Radiative effects of inhomogeneous clouds

G.A. Titov and E.I. Kassianov

Institute of Atmospheric Optics,
Siberian Branch of the Russian Academy of Sciences, Tomsk, Russia
Pacific Northwest National Laboratory, USA

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The state and the prospects of the radiative transfer theory in inhomogeneous clouds are discussed. Urgent problems of atmospheric optics are emphasized. It is shown that for developing the radiation parameterizations and solving remote sensing problems, the stochastic geometry and inhomogeneous internal cloud structure should be adequately taken into account.

1. Introduction

Study of global climate and prediction of likely climate change scenarios are impossible without adequate treatment and correct parameterization of cloud–radiation interaction. Detailed knowledge of physical, optical, and radiative cloud properties, as well as of the spatial cloud inhomogeneity, is highly important not only for assessment of cloud impact on climate, but also for interpreting data of remote sensing. The radiative properties of clouds have been studied in a number of international and national programs such as International Satellite Cloud Climatology Project (ISCCP), United States Global Change Research Program (USGCRP), and GEWEX Cloud System Study (GCSS).

Cloud inhomogeneity is caused by both cloud field stochastic geometry (irregular boundaries, amount, sizes, and spatial positions of clouds) and inhomogeneous internal cloud structure (fluctuations of liquid water content, phase composition, and particle size spectrum). As a result, no one-to-one relationship between radiation intensity and cloud parameters can be established because of the stochastic cloud field geometry. However, the mean radiative properties are predictable. Normally no detailed description of radiation field is required in climate models, and only mean radiative parameters are needed to predict long-term trends in climate. Therefore, the problem of statistical radiative transfer in clouds has been of a great concern as being aimed at establishing the relationship between the statistical parameters of clouds and radiation. Although the necessity of such a statistical approach has long been recognized and a number of models and methods have already been developed,[1–10] many issues still remain unresolved.

The study of radiative transfer through inhomogeneous clouds has been being conducted within two, quite vast and overlapping research areas. The first one includes investigations, dating at least to the paper by Avaste and Vainikko[11] in which they estimated the influence of broken clouds on radiative transfer.[12–16] Here and below, by broken clouds we understand a cloud field with stochastic geometry while deterministic optical parameters inside an individual cloud. It should be stressed that the statistical theory of radiative transfer in broken clouds has been intensively developed at the Institute of Atmospheric Optics SB RAS for over 20 years.[18–20,22] A considerable contribution to the stochastic radiative transfer theory has been made by professor Pomraning and his colleagues (see, e.g., Ref. 17 and bibliography therein).

Even more activity has been invested recently in the second research area dealing with the radiation interaction with inhomogeneous stratocumulus (*Sc*) clouds,[23–27] especially after initiation of the First ISCCP Regional Experiment (FIRE), during which first data on the horizontal distribution of liquid water path (optical depth) of marine *Sc* have been collected[28] and its influence on the mean albedo studied.[29,30]

It is quite clearly that it is impossible to review both research areas in full detail in a single paper, so only brief description of the main results obtained in both of these areas, in particular, at the Institute of Atmospheric Optics SB RAS are presented in this paper.

2. Inhomogeneous stratocumulus clouds

Typically marine *Sc* clouds have considerable horizontal extension and small geometrical thickness; so they are most frequently treated as plane parallel.[24,29,30] Experimental data show that the *horizontal* variations of optical depth (liquid water path) are well described by the lognormal distribution and power-law spectrum with the exponent corresponding to the Kolmogorov–Obukhov law.[28] The observed horizontal distribution of optical depth τ is modeled using a two-parameter fractal model generated by multiplicative cascade processes.[31] Based on this plane parallel model, the effect of horizontal inhomogeneities in τ on the solar radiative transfer has been studied.[29,30,32–35]

The numerical realizations of the random fields with prescribed one-dimensional distribution and spectral density (correlation function) can be constructed based on the methods of spectrum analysis and randomization.[36–38] The latter, noticeably superior over fractal models,[39] have been used by ourselves to

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study the sensitivity of radiative properties of *Sc* clouds to their inhomogeneous spatial structure[40–49] (Fig. 1).

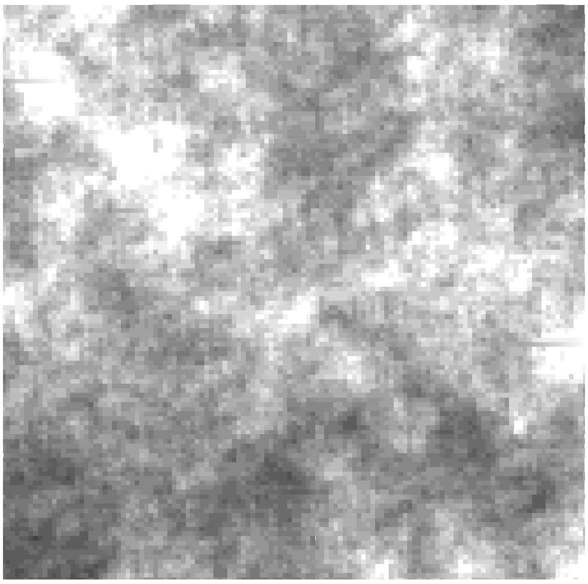

Fig. 1. Computer realization of two-dimensional field of the marine stratocumulus clouds optical depth.

We would like to note here that, in contrast to fractal models, the spectral methods allow one to construct numerical realizations of the fields whose spectra have scale breaks.[50] The occurrence of such fields in the atmosphere has been revealed from satellite observations (see section 2.1). The transfer of solar radiation through inhomogeneous *Sc* clouds was studied using two cloud models as given in Refs. 43 and 46 in detail. The first (*WP*) model assumes plane parallel geometry and random horizontal distribution of optical depth (extinction coefficient). In the second (*GWP*) model, the fluctuations of both the extinction coefficient and height of cloud top boundary are simultaneously taken into consideration. A few-parameter model of stochastic upper boundary of *Sc* clouds was constructed based on data of airborne laser sensing obtained at the Institute of Atmospheric Optics SB RAS.

Next sections address two most controversial issues concerning the horizontal radiative fluxes (horizontal transport).

2.1. Scale break inferred from satellite observations

The power-law spatial spectrum of reflected radiation, inferred from satellite (Landsat) data, has two different slopes at large (from ~ 100 km to ~ 200–500 m) and small (less than ~ 200–500 m) scales.[28,51,52] The radiative field is much smoother (the exponent is larger) at smaller than larger scales. The satellite-based radiation measurements are an important source of vital geophysical information, e.g., on the turbulence inside a cloud; so the causes for such an BunusualB spectral behavior of reflected radiance are of great recent concern.[28,32,34,51–53] The different spatial distributions of clouds at large and small spatial scales may be one of the likely explanation for the observed scale break. For instance, Reference 53 argues that (1) the horizontal wind may influence considerably the *cloud structure*; and (2) this influence depends on the spatial scale. The horizontal wind can also smooth out the surface of small-size clouds.[54]

Alternatively, the scale break is explained not by the scale-sensitive spatial distribution of clouds, but rather by the *radiative* smoothing effect,[32] i.e., smoothing of small-scale radiative field fluctuations by the horizontal radiative fluxes (radiative horizontal transport). Note that the reflected Landsat radiances have been measured with high (~ 0.05 km) horizontal resolution. It was found[32,34] that the characteristic scale η, where the scale break takes place, is proportional to $\sqrt{\overline{\rho^2}}$, where $\overline{\rho^2}$ is the second moment of distribution of distances between photon entry and exit points. In diffuse approximation, it was found that

$$\sqrt{\overline{\rho^2}} \approx \begin{cases} h[(1-g)\tau]^{-1/2} & \text{for albedo,} \\ h & \text{for transmittance,} \end{cases} \qquad (1)$$

where h is the cloud thickness; g is the asymmetry factor; and τ is the optical depth. For inhomogeneous clouds, one can use, instead of τ and h, their average (over a realization) values, $\overline{\tau}$ and $\overline{h}$. From Eq. (1) we can conclude that $\sqrt{\overline{\rho^2}}$ depends on standard parameters that determine the radiative transfer in homogeneous clouds – h, g, and τ. Our results suggest that the horizontal radiative fluxes strongly depend both on the horizontal gradient of optical depth,[43,48] and on the irregular geometry of cloud top boundary.[46,47] In particular, for fixed $\overline{\tau}$ and $\overline{h}$ values, variations in the cloud top boundary height may cause approximately an order of magnitude increase in the variance of the horizontal transport.[46] Let us assume that the horizontal transport actually determines the characteristic scale η. This means that with variations of the geometrical (optical) parameters of a cloud field the characteristic scale η and $\sqrt{\overline{\rho^2}}$, proportional to it, both must change. However, from Eq. (1) it follows that $\sqrt{\overline{\rho^2}}$ does not depend on parameters characterizing the spatial variability of the cloud layer. For this reason, no one-to-one relationship exists between η and $\sqrt{\overline{\rho^2}}$, and, thus, $\sqrt{\overline{\rho^2}}$ cannot determine η. Reference 49 clearly demonstrates that the explanation of scale break in terms of the smoothing effect of the horizontal radiative fluxes, as well as the hypothesis that the scale η and $\sqrt{\overline{\rho^2}}$ are interrelated, has no good physical grounds.

2.2. Anomalous cloud absorption problem

Cloud absorption measured in the field may exceed considerably (by more than a factor of two) the radiation calculations (see, e.g., Ref. 55). What is the reason for such a large discrepancy? Why *negative* cloud absorption values may occasionally be found when traditional measurement techniques are applied[55–57]? Answering these key questions has been the topic of many theoretical and experimental studies over the last 40 years.

Most of the radiation calculations are based on plane parallel model (plane parallel geometry and horizontal homogeneity of optical properties of a cloud field); so it is expected that the stochastic cloud geometry and/or inhomogeneous internal cloud structure are mainly responsible for the excess absorption that is being observed in the experiments. Our results show that, for fixed $\bar{\tau}$ and $\bar{h}$, the stochasticity of cloud top boundary and horizontal fluctuations of extinction coefficient have *little* influence on the mean absorption of *Sc* clouds.[46] This finding also applies to broken clouds (see, e.g., Ref. 58). At $\bar{\tau}$ less than 30–40, absorption of cumulus clouds, whose optical and geometrical properties are most variable in space and show most strong fluctuations, is, on the average, about 1/5 of the total atmospheric absorption and differs quite insignificantly (by less than 2 – 3% of incident solar radiation) from a plane parallel estimate. This fact indicates that, *on the average*, the stochastic cumulus clouds are unable to absorb *much more* solar radiation than the deterministic radiative transfer theory predicts.

Alternatively, the large difference between calculated and observed absorption may be due to incorrect interpretation of the experimental data. The results presented below clearly illustrate considerable influence of the horizontal transport on the accuracy of cloud absorption retrievals. Now we will give an outline of the problem on determining the absorption in inhomogeneous clouds.[44,48] For simplicity, we assume that the clouds are located over a nonreflecting underlying surface, while their optical characteristics depend only on the horizontal coordinate x. Then, the radiative energy conservation law in inhomogeneous clouds has the form

$$R(x) + T(x) + A(x) = 1 - E(x). \qquad (2)$$

In this equation, the unknown functions are albedo $R(x)$, transmission $T(x)$, absorption $A(x)$, and horizontal transport $E(x)$; of which only albedo and transmission are measured in practice. Therefore, instead of actual absorption $A(x)$, from Eq. (2) we can only determine the reconstructed absorption:

$$A'(x) = A(x) + E(x) = 1 - R(x) - T(x). \qquad (3)$$

From Eq. (3) it follows that if the horizontal transport, being zero in the plane parallel model, is comparable, by the order of magnitude, with $A(x)$, then the reconstructed absorption, $A'(x)$, may considerably diverge from the actual one, $A(x)$. In Fig. 2, $A(x)$ is plotted versus $A'(x)$. Since the horizontal transport $E(x)$ may take either positive or negative sign, the same may happen to the reconstructed absorption $A'(x)$. Due to the stochastic geometry of the cloud top boundary, the ranges of possible $A(x)$ and $A'(x)$ values may increase by more than two and three times, respectively.[46] As seen, the single net-flux measurements *cannot* provide reliable cloud absorption estimates at small (~ 0.05 km) spatial scales. The $A(x)$ estimates can be improved using two approaches based on (i) spatial averaging of radiative characteristics; and (ii) synchronous flux measurements in the visible and near-IR spectral regions.[44,48] The simultaneous measurements of the visible and near-IR fluxes can be used to study small-scale (~ 0.05 km) variations of absorption by plane parallel clouds.[44,48] Due to the stochastic geometry of *Sc* cloud top boundary, the retrieval accuracy may degrade by about an order of magnitude.[46]

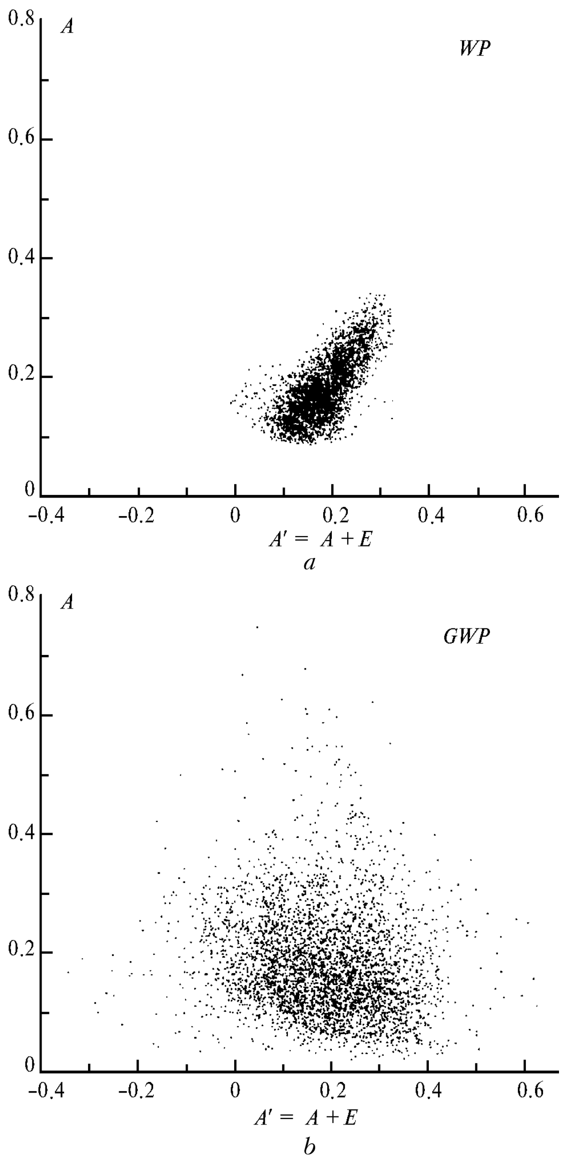

Fig. 2. Absorption A versus reconstructed absorption $A' = A + E$ for solar zenith angle 60° and single scattering albedo $\omega_0 = 0.99$. Computations were made for a cloud realization consisting of 2^{12} pixels with the same horizontal size $\Delta x = 0.05$ km.

From the results presented here it follows that incorrect interpretation of field data is one of the main reasons for existing discrepancy between the theory and experiment.

2.3. Three-dimensional stratocumulus clouds

The above analysis is based on quite a simple, *one-dimensional* (1D) (i.e., depending on just the *horizontal* coordinate) cloud models accounting for either fluctuations of cloud top height or extinction coefficient, or both. It was assumed that the cloud top height and extinction coefficient are *independent* random processes; whereas in real clouds, optical properties (extinction coefficient, single scattering albedo, and scattering phase function) *all* vary in both horizontal and vertical directions and, moreover, the variations of optical and geometrical characteristics in 3D space are *interrelated*.

The 3D Large Eddy Simulation (LES) cloud models with explicit microphysics proved to be a highly promising tool allowing correct treatment of this complex relationship in the radiation studies (see, e.g., Refs. 59 and 60). We used the Cooperative Institute for Mesoscale Meteorological Studies (CIMMS) LES cloud model[61,62] to study the radiative effects of 3D stratocumulus clouds.[63–66] The obtained results confirmed main conclusions that have been drawn earlier with the use of simpler cloud models and demonstrated that the radiative properties of *Sc* clouds depend strongly on their *vertical* stratification. In particular, it was shown that the mean fluxes, calculated for two cloud fields with the same 2D optical depth distribution but different vertical structures, may substantially differ.[66]

The LES models may provide information not only on the 3D distribution of liquid water (particle size spectrum), but also on the water vapor. The latter is critical when radiative properties of inhomogeneous clouds are calculated in the IR spectral range, especially in the water vapor absorption bands. The utility of these data is well illustrated by the following example. Presently, the absorption retrieval methods frequently assume that the variations of the horizontal transport E are determined by the corresponding variations in the *scattering* properties (scattering coefficient and scattering phase function),[33,57,67] whose spectral behavior can be safely neglected. We showed that *absorption* by water droplets and water vapor has considerable influence on E and determines its spectral dependence. The neglect of this dependence may introduce serious errors in cloud absorption estimates.[64,65]

3. Broken clouds

In the atmosphere there often occur fields of cumulus cloud that only partially cover the sky. Numerous individual clouds vary in size and shape and have different positions (Fig. 3). The irregular cloud geometry has stronger influence on the solar radiative transfer than inhomogeneous internal cloud structure[9,20,21,66]; therefore, the fluctuations of optical properties inside an individual cloud can be neglected in the first approximation. One possible and promising theoretical treatment of the radiative transfer in broken clouds, having random geometry and deterministic optical parameters, has been currently developed at the Institute of Atmospheric Optics SB RAS. The main achievements in this research area over the past two decades (1975–1995), as well as a vast bibliography, have been summarized in Refs. 20–22.

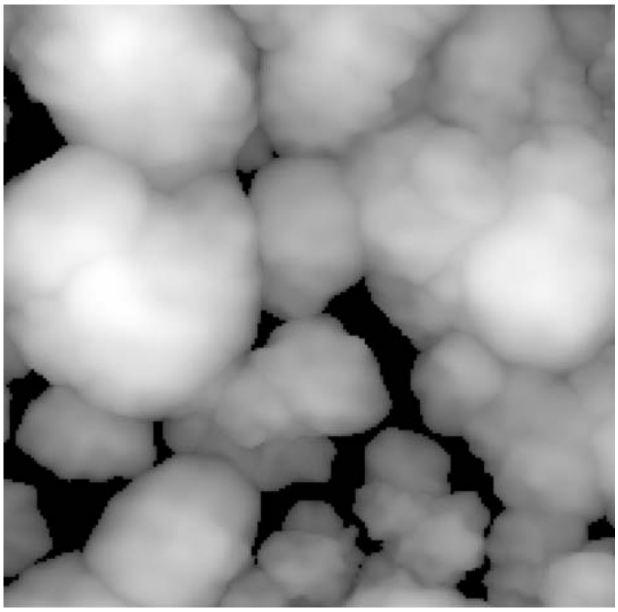

Fig. 3. A computer realization of 2D optical depth field of cumulus clouds.

In this section, only main results, obtained at the Institute of Atmospheric Optics in recent five years, will be discussed briefly.

3.1. Parameterization of radiative properties of single-layer broken clouds

The radiation codes of general circulation models (GCMs) cannot be improved without adequate treatment of interaction of radiation with broken clouds. The mean radiative fluxes F_{bc} in broken clouds can be represented as a linear combination of overcast and clear-sky fluxes, F_{pp} and F_{clr}, taken with weights N_e and $(1 - N_e)$, respectively. Here N_e is the effective cloud fraction. The term BeffectiveB is used to indicate that the parameter N_e depends not only on the three-dimensional geometry and optical properties of the broken clouds, but also on the solar zenith angle and surface albedo A_s. The F_{pp} and F_{clr} values can be readily calculated from the well-developed deterministic radiative transfer theory; therefore, for a proper F_{bc} determination, N_e value should be correctly specified. A fast and convenient method of calculating N_e, as well as a new, N_e-based parameterization of the radiative regime in single-layer broken clouds, is presented in Ref. 68. Using the created numerical model of N_e, the dependence of the mean fluxes on optical and geometrical parameters of broken clouds, solar zenith angle, and surface albedo A_s can be thoroughly studied[69] (Fig. 4). It is shown that the stochastic cloud geometry can have a considerable influence on N_e and, hence, the radiation parameterizations developed for the cumulus clouds should take it adequately into account. This radiation parameterization has several advantages over other radiation models; and it is most important that it can be readily incorporated into the existing GCM radiation codes without serious changes of the latter.

3.2. Multilayer broken clouds

Cloud field may consist of several cloud layers,[70,71] and the broken clouds may be simultaneously present at all atmospheric levels. So, calculation of radiative properties of multilayer clouds may be frequently needed.

In the discussion that follows we will address a generalization of the statistical approach, developed for single-layer broken clouds, to N statistically *independent* layers. The main idea of such a generalization will be explained by the simplest example of unscattered radiation. For integrity of presentation, we remind the main steps in derivation of equation for mean intensity of unscattered radiation in one-layer broken clouds.[22,72]

Suppose that the broken clouds occupy a layer Λ: $0 \le z \le H$. Let a parallel solar flux be incident on the plane $z = 0$ in the direction $\boldsymbol{\omega} = (a, b, c)$. For simplicity, we assume that the flux has a unit intensity and that extinction coefficient $\sigma(\mathbf{r}) = \sigma = \text{const}$. The intensity of unscattered radiation $j(\mathbf{r})$ at the point $\mathbf{r}$ in the direction $\boldsymbol{\omega}$ is a solution of the stochastic transfer equation

$$\boldsymbol{\omega}\nabla j(\mathbf{r}) + \sigma\kappa(\mathbf{r})\, j(\mathbf{r}) = 0 \quad (4)$$

with the boundary condition

$$j(\mathbf{r}_0) = j(x, y, 0) = 1. \quad (5)$$

In contrast to the deterministic transfer equation, stochastic equation (4) contains a *random* indicator field $\kappa(\mathbf{r})$ that characterizes the irregular geometry of broken clouds. As defined, $\kappa(\mathbf{r}) = 1$ inside the clouds, and $\kappa(\mathbf{r}) = 0$ in gaps among the clouds. The statistically *homogeneous* model[19] is described completely by unconditional and conditional probabilities of the cloud presence, $\langle\kappa(\mathbf{r})\rangle$ and $V(\mathbf{r}, \mathbf{r}') = P\{\kappa(\mathbf{r}) = 1 / \kappa(\mathbf{r}') = 1\}$ (Markov approximation). The input model parameters are related to the cloud fraction p and the mean horizontal cloud size D as $\langle\kappa(\mathbf{r})\rangle = p$ and $V(\mathbf{r}, \mathbf{r}') = f(p, D) = (1 - p) \exp(- A(\boldsymbol{\omega})|\mathbf{r} - \mathbf{r}'|) + p$, where $A(\boldsymbol{\omega}) \sim 1/D$.

By inverting the differential operator in Eq. (4) for $c \ne 0$, we obtain

$$j(\mathbf{r}) + \frac{\sigma}{c}\int_0^z \kappa(\mathbf{r}')\, j(\mathbf{r}')\, d\xi = 1, \quad (6)$$

where $\mathbf{r}' = \mathbf{r} + \boldsymbol{\omega}(\xi - z)/c$. Let us average Eq. (6) over the ensemble of $\kappa(\mathbf{r})$ field realizations

$$\langle j(\mathbf{r})\rangle + \frac{\sigma}{c}\int_0^z \langle\kappa(\mathbf{r}')\, j(\mathbf{r}')\rangle\, d\xi = 1. \quad (7)$$

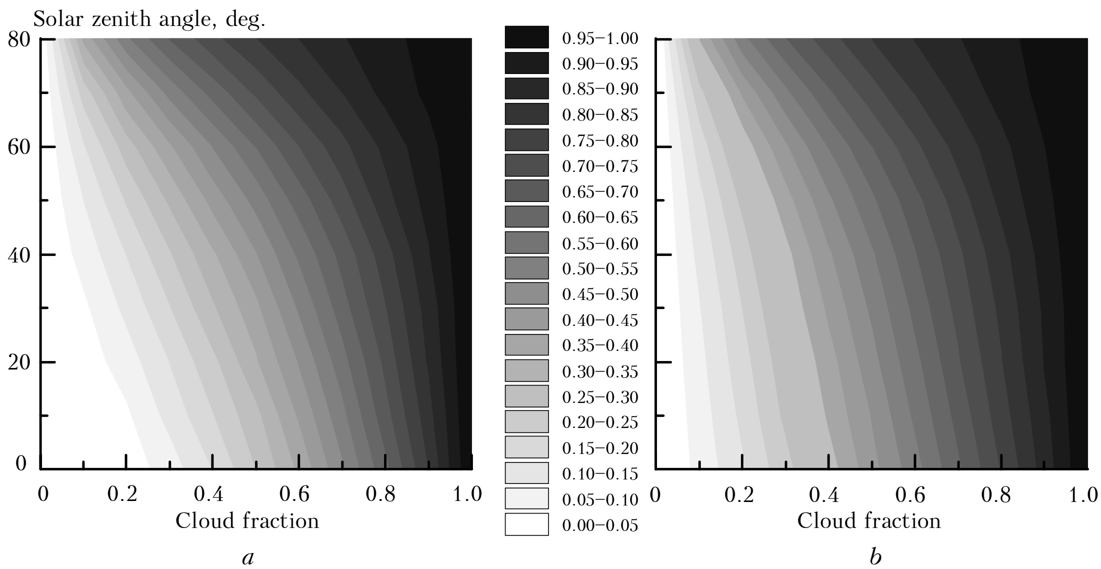

Fig. 4. Effective cloud fraction N_e as a function of solar zenith angle and cloud fraction for the optical depth $\tau = 15$, aspect ratio $\gamma = 2$ (defined here as $\gamma = \Delta H/D$, where ΔH is the geometrical thickness of the cloud layer and D is the mean horizontal cloud size), and surface albedo $A_s = 0$ (*a*) and 0.4 (*b*).

Multiply then Eq. (6) by $\kappa(\mathbf{r})$ and average once more

$$\langle\kappa(\mathbf{r})\, j(\mathbf{r})\rangle + \frac{\sigma}{c}\int_0^z \langle\kappa(\mathbf{r})\,\kappa(\mathbf{r}')\, j(\mathbf{r}')\rangle\, d\xi = \langle\kappa(\mathbf{r})\rangle. \quad (8)$$

Using formula for correlation splitting, $\langle\kappa(\mathbf{r})\,\kappa(\mathbf{r}')\, j(\mathbf{r}')\rangle = V(\mathbf{r}, \mathbf{r}')\,\langle\kappa(\mathbf{r}')\, j(\mathbf{r}')\rangle$, Equation (8) can be written as follows:

$$\langle\kappa(\mathbf{r})\, j(\mathbf{r})\rangle + \frac{\sigma}{c}\int_0^z V(\mathbf{r}, \mathbf{r}')\,\langle\kappa(\mathbf{r}')\, j(\mathbf{r}')\rangle\, d\xi = \langle\kappa(\mathbf{r})\rangle. \quad (8a)$$

The closed system of equations (7) and (8a) can be solved using the Laplace transform. The mean intensity $\langle j(\mathbf{r})\rangle$ is calculated from simple formula (see, e.g., Ref. 22, formula (7.10)).

Let us turn to the derivation of equations for the mean intensity of unscattered radiation in a cloud field composed of two layers of broken clouds (Fig. 5). We assume that these layers are statistically homogeneous and *independent*. In this case, $\langle\kappa(\mathbf{r})\rangle = p_1$ and $V(\mathbf{r}, \mathbf{r}') = f(p_1, D_1)$ for the first layer ($0 \le z \le h$), and $\langle\kappa(\mathbf{r})\rangle = p_2$ and $V(\mathbf{r}, \mathbf{r}') = f(p_2, D_2)$ for the second layer ($h < z \le H$).

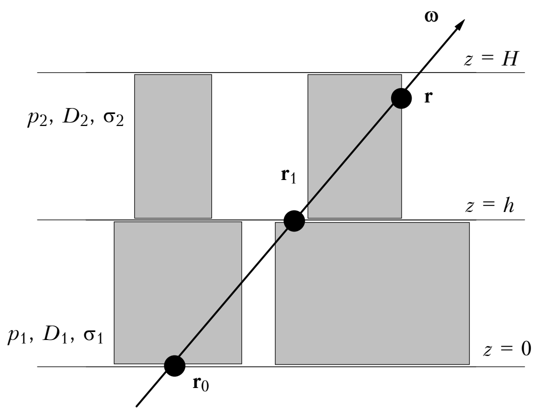

Fig. 5. Schematic illustration of the two-layer cloud model.

If the point $\mathbf{r}_1$ belongs to the first layer ($0 \le z \le h$), then the system of equations for the mean intensity $\langle j(\mathbf{r}_1, \mathbf{r}_0)\rangle$ is given by Eqs. (7) and (8a). Otherwise, for a point $\mathbf{r}$ located in the second layer ($h < z \le H$), the equations for mean intensity (7) and (8a) modify to the ones having other integration limits and different right-hand sides: 1 in Eq. (7) changes to $\langle j(\mathbf{r}_1, \mathbf{r}_0)\rangle$, and $\langle\kappa(\mathbf{r})\rangle$ in (8a) is replaced by the product of the means $\langle\kappa(\mathbf{r})\rangle\,\langle j(\mathbf{r}_1, \mathbf{r}_0)\rangle$. Obviously, $\langle j(\mathbf{r}, \mathbf{r}_0)\rangle = \langle j(\mathbf{r}, \mathbf{r}_1)\rangle\,\langle j(\mathbf{r}_1, \mathbf{r}_0)\rangle$ in this case. When the functions $\langle j(\mathbf{r}_1, \mathbf{r}_0)\rangle$ and $\langle j(\mathbf{r}, \mathbf{r}_1)\rangle$ are calculated by standard formula,[22] the parameters for the first and second layers, respectively, should be used. Generalization to N (≥ 2) statistically independent layers is quite straightforward. The mean intensity of unscattered radiation for the Nth layer is calculated by

$$\langle j(\mathbf{r}, \mathbf{r}_0)\rangle = \langle j(\mathbf{r}, \mathbf{r}_{N-1})\rangle \prod_{n=1}^{N-1} \langle j(\mathbf{r}_n, \mathbf{r}_{n-1})\rangle.$$

The mean *diffuse* intensity is calculated quite easily also. In each of the cloud layer, the photon trajectories are simulated by standard algorithms (see, e.g., Refs. 19 and 22) using parameters corresponding to a given cloud layer, while the local estimate is made with the account of the statistically homogeneous and *independent* cloud layers located above (reflection, $c > 0$) or below (transmission, $c < 0$) this layer. For instance, contribution of photons reflected from the underlying surface to the mean intensity of reflected radiation at the top of the Nth layer (local estimate) is proportional to the product of the mean intensities calculated for each of the N layers:

$$\langle j(\mathbf{r}, \mathbf{r}_{N-1})\rangle \prod_{n=1}^{N-1} \langle j(\mathbf{r}_n, \mathbf{r}_{n-1})\rangle.$$

For the single-layer clouds, this contribution is proportional to just one of the factors, namely $\langle j(\mathbf{r}_1, \mathbf{r}_0)\rangle$. Note that the algorithms of calculating the brightness fields of *multilayer* atmosphere,[73] located over a reflecting surface, have been successfully used to solve an important applied problem.[74] The radiative properties of a system, consisting of two statistically independent layers of broken clouds, have been studied in Ref. 75.

This method is only applicable when the spatial distributions of clouds at individual atmospheric levels do not depend on how the clouds are distributed over the other levels (the hypothesis of random cloud overlap). However, the total cloud fractions calculated for two- and three-layer clouds assuming random cloud overlap frequently disagree with the measurements, with the largest differences observed for two-layer broken clouds.[70] This suggests that the multilevel cloud systems may contain statistically *dependent* broken-cloud layers; so, different combinations of maximum and random cloud overlaps are normally used in GCMs to simulate this.[76,77] In single-layer 3D broken clouds, the geometrical parameters (cloud fraction, mean horizontal cloud size, etc.) vary with altitude, and values of these parameters at different altitudes are interrelated.

In this connection, there is a need to generalize the ideas and methods developed within the statistically *homogeneous* model[19] to the case of statistically *inhomogeneous* broken clouds. The Bstatistical *inhomogeneity*B will be understood in the meaning that the unconditional probability $\langle\kappa(\mathbf{r})\rangle$ depends on the vertical coordinate while the conditional probability $V(\mathbf{r}, \mathbf{r}')$ depends on the *positions* of points $\mathbf{r}$ and $\mathbf{r}'$. Remind that, in the statistically homogeneous model, $\langle\kappa(\mathbf{r})\rangle = p = \text{const}$, while $V(\mathbf{r}, \mathbf{r}')$, for a fixed $\boldsymbol{\omega}$, depends only on the *distance* between points $\mathbf{r}$ and $\mathbf{r}'$.[19] For a cloud field consisting of statistically homogeneous and *independent* fields $V(\mathbf{r}, \mathbf{r}') = \langle\kappa(\mathbf{r})\rangle$.

3.3. Statistically inhomogeneous model

The broken clouds can be represented as a Markovian mixture of cloudy ($\kappa(\mathbf{r}) = 1$) and noncloudy ($\kappa(\mathbf{r}) = 0$) segments. To obtain the conditional probability $V^*(\mathbf{r}, \mathbf{r}')$ of the cloud presence, we will use the piecewise constant approximation of probabilities of transition from cloud to clear sky and backward. It is assumed that the probability of transition, on a short distance $\Delta l = \Delta z / c$, from clear sky to cloud ($0 \to 1$) is $\mu_i \Delta l$ and from cloud to clear sky ($1 \to 0$) is $\lambda_i \Delta l$, $i = 1, 2, \dots, N$, where N is the number of statistically homogeneous and *dependent* layers. Note that $1/\lambda$ and $1/\mu$ can be interpreted as mean chord lengths in the cloudy and cloud-free segments, respectively, and they can depend on the direction $\boldsymbol{\omega}$ (see, e.g., Refs. 16 and 17). The conditional probability of cloud presence $V^*(\mathbf{r}, \mathbf{r}') = V^*(l) = V^*(z, \xi)$ satisfies the Chapman–Kolmogorov equation (see, e.g., Ref. 78):

$$\frac{\partial V^*(l)}{\partial l} = -(\lambda + \mu)\, V^*(l) + \mu, \quad V^*(0) = 1, \quad (9)$$

where $l = |\mathbf{r} - \mathbf{r}'|$; and λ and μ are piecewise constant functions of l or z. General solution of Eq. (9) has the form

$$V^*(l) = \exp\left\{ -\int_0^l [\lambda(u) + \mu(u)]\, du \right\} + \int_0^l \mu(v) \exp\left\{ -\int_v^l [\lambda(u) + \mu(u)] \right\} dv . \quad (10)$$

Let the point $\mathbf{r}$ belong to the first layer ($0 \le z \le h$). Then, using Eq. (10), we obtain

$$V^*(z, \xi) = V_1^*(l_1) = (1 - p_1) \exp[-A_1(z - \xi)/c)] + p_1, \quad (11)$$

$$p_1 = \mu_1/(\lambda_1 + \mu_1), \quad A_1 = \lambda_1 + \mu_1. \quad (12)$$

Here p_1 can be interpreted as the cloud fraction in the first layer (field); and the parameter A_1 is inversely proportional to its correlation length.[15,16] Note that formula (11) is the conditional probability of the cloud occurrence in a statistically *homogeneous* field.[19]

Let $\mathbf{r}$ belong to the second layer ($h < z \le H$). In this case, equation (10) becomes

$$V_2^*(z, \xi) = \exp[-A_2(z - z_1)/c]\,[V(z_1, \xi) - p_2] + p_2, \quad (13)$$

$$p_2 = \mu_2/(\lambda_2 + \mu_2), \quad A_2 = \lambda_2 + \mu_2. \quad (14)$$

It can be shown that

$$V_i^*(z, \xi) = \exp[-A_i(z - z_{i-1})/c][\, V_{i-1}^*(z_{i-1}, \xi) - p_i] + p_i, \quad i = 2, \dots, N, \quad (15)$$

$$p_i = \mu_i/(\lambda_i + \mu_i), \quad A_i = \lambda_i + \mu_i, \quad i = 2, \dots, N. \quad (16)$$

Suppose that the conditional probabilities $V_i^*(z, \xi)$ and unconditional probabilities p_i, $i = 1, 2, \dots, N$, are known (either from model results or from field observations). Then, from Eq. (16) we can determine the unknown parameter A_i, $i = 1, 2, \dots, N$.

3.4. Statistically inhomogeneous model: mean intensity

Let us now turn to the derivation of closed equations for the mean intensity of unscattered radiation. For simplicity, we first consider a cloud field consisting of two layers. Let a parallel solar flux be incident on the plane $z = 0$ in the direction $\boldsymbol{\omega}$ (see Fig. 5). If the point $\mathbf{r}_1$ belongs to the *first* layer ($0 \le z \le h$), then coupled equations (7) and (8a) can be used to calculate the mean intensity $\langle j(\mathbf{r}_1, \mathbf{r}_0)\rangle$.

Let us determine the mean intensity $\langle j(\mathbf{r}, \mathbf{r}_0)\rangle$ in the case when the point $\mathbf{r}$ belongs to the *second* layer ($h < z \le H$). The random value $\langle j(\mathbf{r}, \mathbf{r}_0)\rangle$ of unscattered radiation intensity satisfies the stochastic equation

$$j(\mathbf{r}, \mathbf{r}_0) + \frac{\sigma_2}{c} \int_h^z \kappa(\mathbf{r}')\, j(\mathbf{r}', \mathbf{r}_0)\, d\xi = j(\mathbf{r}_1, \mathbf{r}_0). \quad (17)$$

Let us average Eq. (17) over the ensemble of $\kappa(\mathbf{r})$ field realizations

$$\langle j(\mathbf{r}, \mathbf{r}_0)\rangle + \frac{\sigma_2}{c} \int_h^z \langle \kappa(\mathbf{r}')\, j(\mathbf{r}', \mathbf{r}_0)\rangle\, d\xi = \langle j(\mathbf{r}_1, \mathbf{r}_0)\rangle. \quad (18)$$

Multiplication of Eq. (17) by $\kappa(\mathbf{r})$, averaging, and applying of the formula for correlation splitting yield

$$\langle \kappa(\mathbf{r})\, j(\mathbf{r}, \mathbf{r}_0)\rangle + \frac{\sigma_2}{c} \int_h^z V(\mathbf{r}, \mathbf{r}') \langle \kappa(\mathbf{r}')\, j(\mathbf{r}', \mathbf{r}_0)\rangle\, d\xi = \langle \kappa(\mathbf{r})\, j(\mathbf{r}_1, \mathbf{r}_0)\rangle. \quad (19)$$

Following the same argument, we arrive at the expression for function $\langle \kappa(\mathbf{r})\, j(\mathbf{r}_1, \mathbf{r}_0)\rangle$:

$$\langle \kappa(\mathbf{r})\, j(\mathbf{r}_1, \mathbf{r}_0)\rangle + \frac{\sigma_1}{c} \int_0^h V_2^*(\mathbf{r}, \mathbf{r}') \langle \kappa(\mathbf{r}')\, j(\mathbf{r}', \mathbf{r}_0)\rangle\, d\xi = p_2. \quad (20)$$

Solution of the system of equations (18)–(20) has the form

$$\langle j(z, 0)\rangle = \langle j(z, h)\rangle \langle j(h, 0)\rangle + \varepsilon(z, h)\, \alpha(h, 0), \quad (21)$$

where

$$\langle j(z, 0)\rangle = \langle j(\mathbf{r}, \mathbf{r}_0)\rangle, \quad \langle j(z, h)\rangle = \langle j(\mathbf{r}, \mathbf{r}_1)\rangle, \quad \langle j(h, 0)\rangle = \langle j(\mathbf{r}_1, \mathbf{r}_0)\rangle;$$

$$\varepsilon(z, h) = \frac{\sigma_2 p_2}{\lambda_2^{(2)} - \lambda_1^{(2)}} \left[\exp\left(-\lambda_2^{(2)} \frac{z - h}{c}\right) - \exp\left(-\lambda_1^{(2)} \frac{z - h}{c}\right) \right]; \quad (22)$$

$$\alpha(h, 0) = \frac{1}{p_2} \{p_2(1 - \langle j(h, 0)\rangle) - p_1[1 - \nu(h, 0)]\} . \quad (23)$$

The functions $\langle j(h, 0)\rangle$, $\langle j(z, h)\rangle$, $\nu(h, 0) = \langle \kappa(\mathbf{r}_1)\, j(\mathbf{r}_1, \mathbf{r}_0)\rangle / p_1$ and the coefficients $\lambda_1^{(2)}$, $\lambda_2^{(2)}$ are calculated from formulas obtained for the statistically *homogeneous* model.[22] In coefficients $\lambda_1^{(2)}$, $\lambda_2^{(2)}$, the superscript (2) indicates that they should be calculated using the parameters for the second layer.

These results can be readily generalized to a multilayer cloudy atmosphere consisting of N statistically homogeneous and *independent* layers. For a point $\mathbf{r}$ belonging to the ith layer, $i = 2, 3, \ldots, N$, the formula for the mean intensity of unscattered radiation assumes the form

$$\langle j(z, 0)\rangle = \langle j(z, z_{i-1})\rangle \langle j(z_{i-1}, 0)\rangle + \varepsilon(z, z_{i-1})\, \alpha(z_{i-1}, 0) , \quad i = 2, 3, \ldots, N, \quad (24)$$

where

$$\langle j(z, z_{i-1})\rangle = \langle j(\mathbf{r}, \mathbf{r}_{i-1})\rangle ,$$

$$\langle j(z_{i-1}, 0)\rangle = \langle j(\mathbf{r}_{i-1}, \mathbf{r}_0)\rangle , \quad i = 2, 3, \ldots, N;$$

$$\varepsilon(z, z_{i-1}) = \frac{\sigma_i p_i}{\lambda_2^{(i)} - \lambda_1^{(i)}} \times \left[\exp\left(-\lambda_2^{(i)} \frac{z - z_{i-1}}{c}\right) - \exp\left(-\lambda_1^{(i)} \frac{z - z_{i-1}}{c}\right)\right],$$

$$i = 2, 3, \ldots, N; \quad (25)$$

$$\alpha(z_{i-1}, 0) = \frac{1}{p_i} \{p_i(1 - \langle j(z_{i-1}, 0)\rangle) - p_{i-1}[1 - \nu(z_{i-1}, 0)]\} , \quad i = 2, 3, \ldots, N. \quad (26)$$

The considerations analogous to those outlined above can also be used to produce equations for the mean intensity of diffuse radiation.

The results presented here clearly illustrate that the problem of radiative transfer in statistically *inhomogeneous* broken clouds can be successfully solved by using ideas and methods developed for statistically *homogeneous* model.

4. Conclusion

The deterministic radiative transfer theory has a long history, during which simplest one-dimensional radiation models have evolved to multidimensional ones. The evolution of the statistical radiative transfer theory for inhomogeneous clouds is very much the same, and the development stages include (1) construction of cloud models and development of the relevant techniques for radiation calculations; (2) study of the sensitivity of model-derived radiative properties to the variations of cloud parameters; and (3) use of this knowledge in newly developed GCM radiation parameterizations and for improvement of the remote sensing techniques.

Recently, *one-* and *two*-dimensional models, that treat correctly the *horizontal* variability of optical depth τ, have been developed and, using these models, the dependence of radiative properties of the cumulus and stratocumulus clouds on the horizontal distribution of τ has been thoroughly studied. These results were used to develop new parameterizations of the radiation regime of one-layer broken clouds. First attempts have been made to study the radiation regime of two-layer broken clouds. Based on the airborne laser sensing data (IAO SB RAS), the model of stochastic geometry of the top boundary of stratocumulus clouds has been developed. The combined and individual effects of variations of extinction coefficient and height of the top boundary of stratocumulus clouds on the small- and large-scale spatial variations of absorption and vertical and horizontal fluxes of solar radiation have been investigated. It has been shown that the stochastic cloud geometry and inhomogeneous internal structure of clouds can strongly influence the radiative transfer.

Most promising approach to studying the radiative effects of *three-dimensional* cloud fields has been to use the numerical cloud-scale models with explicit microphysics (LES models). This can provide a deeper insight into the physical nature of cloud–radiation interaction and identify 3D cloud properties most strongly influencing the radiative transfer. Of course, the results of these studies will help to formulate and solve remote sensing problems, and to create and test the radiation models.

New parameterizations will be constructed based on realistic 3D cloud models with too large computer demands. It is unlikely, hence, that in the near future, the newly developed radiation parameterizations will cover the full range of cloud variability scales. Therefore, to calculate the statistical radiative characteristics of inhomogeneous clouds, it is necessary to construct few-parameter cloud models and develop approximate methods based on the analytical averaging of stochastic transfer equation. The existing statistically *homogeneous* model of broken clouds takes into account only *horizontal* inhomogeneity of a cloud field (the random amount, positions, sizes, and base shapes of vertically homogeneous clouds). The developed statistically inhomogeneous model of broken clouds is a logical development of the statistically *homogeneous* model and allows one to take into account both the horizontal and vertical cloud structure. This statistically *inhomogeneous* model, as well as the equations for mean intensity of unscattered radiation based on it, has been an important milestone in the development of statistical transfer theory.

The development and validation of new cloud models and improved parameterizations cannot be advanced without experimental data on the probabilistic properties of three-dimensional cloud fields and their radiative properties. This information can be obtained from *integrated* and *simultaneous* measurements. The processes in inhomogeneous cloud and radiative fields are presently an area of an active theoretical and experimental research. Hopefully, even more considerable and impressive progress will be achieved in the development of statistical transfer theory in the near future.

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