MEASUREMENT OF THE TURBULENT ENERGY DISSIPATION RATE WITH A SCANNING DOPPLER LIDAR

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We propose here a technique for determining the turbulent energy dissipation rate from Doppler lidar data. The technique has been used for reconstructing the dissipation rate profile to a height of 650 m.

INTRODUCTION

The known methods for determination of the turbulence kinetic energy dissipation rate ε_T in the atmosphere are based on the relationships, resulting from the fundamental laws of the turbulent energy transformation in the inertial interval of scales of a heterogeneous medium, established by A.N. Kolmogorov and A.M. Obukhov. In this case the data on the dissipation rate can be extracted from the measurements of structural functions or the wind velocity fluctuation spectra, which are determined by the Kolmogorov–Obukhov "2/3" law in the inertial interval.[1] In the case of time structural functions or spectra we follow the hypothesis of "frozen" turbulence.[2]

Remote sensing, by Doppler lidars,[3–10] has opened new opportunities for a detailed study of the atmospheric dynamic processes as compared with the commonly used airborne devices or devices installed on meteorological masts.[1,2,11,12] Thus, for example, from the Doppler spectral width of the lidar return one can determine the turbulent energy dissipation rate at certain altitude[3,4,7,10] when the longitudinal size of a sounded volume does not exceed the maximum size of the turbulent inhomogeneities in the inertial interval. However, in the case of a cw Doppler lidar the use of such a method has a restriction on the height of sounding because with the path length increase the longitudinal size of the volume sounded increases[13] and can exceed the maximum size of turbulent inhomogeneities in the inertial interval. The use of the methods for determining ε_T from time structural functions and spectra measured with a Doppler lidar[10] is not always possible due to violation of the conditions of applicability of the Taylor hypothesis on turbulence "freezing".

In this paper we propose a technique for measuring the dissipation rate of turbulent energy with a cw Doppler lidar capable of making conic scanning. In this case, as well as in the use of the method of determining the dissipation rate from the Doppler spectrum width, the turbulence "freezing" hypothesis is not needed, but in contrast to the above method there are no any restrictions on the sounding altitude.

SOUNDING GEOMETRY AND BASIC RELATIONS

Figure 1 shows the sounding geometry with a ground-based cw Doppler lidar with conic scanning. The lidar is at the center of the Cartesian coordinates $\mathbf{r} = \{z, x, y\}$. The sounding beam, focused at the distance R from the lidar, is inclined at an angle φ to the horizontal plane and rotates about the vertical axis z with the angular rate ω_0 forming the cone with the height $h = R\cos\varphi$ and the radius at its base $a = R\cos\varphi$. The estimates of the radial speed $V_D(\theta)$, in the direction of the azimuth angle θ averaged over the sounding volume formed by a coherent lidar in the vicinity of the focus, were performed from Doppler spectra of lidar return measured in equal intervals t_0.

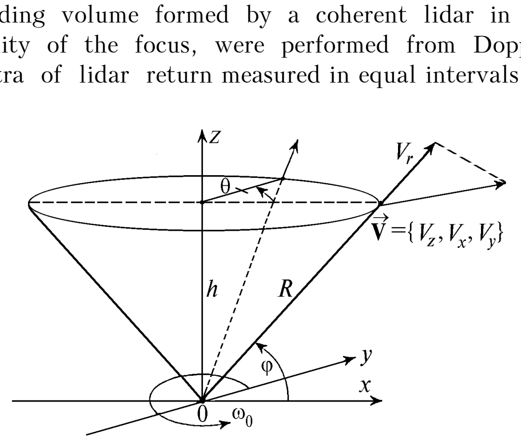

FIG. 1. Geometry of lidar with conical scanning.

The speed V_D is expressed by the expression[9]

$$V_D(\theta) = \int\limits_0^\infty \mathrm{d}z'\, Q_s(z')\, V_r(z', \theta), \qquad (1)$$

where z' is the distance from the lidar to an arbitrary point at the beam axis; $Q_s(z') = \{\pi\, k\, a_0^2\, [(1 - z'/R)^2 + z'^2/(ka_0^2)^2]\}^{-1}$ is the function characterizing the spatial resolution; a_0 is the beam radius in the plane of a transceiving telescope; $k = 2\pi/\lambda$, λ is the wavelength; V_r is the projection of the wind velocity vector $\mathbf{V}(\mathbf{r}) = \{V_z(\mathbf{r}), V_x(\mathbf{r}), V_y(\mathbf{r})\}$ on the beam axis (radial velocity). In accordance with the sounding geometry, the connection between V_r and $\mathbf{V}$ is:

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$$V_r(z', \theta) = \mathbf{S}(\theta)\,\mathbf{V}(z'\,\mathbf{S}(\theta))\ , \qquad (2)$$

where

$\mathbf{S}(\theta) = \{\sin\varphi,\ \cos\varphi\,\cos\theta,\ \cos\varphi\,\sin\theta\}$.

We assume that the wind velocity field is statistically homogeneous, that is, the ensemble average expression $\langle \mathbf{V}(\mathbf{r})\rangle = \langle \mathbf{V}\rangle$ does not depend on the coordinates. Then from Eq. (1), after averaging, we obtain:

$$\langle V_D(\theta)\rangle = \langle V_r(\theta)\rangle \int\limits_0^\infty \mathrm{d}z\, Q_s(z). \qquad (3)$$

For an acceptable spatial resolution the sounding range R must satisfy the condition $R \ll ka_0^2$ (the near zone of diffraction). When fulfilling the above condition, the second multiplier in Eq. (3) equals unity, and the longitudinal size of a sounding volume $\Delta z = \int\limits^\infty \mathrm{d}z\, Q_s(z)/Q_s(R)$ is described by the formula[9]:

$$\Delta z = \frac{\lambda}{2}\frac{R^2}{a_0^2}. \qquad (4)$$

From Eqs. (2) and (3) we derive the sinusoidal dependence of the mean value of the Doppler velocity on the azimuthal angle θ:

$$\langle V_D(\theta)\rangle = \langle V_r(\theta)\rangle = \mathbf{S}(\theta)\,\langle \mathbf{V}\rangle \equiv \sin\varphi\,\langle V_z\rangle + \cos\varphi\,\cos\theta\,\langle V_x\rangle + \cos\varphi\,\sin\theta\,\langle V_y\rangle. \qquad (5)$$

The estimate of the mean value of the Doppler velocity $\hat{V}_D(\theta)$ from the data on $V_D(\theta_i)$ $(i = 1, 2,\ldots,n)$ measured for one complete scan is derived using the fitting of $V_D(\theta_i)$ by the method of least squares to the relationship:

$$\hat{V}_D(\theta) = \mathbf{S}(\theta)\,\hat{\mathbf{V}}. \qquad (6)$$

where, at a sufficiently large n, the estimated components of wind velocity $\hat{\mathbf{V}} = \{\hat{V}_z,\ \hat{V}_x,\ \hat{V}_y\}$ are described by the expression[6]

$$\hat{\mathbf{V}} = \frac{1}{2\pi}\int\limits_0^{2\pi} \mathrm{d}\theta\, V_D(\theta)\,\mathbf{A}(\theta); \qquad (7)$$

$$\mathbf{A}(\theta) = \left\{\frac{1}{\sin\varphi},\ \frac{2\cos\theta}{\cos\varphi},\ \frac{2\sin\theta}{\cos\varphi}\right\}.$$

Fluctuations of such an estimate $\hat{V}_D - \langle\hat{V}_D\rangle$ are mainly due to the turbulent eddies, whose dimensions exceed the diameter of the scanning cone base, and the deviations from the estimate of the mean value $\tilde{V}_D(\theta) = V_D(\theta) - \hat{V}_D(\theta)$ are, on the contrary, due to smaller eddies.

STRUCTURAL FUNCTION OF WIND VELOCITY

The mean square of the difference in deviations $\tilde{V}_D(\theta)$ measured at the angles θ_1 and θ_2 is

$$D(\theta_1, \theta_2) = \langle[\tilde{V}_D(\theta_1) - \tilde{V}_D(\theta_2)]^2\rangle \qquad (8)$$

(structural function of wind velocity). After cumbersome rearrangements with the use of Eqs. (1)–(3) and (5)–(7) it can be represented as

$$D(\theta_1, \theta_2) = (2\pi)^{-2} \int\!\!\int\limits_0^{2\pi} \mathrm{d}\theta_3\, \mathrm{d}\theta_4$$

$$\int\!\!\int\limits_0^\infty \mathrm{d}z_1\, \mathrm{d}z_2\, Q_s(z_1) Q_s(z_2)\, F\{D_{ij}(z_k\mathbf{S}(\theta_l) - z_p\mathbf{S}(\theta_m))\}, \qquad (9)$$

where F is the linear function of the tensor components

$D_{ij}(\mathbf{r}) = \langle[V'_i(\mathbf{r}_0 + \mathbf{r}) - V'_i(\mathbf{r}_0)]\,[V'_j(\mathbf{r}_0 + \mathbf{r}) - V'_j(\mathbf{r}_0)]\rangle$;

$V'_i = V_i - \langle V_i\rangle$; $i, j = z, x, y$; $k, p = 1, 2$;

$l, m = 1, 2, 3, 4$.

For locally isotropic turbulence the tensor D_{ij} can be expressed in terms of the longitudinal structural function $D_{LL}(r)$ as[2]

$$D_{ij}(\mathbf{r}) = D_{LL}(r)\delta_{ij} + \frac{r}{2}\frac{\mathrm{d}D_{LL}(r)}{\mathrm{d}r}\left[\delta_{ij} - \frac{r_i r_j}{r^2}\right], \qquad (10)$$

where $D_{LL}(r)$ in the inertial interval of turbulence $r \ll L_V$ (L_V is the outer scale of turbulence) is described by the Kolmogorov formula

$$D_{LL}(r) = C\,\varepsilon_T^{2/3}\, r^{2/3}, \qquad (11)$$

$r = |\mathbf{r}|$, $\delta_{ij} = 1$ at $i = j$ and $\delta_{ij} = 0$ at $i \neq j$, $C \approx 2$ is the universal Kolmogorov constant, ε_T is the turbulent energy dissipation rate.

Having substituted Eqs. (10) and (11) in Eq. (9) and passing to the limit at small angles $|\theta_1 - \theta_2| \ll \pi/2$, we find

$$D(\theta_1 - \theta_2) = C\,\varepsilon_T^{2/3} \int\!\!\int\limits_0^\infty \mathrm{d}z_1\, \mathrm{d}z_2\, Q_s(z_1)\, Q_s(z_2) \times$$

$$\times\{[(z_1 - z_2)^2 + (\theta_1 - \theta_2)^2\, z_1\, z_2 \cos^2\varphi]^{1/3} \times$$

$$\times\left[1+\frac{1}{3}\,\frac{(\theta_1-\theta_2)^2 z_1 z_2 \cos^2\varphi}{(z_1-z_2)^2+(\theta_1-\theta_2)^2 z_1 z_2 \cos^2\varphi}\right]-$$

$$-\,|z_1-z_2|^{2/3}\Big\}. \qquad (12)$$

at $\Delta z \to 0$ Eq. (12) reduces to the expression

$$D(\theta_1-\theta_2)=\frac{4}{3}\,C\,\varepsilon_T^{2/3}\,(|\theta_1-\theta_2|\;R\cos\varphi)^{2/3} \qquad (13)$$

for the transverse structural function of wind velocity.[2] From that it follows that the formula (12) can be used only on condition that the sector arc length in the scanning cone base $|\theta_1-\theta_2|R\cos\varphi$ does not exceed the size of the inertial interval, i.e.

$$|\theta_1-\theta_2|\;R\cos\varphi \ll L_V. \qquad (14)$$

It should be noted that at small longitudinal dimensions of the sounding volume Δz the situation may occur when the radial velocity averaging over time t_0 should be taken into account. However, in the experiments made by the authors, the condition $\Delta z \gg \omega_0\, t_0\, R\cos\varphi$ can be realized and therefore this averaging will be neglected.

Thus, from the results of measurements of structural function $D(\theta_1-\theta_2)$ one can estimate, by formula (12), the value of the turbulent energy dissipation rate ε_T. In this case the only limitation is the condition (14).

EXPERIMENT

The procedure of the Doppler lidar measurements of the altitude profile $\varepsilon_T(h)$ at conical scanning is as follows. After focusing the beam at the distance R and at the angle of elevation φ, the conical scanning by a laser beam is performed (see Fig. 1), during which the Doppler spectra of laser returns are measured every 50 ms. The time of one scan is 7 s. Similar measurements at different R and φ yield the data arrays, related to the corresponding altitudes $h = R\sin\varphi$. For averaging over the ensemble of square difference $\tilde V_D(\theta_1)-\tilde V_D(\theta_2)$ aimed at estimating the structure function $D(\theta_1,\theta_2)$, the repeated scannings are needed for every altitude. Such repetitions were performed both at continuous scaning and after each measurement cycle at all altitudes h.

From the data of a single scan the values $V_D(\theta_j)$ were determined. Based on the above values and using the method of least squares we estimate the mean values of the Doppler velocity $\hat V_D(\theta_j)$ and after determining the differences $V_D(\theta_j)-\hat V_D(\theta_j)$ by formula (8) the structure function $D(\theta_1,\theta_2)$ was calculated for small angles $|\theta_1-\theta_2|$. From such functions using the formula (12) we determined the values of the turbulent energy dissipation rate for every altitude.

Figure 2 presents an example of the estimates of $V_D(\theta)$ [by dots], $\tilde V_D(\theta)$ [dashed line] and the difference $\tilde V_D(\theta)=V_D(\theta_j)-\hat V_D(\theta_j)$ [two solid curves in the ranges of positive and negative values of $V_D(\theta_j)$] from the experimental data obtained at one scan. In that and another ranges there are 60 points with almost equal angular distances between the points $\Delta\theta = 2.7°$.

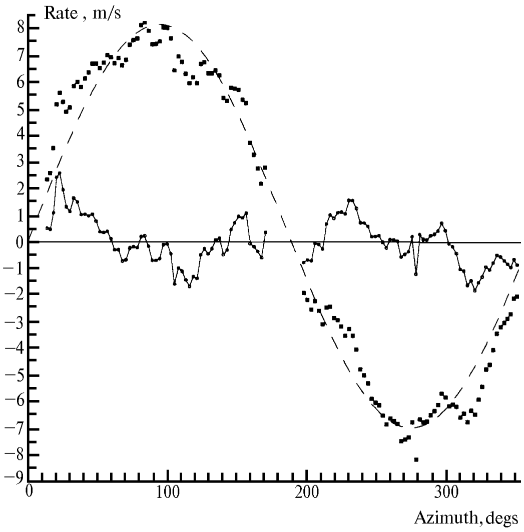

FIG. 2. Dependence of V_D (dots), $\hat V_D$ (dashed curve) and $\tilde V_D$ (solid curves) on the azimuthal angle θ.

The estimates of $V_D(\theta_j)$ are accurate to the errors $V_n(\theta_j)$ due to noise; the mean value of the errors being equal to zero. They correlate neither with the radial velocity $V_r(z',\theta_j)$ nor with each other ($\langle V_n(\theta_j)V_n(\theta_i)\rangle = 0$ at $i \ne j$). Therefore, the measured structure function $D(\theta_1,\theta_2)$ contains, along with the information component (12), an additive contribution $2\sigma_n^2$, where $\sigma_n^2=\langle V_n^2\rangle$ is the variance of the noise component of the estimated velocity. However, as the experiments have shown, at very large signal-to-noise ratio this contribution can be neglected.

From the data obtained using N full scans, for the angular interval of 180° we have $2N$ successions $\tilde V_D(j\Delta\theta)$. Using all points of every such a succession, one can obtain $2N$ estimates of the structure function at $|\theta_1-\theta_2| \ll 180°$, and then they can be averaged at the corresponding decrements of the angles $|\theta_1-\theta_2|$. The values ε_T and σ_n^2 are estimated by the formulas

$$\varepsilon_T=\left[\frac{1}{(n-1)}\sum_{k=1}^{n-1}\frac{1}{(n-k)}\sum_{j=1}^{n-k}\frac{[D((j+k)\Delta\theta)-D(j\Delta\theta)]}{[G((j+k)\Delta\theta)-G(j\Delta\theta)]}\right]^{3/2}; \qquad (15)$$

$$\sigma_n^2=\frac{1}{2n}\sum_{j=1}^{n}\,[D(j\Delta\theta)-\varepsilon_T^{2/3}\,G(j\Delta\theta)], \qquad (16)$$

where $D(j\Delta\theta)$ are the experimental values of the structure function; $G(j\Delta\theta)$ is the factor at $\varepsilon_T^{2/3}$ in the right-hand side of the formula (12). According to the requirement of the theory applicability, $n\Delta\theta \ll \pi/2$ and $n\Delta\theta R\cos\varphi \ll L_V$, in our experiment the number $n = 10$.

RESULTS OF THE EXPERIMENT

The experiment performed using a CO_2 Doppler lidar (Institute of Optoelectronics of German Aerospace Agency) was carried out in a narrow alpine valley not far from Garmisch–Partenkirchen (FRG). The valley width is about 1 km or 1.5 km. When performing measurements on October 12, 1993, we observed sharp changes in the dynamic state of the atmosphere over measurement site. In particular, at 14:00 PM a fast change of wind direction to an opposite one took place. Then during one hour we observed a relatively stationary state of a strongly turbulent air flow. In this case the wind velocity at altitudes $h \geq 150$ m exceeded 10 m/s and was 14–15 m/s. During that hour we made six measurements at altitudes $h = 50$ m ($R = 100$ m, $\varphi = 30°$); $h = 150$ m ($R = 300$ m, $\varphi = 30°$); $h = 250$ m ($R = 500$ m, $\varphi = 30°$); $h = 350$ m ($R = 700$ m, $\varphi = 30°$); $h = 450$ m ($R = 636$ m, $\varphi = 45°$); $h = 550$ m ($R = 778$ m, $\varphi = 45°$); $h = 650$ m ($R = 919$ m, $\varphi = 45°$) to which the following longitudinal dimensions of a sounded volume corresponded $\Delta z = 9.2$; 83; 230; 450; 372; 557; and 777 m. In every such measurement at $h = 50$ m we made three scannings, at $h = 150$ m two scannings were carried out and one scaning was performed at other altitudes.

The measurement data are presented in Fig. 3 and Fig. 4 by dots, namely, the experimental values of the structure function $D(j\Delta\theta)$ at $h = 50$ m ($\Delta z = 9.2$ m) and $h = 550$ m ($\Delta z = 557$ m), respectively. Relative errors of these estimates of the structure functions were 9% and 13% respectively. Bars in the figure show the 90% confidence intervals. Having used Eqs. (15) and (16) and these experimental data we have the estimates $\varepsilon_T = 0.039$ m^2/s^3; $\sigma_n^2 = 0.04$ м^2/c^2 for $h = 50$ m and $\varepsilon_T = 0.012$ m^2/s^3, $\sigma_n^2 = 0{,}09$ m^2/s^2 for $h = 550$ m $\sigma_n^2 = 0.09$ m^2/s^2 for $h = 550$ m. Figures 3 and 4 present the results of the calculation by the formula (12) with regard for the noise component $2\sigma_n^2$. From the figures we notice that the experimental and theoretical dependences of the structural function on the azimuthal angle are in agreement. The same good agreement of the theory and the experiment is observed for the other altitudes of sounding. The results of calculation by the formula (13) for the structural function in the case of point measurements are given in Figs. 3 and 4 as dashed lines. A comparison of dashed lines with solid curves shows the degree of the influence of spatial averaging along a beam axis on the characteristics being considered.

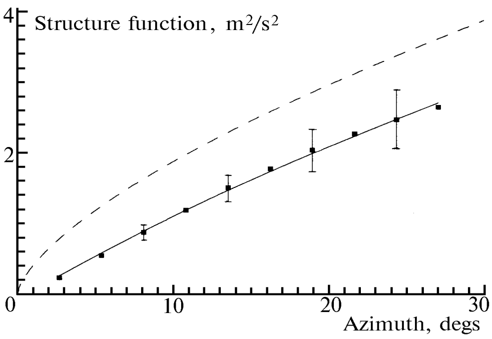

FIG. 3. Structure function of wind velocity measured with the scanning Doppler lidar at $h = 50$ m *and* $\Delta z = 9.2$ m. *Dots denote the experiment, solid curve is for the theory, and dashed curve is for calculation by the formula (13).*

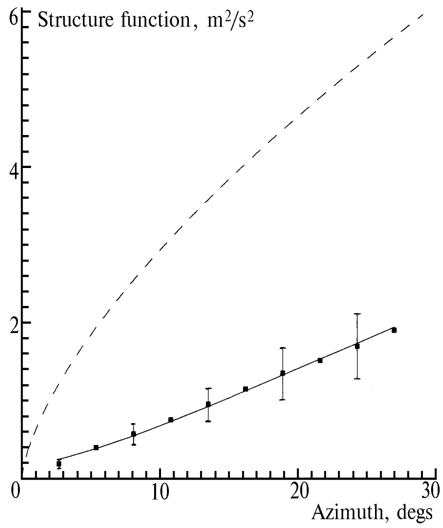

FIG. 4. Structure function of wind velocity measured with the scanning Doppler lidar at $h = 550$ m *and* $\Delta z = 557$ m. *Dots show the experimental data, the theory is presented by solid curve, the calculations by the formula (13) are shown by dashed curve.*

Figure 5 shows the profile of the turbulent energy dissipation rate reconstructed from the data of the scanning Doppler lidar. It is evident that, on the whole, the turbulent energy dissipation rate decreases with altitude what qualitatively agrees with the known theoretical[14] and experimental[1] results. It is probable that "the inversion" observed in this case at altitudes $h = 250$ m and $h = 550$ m is due to the statistical errors because of incomplete averaging of the structure function fluctuations estimated. On the average, relative error of estimates of the structure functions is about 13%. Taking into account the fact that the estimate of the dissipation rate is proportional to $D^{3/2}(\theta)$, its relative error is 20%. Relatively large absolute values of ε_T value, in our opinion, are due to the complex relief of the mountain valley and specific properties of weather conditions during the experiment. It should be noted that starting from $h = 250$ m and higher the longitudinal dimension of a volume Δz sounded greatly exceeds the maximum size of turbulent inhomogeneities in the inertial interval and becomes comparable and even larger than the outer scale of turbulence, L_V. In contrast to the method of determining ε_T from the Doppler spectrum width, where the condition $\Delta z \ll L_V$ is required, in the approach proposed it is sufficient to impose a weaker inequality (14).

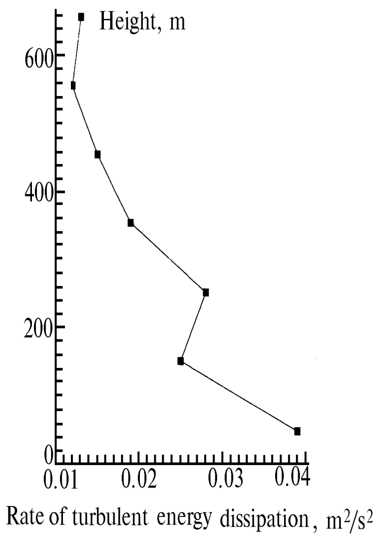

FIG. 5. The reconstructed profile of the turbulent energy dissipation rate.

CONCLUSION

This paper presents the results of theoretical and experimental investigations of the wind velocity structure function, measured with a cw Doppler lidar at conic scanning. It has been shown that due to an essential spatial averaging of the wind velocity fluctuations over the volume sounded a considerable slowing down of the structure function increase is observed with the growth of spatial separation (azimuthal angle) of the observation points as compared with the case of point measurements of the structure function when the “2/3” Kolmogorov-Obukhov law holds. The theoretical and experimental results obtained are in a good agreement. Based on the obtained relationships for the spatial structure function of wind velocity measured with the scanning lidar, the method is proposed for determining the turbulent energy dissipation rate from the Doppler lidar data. Using this method, the dissipation rate profile has been reconstructed up to the height $h = 650$ m. The proposed method is free from the limitations on the value of the longitudinal size of the volume sounded characteristic of the known methods what essentially extends the applicability range of Doppler lidars to investigating the turbulence in the atmospheric boundary layer.

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