Light scattering by hexagonal ice cylinders

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Received May 2, 2007

Light scattering by hexagonal ice cylinders with the refractive index corresponding to ice particles in the visible spectral region is considered. The angular dependence of elements of the scattering matrix for randomly and horizontally oriented hexagonal cylinders has been calculated numerically. Orientation averaging of optical characteristics over an ensemble of particles has been performed analytically with use of the T-matrix method. Polarization characteristics of hexagonal and circular cylinders are compared.

Crystal clouds cover about 20% of the Earth's surface[1,2] and play an important role in the Earth–atmosphere radiative balance. The knowledge of microphysical and radiative properties of crystal clouds is necessary for interpretation of remote sensing data, for estimation of reflectance and transmittance of light fluxes in a spectral range, and for use in climatic models.[3] The hexagonal shape of ice crystals is used in simulation of optical characteristics of crystal clouds.[4–6]

In theoretical investigations of single and multiple light scattering by isotropic media, the approach based on the expansion of elements of the scattering matrix in terms of the complete orthogonal system of generalized spherical functions is useful and efficient.[7] The use of coefficients of the Fourier series for scattering matrix elements in terms of the generalized spherical functions is a compact and convenient method for storage of information about optical characteristics of a particle ensemble. They, once calculated, can be used many times in solving problems of singe and multiple scattering. In addition, they significantly facilitate the numerical solution of the radiative transfer equation[8] and estimation of scattered radiation fluxes in arbitrary conical solid angles.[9] Now the solutions are known and algorithms are implemented for spherical particles[10] and chaotically oriented axisymmetric particles.[11]

The analytical algorithm of orientation averaging of optical characteristics of randomly oriented particles free of axial symmetry was used[13,14] to calculate the angular dependence and to determine the Fourier coefficients of expansion of the scattering matrix elements of randomly oriented hexagonal cylinders in terms of the generalized spherical functions based on the T-matrix method.[12]

The data of comparative analysis of the computation time, needed for the numerical implementation of the analytical method,[13,14] and the results of Ref. 5, in which the computations for randomly oriented hexagonal cylinders were performed with the use of the T-matrix method and the FDTD (finite-difference time domain) method,[15] are tabulated below.

A hexagonal cylinder is characterized by the diffraction parameter $\rho = kl$ (k is the wave number), by the ratio $\varepsilon = l/d$ of the cylinder length l to the base diameter d equal to the double side of a regular hexagon, and by the relative refractive index m_r.

Table. Time for computation (in s) of optical characteristics of randomly oriented hexagonal cylinders with the relative refractive index $m_r = 1.30778 + i0.166667 \cdot 10^{-7}$ and $\varepsilon = 1$

$\rho = kL$	1*	2**	3***
5	1.25	91.45	3960.0
10	3.66	691.31	48600.0
15	26.96	6978.91	180000.0
20	91.57	20241.95	433800.0

* T-matrix method with the use of the analytical averaging algorithm, computations performed on Intel Celeron Mobile 1.3 GHz; ** T-matrix method,[5] computations performed on DEC VAX Alpha 600 GHz; *** FDTD method,[5] computations performed on SGI Octane 300 GHz.

The analytical algorithm of orientation averaging of optical characteristics for ensembles of nonspherical particles devoid of axial symmetry is much more efficient than the procedure of numerical orientation integration over three Euler angles. On the average, the analytical algorithm for hexagonal cylinders is two orders of magnitude faster than existing analogs.[5]

Computations with the use of the exact theory (T-matrix method) are reference and can be used to check the adequacy of approximate methods and to determine the domain of their correct application. In some papers,[4,5] in estimation of angular dependence of the scattering matrix elements for horizontally and randomly oriented hexagonal cylinders, circular cylinders, approximating the hexagonal shape of particles and having the same volume as hexagonal cylinders, are used.

The angular dependence of normalized elements of the scattering matrix for randomly oriented hexagonal and circular cylinders calculated by the T-matrix method is shown in Fig. 1. The diameter d here is the diameter of the circle inscribed in the base of the hexagonal cylinder.

It should be noted that the analytical algorithm of orientation averaging can be also applied to horizontally oriented hexagonal cylinders.[14]

Let α, β, γ be the Euler angles characterizing the sequential rotation of the laboratory coordinate system (*L*) about the movable coordinate axes *Z*, *Y*, *Z* to the coordinate system (*P*) related to the hexagonal cylinder, where *Z* coincides with the particle axis and passes through the centers of the lower and upper hexagonal bases, while *X* is perpendicular to the base side. If the coordinate systems are right-hand, then the incident radiation is directed along the positive axis *Z* of the laboratory coordinate system.

The angular dependence of the scattering matrix elements for horizontally oriented hexagonal and circular cylinders is shown in Fig. 2 for the following orientation distribution density functions:

$$p(\alpha,\beta,\gamma) = \frac{1}{2\pi}\delta\left(\cos\beta - \cos\frac{\pi}{2}\right)\delta(\gamma - \gamma_0), \qquad (1)$$

$$p(\alpha,\beta,\gamma) = \frac{1}{4\pi^2}\delta\left(\cos\beta - \cos\frac{\pi}{2}\right), \qquad (2)$$

where δ is the Dirac delta. The particle axis makes an angle of 90° with the direction of the incident radiation, and the orientation of the particle axis in the horizontal plane has the uniform distribution, that is, density functions (1) and (2) are independent of α. The orientation of the hexagonal cylinder determined by the rotation about the particle axis is described by γ_0. The independence of the density function (2) of γ means the uniform distribution over γ. The area of projection of the hexagonal cylinder on the plane perpendicular to the direction of the incident wave is maximal at $\gamma_0 = 0$ and minimal at $\gamma_0 = \pi/6$.

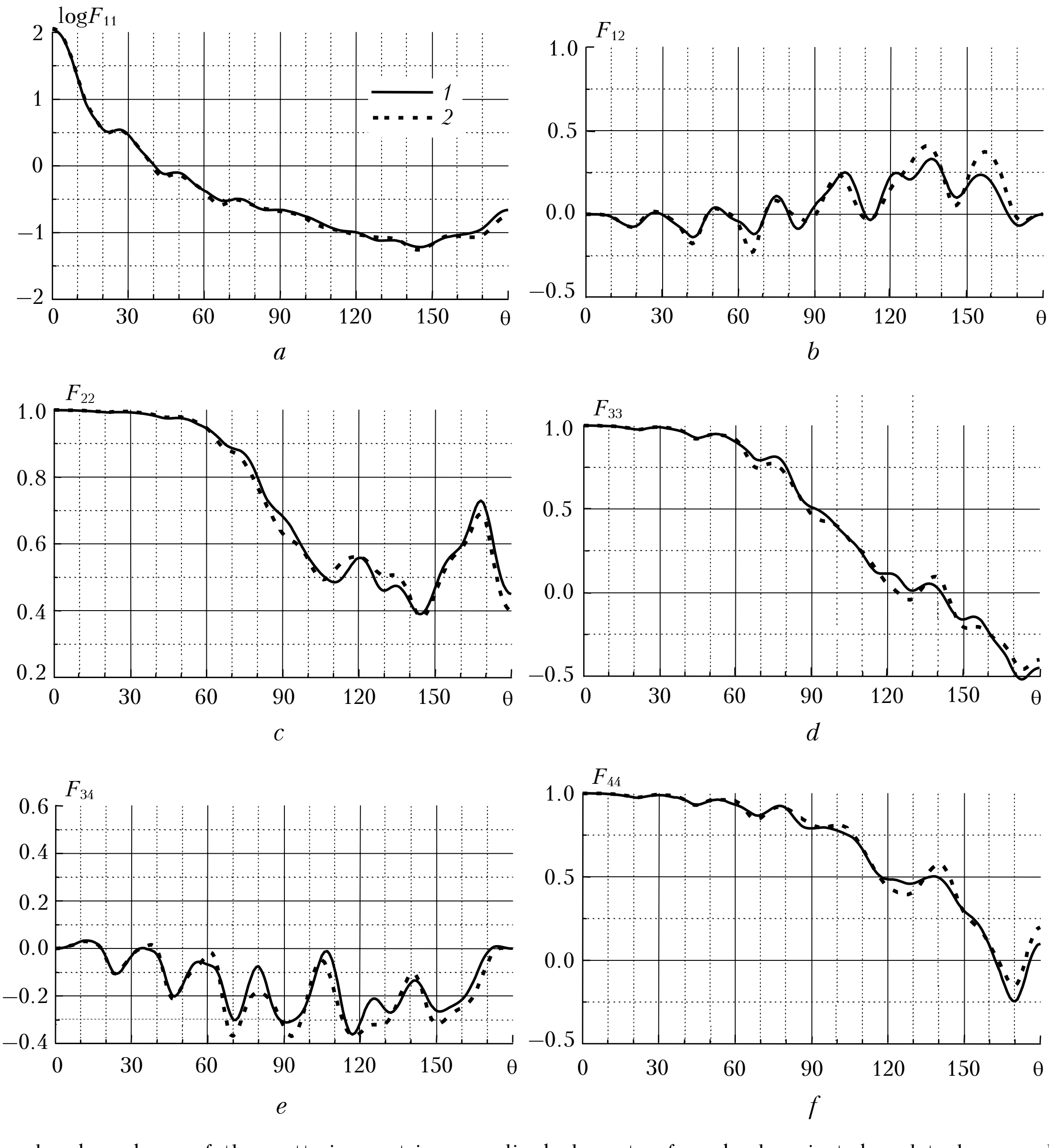

Fig. 1. Angular dependence of the scattering matrix normalized elements of randomly oriented prolate hexagonal (*1*) and circular (*2*) cylinders of the same volume: $\log F_{11}$ (*a*), F_{12} (*b*), F_{22} (*c*), F_{33} (*d*), F_{34} (*e*), F_{44} (*f*); $\rho = 20$; $\varepsilon = 2$; $m_r = 1.313$.

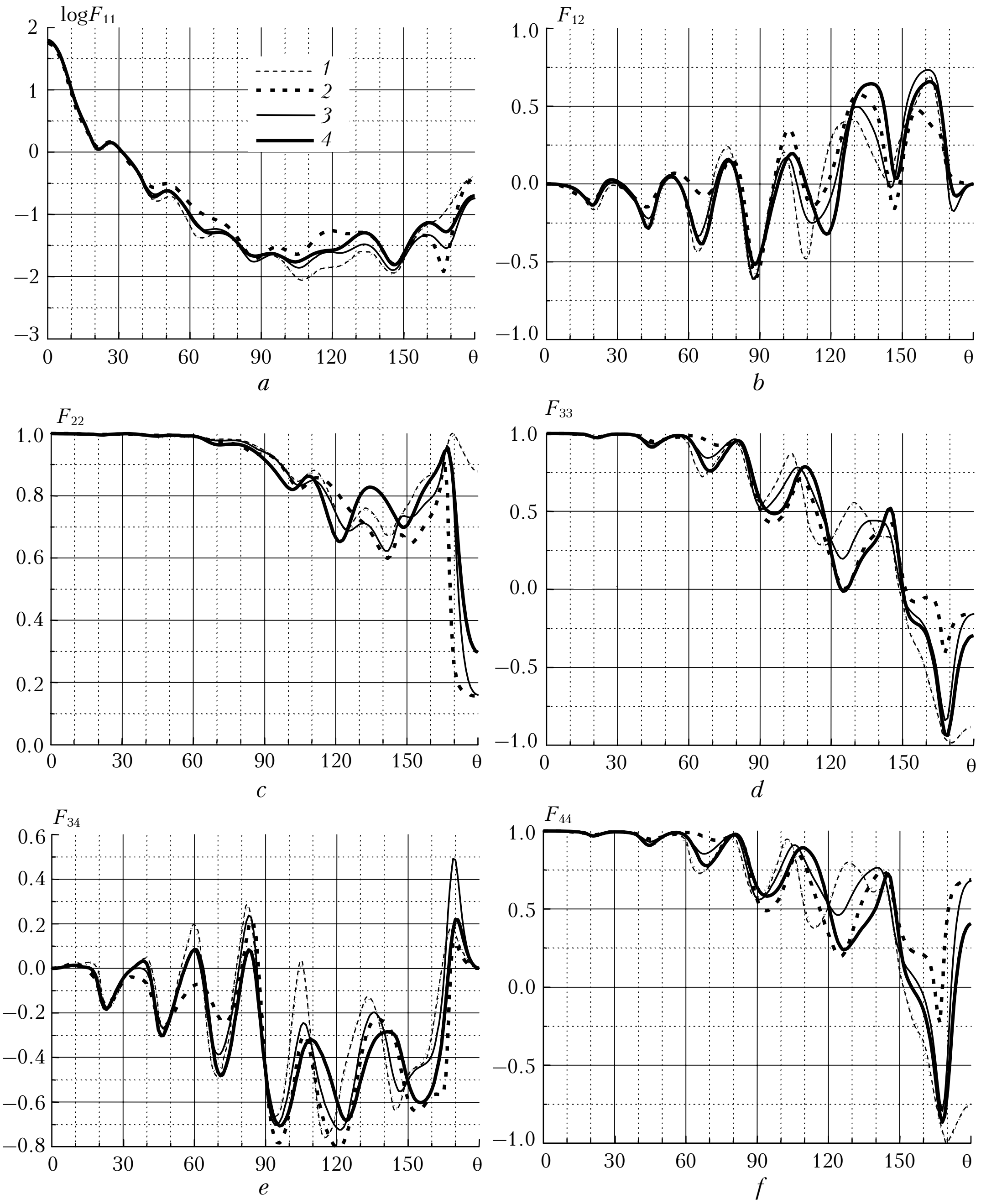

Fig. 2. The same as in Fig. 1, but for horizontally oriented hexagonal cylinders with the orientation distribution (1): $\gamma_0 = 0$ (*1*), $\gamma_0 = \pi/6$ (*2*); for hexagonal (*3*) and circular (*4*) cylinders with the orientation distribution (2).

For horizontally oriented circular cylinders, the application of the density functions (1) and (2) gives the same result, since the orientation of a circular cylinder in space is independent of γ.

In conclusion, it should be noted that the results of Refs. 13 and 14 are the logical continuation and generalization of the results of Refs. 10 and 11 to the case of ensembles of particles devoid of axial symmetry. The numerical implementation of the analytical algorithm has shown that in some cases the polarization characteristics of randomly oriented hexagonal cylinders can be estimated with the aid of circular cylinders of the same volume. For horizontally oriented hexagonal cylinders, there are significant variations in the dependence on γ_0 in the backscattering region for the scattering matrix elements F_{22}, F_{33}, F_{44}.

Acknowledgments

This work was supported in part by the Russian Foundation for Basic Research (Grant No. 07–05–00734a).

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