed software allowed multiparametric analysis of the statistical properties of radiation.

Meteorological parameters on the path were measured synchronously with the determination of laser radiation parameters, and thus we could determine the structure characteristic of the refractive index fluctuations C_n^2.

In the experiments it was found that[7,8]

1. Depending on weather conditions, weak or strong fluctuations are observed along the path, and they correspond to variations of the structure characteristic of the refractive index fluctuations C_n^2 from 10^{-18} to 10^{-10} cm$^{-2/3}$.

Under conditions of weak fluctuations characteristic of close-to-neutral atmospheric stratification (in the absence of convective flows), the beam profile is characterized by high degree of homogeneity. Under these conditions, the deformation of wave front is smooth and does not lead to its topological perturbations. The beam diameter proves to be smaller than the correlation range. Under the conditions of strong fluctuations (at marked atmospheric stratification and, as a rule, in the presence of wind and strong turbulence), the beam structure acquires the quasi-speckle character and its cross section is markedly larger than the correlation range. In this case, topologic perturbations in the form of screw dislocations are observed in the wave front.

2. The slant path is characterized by anisotropy of the beam profile. Under conditions of strong fluctuations, the beam, as a whole, and some of its speckles become elongated. The axis ratio is roughly 1:3 being largely determined by the presence of vertical streams.

3. Under conditions of sharp temperature gradients occurring most frequently in the transition fall–winter and winter–spring periods, the modes of strong and weak fluctuations are unstable. Instabilities manifest themselves at different stratification of the surface layer. In the experiments described here, they were observed as alternation of two structure states of a narrow collimated laser beam with the Fresnel number close to unity.

One of these states – quasiregular – possesses the properties characteristic of the weak fluctuation mode.

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The second state – stochastic, the transition to which is stepwise, is characterized by all peculiarities inherent in the mode of strong fluctuations. Alternation of the states is of a quasiregular character. In this case, the duration of each state can vary from seconds to tens of seconds depending on the meteorological conditions along the path. The alternation of the structure states is most pronounced on the horizontal path.

4. Local intensity fluctuations in the quasiregular states are significantly different than those in the stochastic states. The differences manifest themselves in a sharp increase of the fluctuation variance at the transition to the stochastic state, as well as in the appearance of asymmetry in the intensity deviations from the mean level. In spite of these differences, the intensity fluctuations in both quasiregular and stochastic states are described by the lognormal distribution to a sufficient accuracy.

5. The stochastic state of the beam is characterized by a marked turbulent widening. The beam cross size in this state exceeds, usually by 1.5–2 times, its size in the quasiregular state. Some examples of the experimental data on the mean diameter D of the beam on the horizontal path, as well as the mean square displacement of the beam centroid ρ_c^2 in different structure states, are given below in the Table.

Table. Beam parameters in different structure states

Beam parameters	State			
	Quasiregular		Stochastic	
	Exp.	Theory	Exp.	Theory
Beam diameter D, cm	3 ± 0.5	2.1	8 ± 1	7.2
RMS displacement of the beam centroid ρ_c^2, cm^2	8.2 ± 2.5	4.02	3 ± 1	3.6

The tabulated data are for the horizontal path; they were obtained in the spring period, when the temperature at the altitude of the path was −1°C. At the altitude of 2 m above the surface, the temperature was −2°C and the wind speed was about 1 m/s. The parameter $C_n^2 = (2 \pm 0.5)\cdot 10^{-14}$ cm$^{-2/3}$ corresponds to these conditions. It should be noted that we observed in these experiments a marked decrease in the displacement of the beam centroid at stochastization of radiation, but these observations were not always confirmed by further studies. In some cases, the data on the displacement of the beam centroid in different structure states coincided within the measurement error.

The experimentally observed sporadic stochastization of radiation is directly related to the alternation of the state of the small-scale turbulence. This alternation can be taken into account using the following model[9]:

$$\Phi_n(K) = 0.033\, C_n^2\, K^{-11/3} \exp\,(K^2/K_m^2), \tag{1}$$

where K is the wave number of the inhomogeneities; $K_m = 5.92/\ell_0$, ℓ_0 is the inner scale of turbulence.

Within the framework of this model, the intermittence of turbulence may be caused by sporadic changes of C_n^2, K_m, and the exponent of the power dependence of the spectrum $\Phi_n(K)$. The interpretation of the above results is based on the assumption that the change of the parameter K_m has a decisive effect on the change of the laser beam structure at transition from one state to another. It should be noted that the outer scale of turbulence L_0, as well as its variations, on the experimental paths passing rather high above the surface does not affect markedly the beam characteristics.[10]

The diffraction widening of the laser beam with the Gaussian amplitude profile is described using the diffraction radius of the beam[11]:

$$a_g = ag/\Omega. \tag{2}$$

Here a is the effective radius of the beam at the transmitting aperture; $\Omega = ka^2/L$ is the Fresnel number of the beam; L is the path length; k is the wave number; g is the generalized diffraction parameter equal to $\sqrt{2}$ for a narrow collimated beam. It follows from Eq. (2) that a narrow collimated beam ($\Omega = 1$) has minimum diffraction size. Therefore, the reaction of this beam to variations of the inner scale of turbulence ℓ_0 occurring near its mean cross size is most clear.

If ℓ_0 markedly exceeds a, then the turbulent widening of the beam proves to be insignificant. At the same time, turbulence leads to large random displacements of the beam centroid. The rms displacement of the centroid in this case is determined by the equation[12]:

$$\rho_c^2 = 2.19\, C_n^2\, \ell_0^{-1/3}\, L^3. \tag{3}$$

The relative variance of fluctuations of the intensity logarithm proves to be equal to[10]:

$$\sigma^2 = 3.2\, C_n^2\, L^3\, \ell_0^{-7/3}. \tag{4}$$

If $\ell_0 \ll a$, then the turbulent widening of the beam is significant and σ^2 is close to unity. This case can be described within the framework of the theory of spatially bounded beams[10,12] that is valid for the developed turbulence. The effective size of the beam D caused by diffraction and turbulent widening is described by the equation[10,11]:

$$D = 2a\,[g^2 + 0.46\,(2.8\,\beta_0^2)^{6/5}]^{1/2}, \tag{5}$$

where $\beta_0^2 \approx 1.24\, C_n^2\, k^{7/6}\, L^{11/6}$.

To estimate the rms displacement of the beam in the case of its turbulent widening, the following equation is used[11]:

$$\rho_c^2 = 1.54\, a^2\,(2.8\,\beta_0^2)^{4/5} - 1.78\, a^2\,(2.8\,\beta_0^2)^{5/8}. \tag{6}$$

The correlation length of the intensity fluctuations r_c of the turbulent-widened beam can be found from the equation[11]:

$$r_c = r_0\left[\left(2 + \frac{4}{3}q\right)\Big/\left(\frac{4}{3} + \frac{1}{3}q\right)\right]^{1/2}, \tag{7}$$

where $q = 1.22\ \beta_0^{12/5}$; $r_0 = \sqrt{L/qk}$ is the length of the spatial coherence of a plane wave. The above equations can form the basis for calculating and optimizing the characteristics of narrow collimated beams along the lidar paths.

In considering the question to what degree the above theoretical model is capable of describing the experimental data, we proceed from the fact that the parameter C_n^2 entering into the theoretical equations corresponds to the values determined from the meteorological data and is averaged over sufficiently long time interval (far exceeding the duration of different structure states of the beam). The data tabulated above correspond to $C_n^2 = (2 \pm 0.5)\cdot 10^{-14}\ \text{cm}^{-2/3}$. Equation (4) allows the inner scale to be determined. For the quasiregular state of the beam with the variance of the intensity fluctuations $\sigma^2 = 0.1–0.3$, ℓ_0 varies from 4 to 15 cm.

Thus, the quasiregular state of the beam corresponds to the case that the beam size (in our experiments, the diameter of the beam was $2a = 1.5$ cm at the exit aperture and $D \approx 3$ cm at the entrance aperture) is markedly smaller than the inner scale of turbulence, and Eq. (3) can be used to describe this state. The mean square displacement of the beam centroid determined in such a way is $\rho_c^2 = 4.02\ \text{cm}^2$. The difference between the theoretical and experimental values can be thought acceptable, taking into account the quadratic character of ρ_c^2.

As experiments show, the transition to the stochastic state is characterized by a sharp decrease (by almost an order of magnitude) in the correlation length of the intensity fluctuations and by marked turbulent widening. If we assume that the change of the inner scale is on the same order as the change in the intensity correlation length, then for ℓ_0 in the stochastic state we obtain the characteristic values $\ell_0 \approx (0.2–0.4)$ cm. At such values of the inner scale of turbulence and the value of C_n^2 given above, the correlation length of the intensity fluctuations can be estimated by Eq. (7) as $r_c = 0.2$ cm. Estimation of the turbulent widening by Eq. (5) shows that the transition to the small-scale turbulence considerably increases the beam size. In this case, the experimental and theoretical values of the beam diameter coincide within the measurement error (see Table). The rms beam displacement under conditions of its turbulent widening is calculated by Eq. (6) as $\rho_c^2 = 3.6\ \text{cm}^2$, and this value is also in a close agreement with the experimental value.

All the above-said concerns the characteristics of narrow collimated beams. To reveal the effect of the aperture on the beam structure under the intermittence, the experiments with beams propagating along close-to-parallel paths were conducted. These experiments allowed the characteristics of the beams with different output diameters recorded at the same moments in time to be compared. If the separation between the beams did not exceed several centimeters, then they were formed from an enlarged beam emitted by the main source. In the cases that the separation between the beams was several tens of centimeters, the second beam was generated by an additional laser.

Figure 1 depicts the images of parallel beams having passed along the path. The image in each square corresponds to a narrow collimated beam, and the left frames show beams with the increased diameter. Consecutive video frames in Figs. 1*a–c* correspond to poorly developed small-scale turbulence (the quasiregular state of the narrow collimated beam is indicative of the poor development of the small-scale turbulence). The frames shown in Figs. 1*d–f* were recorded under conditions of the developed small-scale turbulence (the narrow collimated beam is in the stochastic state). It is seen that the beams with the increased diameter even under conditions of weak turbulence can acquire the stochastic structure in some time intervals (see Fig. 1*c*). At the developed small-scale turbulence (see Fig. 1*d–f*), stochastization of beams is observed regardless of their diameters.

Therefore, the synchronism was observed in the stochastization periods of the narrow collimated beams up to the separation of about one meter.

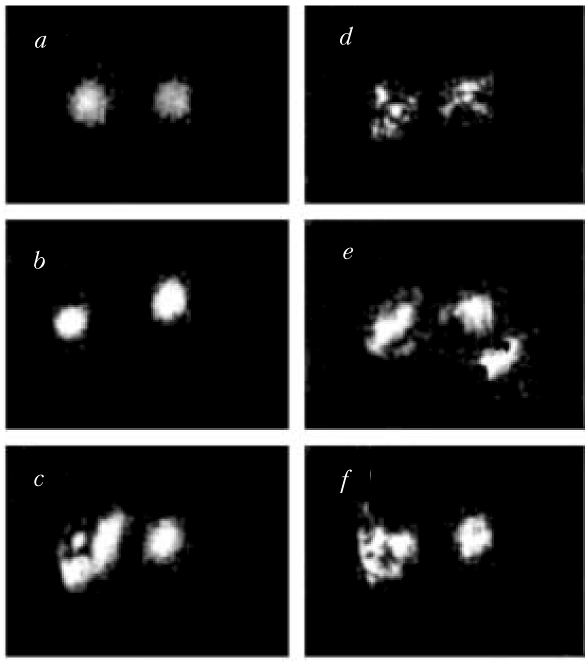

Fig. 1. Effect of atmospheric turbulence on the structure of parallel-propagated laser beams. The frames *a*, *b*, and *c* correspond to poorly developed small-scale turbulence, the frames *d*, *e*, *f* are for the developed small-scale turbulence. The distance between the beam centers in each frame is 5 cm. The Fresnel number is $\Omega = 1$ for the right-hand side beam and 1 (*a*, *d*), 1.6 (*b*, *e*), and 2 (*c*, *f*) for the left-hand beam.

If one of the beams has wider aperture, the synchronism of stochastization breaks at the separation as small as several centimeters. This is illustrated in Fig. 2.

The histograms depicted in this figure correspond to the conditions of Fig. 1. The horizontal axis plots the number of consecutive frames, in which the beams could be in either the stochastic state (shaded areas) or in the quasiregular one. Figures 2*a* and *b* correspond to the case of two narrow collimated beams propagating along the paths, Figs. 2*c* and *d* correspond to the case of one beam having the Fresnel number $\Omega = 1.6$, and Figs. 2*e* and *f* correspond to the case of the beam having the Fresnel number $\Omega = 2$. It is seen from these figures that the increase in the diameter of one beam leads, on the one hand, to the break of synchronism in stochastization and, on the other hand, to the increase of the period of stochastization of the widened beam as compared to the narrow collimated beam. The time ratio between the total duration of the stochastic states t_{st} and the total period of measurements t_Σ estimated based on the results of processing of 10000 frames is $t_{st}/t_\Sigma = 0.55$ for the narrow collimated beam, $t_{st}/t_\Sigma = 0.75$ for the beam with $\Omega = 1.6$, and $t_{st}/t_\Sigma = 0.95$ for the beam with $\Omega = 2$. This estimate indicates that the phenomenon of laser beam stochastization is critical to the value of the beam diameter.

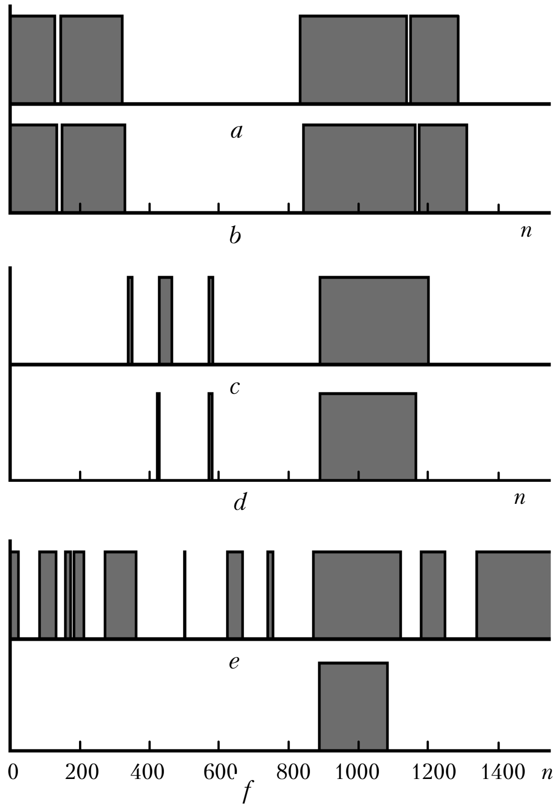

Fig. 2. Synchronism of tne processes of stochastization of beams propagating along close-to-parallel paths: $\Omega = 1$ (*a*, *b*, *d*, *f*), $\Omega = 1.6$ (*c*), and $\Omega = 2$ (*e*).

In processing of the beam images, the objects of the study were also the manifestations of the fractal structure in the intensity fluctuations and displacements of the beam centroid. The possibility of manifestation of the fractal structure in laser radiation fluctuations is connected with the presence of the fractal structure in vortical formations in the atmosphere. Some examples of fractal analysis are depicted in Fig. 3 for intensity fluctuations and in Fig. 4 for fluctuations of the beam centroid displacements for the stochastic state of the beam (*a*, *b*) and for the quasiregular one (*c*, *d*).

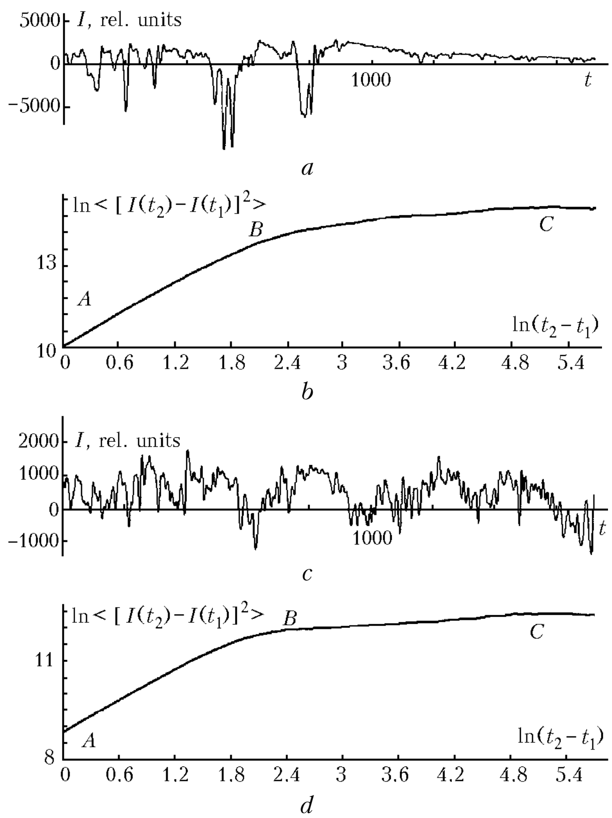

Fig. 3. Analysis of the fractal structure of intensity fluctuations for the beams in the stochastic (*a*, *b*) and quasiregular states (*c*, *d*): structure of the signal intensity (*a*, *c*) and variance of increments of the signal intensity depending on increments of the time interval (*b*, *d*).

The fractal analysis was based on the equation[3,4]:

$$\langle [X(t_2) - X(t_1)]^2 \rangle = \sigma^2 |t_2 - t_1|^{2H}, \qquad (8)$$

where $X(t)$ is the time dependence of the studied signal; σ^2 is its variance; $|t_2 - t_1|$ is the time increment; H is the Hurst parameter connected with the fractal dimension d as follows:

$$d = 2 - H. \qquad (9)$$

Taking the logarithm of the right-hand and the left-hand sides of Eq. (8), we can determine the Hurst parameters from the slope of the curve and, consequently, estimate the fractal dimension.

In Figs. 3 and 4, the curves *a* and *c* are plotted for the signal structure, and the curves *b* and *d* characterize the behavior of the variance of increments of the analyzed parameter depending on time increments in the log-log scale. The time interval is denoted by the number of points, the separation between which is $5 \cdot 10^{-4}$ s when recording intensity fluctuations and $4 \cdot 10^{-2}$ s when recording displacements of the beam centroid.

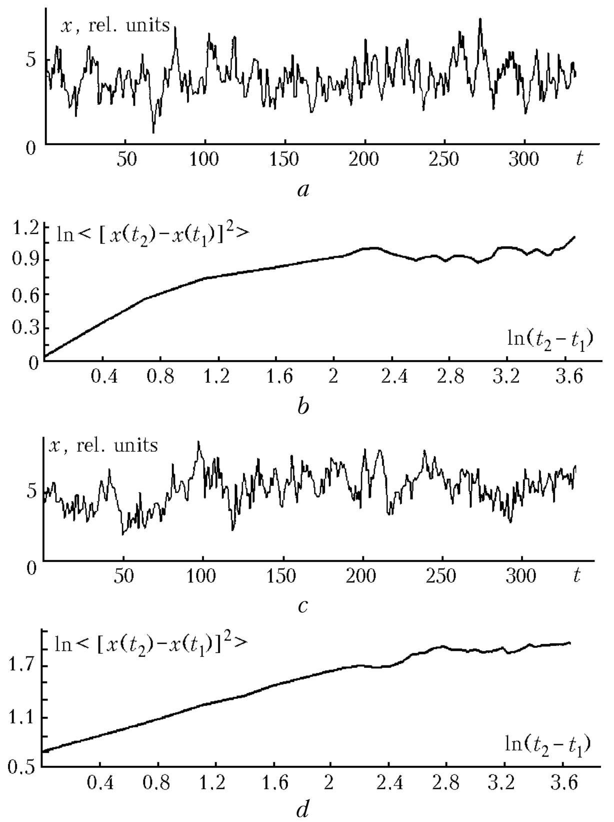

Fig. 4. Analysis of the fractal structure in the displacements of the beam centroid for beams in the stochastic (*a*, *b*) and quasiregular (*c*, *d*) states: displacement of the beam centroid in time (*a*, *c*) and variance of the increments of centroid displacements depending on increments of the time interval (*b*, *d*).

In Fig. 3*d*, two intervals *AB* and *BC*, which can be rather accurately approximated by portions of a straight line, can be distinguished. The slope of the portion *AB* in Fig. 3*d* gives for the Hurst parameter the value $H_{AB} = 0.94$ corresponding to the value of the fractal dimension $d_{AB} = 1.06$. The slope of the segment *BC* corresponds to the value $H_{BC} = 0.28$ ($d_{BC} = 1.72$). The closeness of the value of d to unity in the segment *AB* indicates that the fractal properties of the signal are not observed in this part. This is a consequence of filtering out high-frequency components of the signal in order to suppress the parasitic noise. The fractal properties are most pronounced in *BC* portion, what is confirmed by the value of the fractal dimension. Similar dependence is shown in Fig. 3*b* at $H_{AB} = 0.92$ ($d_{AB} = 1.08$) and $H_{BC} = 0.39$ ($d_{BC} = 1.61$).

The manifestation of the fractal structure can be seen in Figs. 4*b* and *d* as well. Since data on the displacements of the beam centroid were obtained by processing the information recorded in a slow TV standard, the initial part with $d = 1$ caused by filtering out the high-frequency components is absent in Figs. 4*b* and *d*.

Analysis of the fractal structure of fluctuation processes under conditions of intermittent small-scale turbulence showed that some structure states include long periods (comparable with the duration of the states) with the fractal structure of fluctuations. However, formation of the fractal shape of the studied signals is very unstable. This applies to both the width of the scaling zone and the fractal dimension. Although the fractal structure manifests itself more frequently and clearly for the quasiregular state of the beam, it is still impossible to represent this observation in a mathematical form. The instability of the fractal structure in fluctuations of the laser radiation is likely indicative of the instability of statistical parameters characterizing the structure of vortical formations in the surface atmosphere.

Thus, the multiparametric analysis of fluctuation processes in laser radiation, including, in particular, estimation of the variance, turbulent widening, intensity correlation zone, probability densities of the distribution of fluctuations, and the fractal dimension, showed that the turbulence intermittence and the related sporadic stochastization of radiation lead to drastic changes in the radiation characteristics. Alternation of the quasiregular and stochastic states is most pronounced in narrow collimated beams with the close-to-unity Fresnel number. Even at a small increase of the Fresnel number of the beam, the stochastic state proves to be dominant. The experimental data can be explained based on the known theoretical ideas on the assumption that the change of the inner scale of turbulence has the decisive effect on the structure of laser radiation. The fractal structure of vortical formations in the surface layer is reflected in the fractal structure of fluctuating signals. The instability of manifestation of the fractal structure in the fluctuations of intensity and the position of the beam centroid can be considered as an evidence of the instability of cluster vortical formations in the surface atmospheric layer.

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References

1. N.L. Byzova, V.N. Ivanov, and E.K. Garger, *Turbulence in the Atmospheric Boundary Layer* (Gidrometeoizdat, Leningrad, 1989), 283 pp.
2. H.L. Swinney and J.P. Gollub, eds., *Hydrodynamic Instabilities and the Transition to Turbulence* (Springer Verlag, 1981).
3. R.M. Crownover, *Introduction to Fractals and Chaos* (Jones and Bartlett, 1995).
4. A.G. Bershadskii, Usp. Fiz. Nauk **160**, No. 12, 189–194 (1990).
5. T.I. Arsen'yan, P.V. Korolenko, E.A. Kulyagina, A.N. Lyash, S. Pershin, A.I. Odintsov, and N.N. Fedotov, Atmos. Oceanic Opt. **10**, No. 1, 29–33 (1997).

6. T.I. Arsen'yan, L.S. Kornienko, P.V. Korolenko, E.A. Kulyagina, G. Petrova, and N.N. Fedotov, Vestn. Mosk. Univ., Ser. 3. Fiz. Astron., No. 1, 27–30 (1997).
7. T.I. Arsen'yan, A.M. Zotov, P.V. Korolenko, M.S. Maganov, and V.G. Makarov, Vestn. Mosk. Univ., Ser. 3. Fiz. Astron., No. 2, 32–34 (2000).
8. T.I. Arsen'yan, A.M. Zotov, P.V. Korolenko, M.S. Maganova, and V.G. Makarov, Izv. Ros. Akad. Nauk, Ser. Fiz. **63**, No. 10, 2019–2023.
9. V.I. Tatarsky, *Wave Propagation in a Turbulent Medium* (Dover, New York, 1961).
10. A.S. Gurvich, A.I. Kon, V.L. Mironov, and S.S. Khmelevtsov, *Laser Radiation in the Turbulent Atmosphere* (Nauka, Moscow, 1976), 278 pp.
11. V.L. Mironov, *Laser Beam Propagation in the Turbulent Atmosphere* (Nauka, Novosibirsk, 1981), 246 pp.
12. Z.I. Feizulin and Yu.A. Kravtsov, Izv. Vyssh. Uchebn. Zaved., Ser. Radiofiz. **10**, No. 1, 68–73 (1967).