LIMITING FORM OF THE COHERENCE FUNCTION OF A FIELD IN A LAYERED NONUNIFORM MEDIUM

Yu.P. Akulinichev and A.M. Golikov

Tomsk Institute of Automatic Control Systems and Radioelectronics

Received April 13, 1990

It is shown in the Markovian approximation, that as the optical thickness of a layer of nonuniformities increases the spatiotemporal coherence function of the random Green's function becomes Gaussian. It is shown that the form of this function is determined by the geometric parameters of the layer and the component of the wind velocity that is perpendicular to the propagation path. Methods for measuring these parameters by remote sensing in the case of isotropic turbulence are analyzed.

Let the propagation of waves in a layered nonuniform medium in a direction nearly along the Oz axis be described by a parabolic equation.[1]

We assume that the fluctuations of the dielectric constant of the medium are statistically homogeneous in the xoy plane and in time and quasi homogeneous along the oz axis, and that they are characterized by the spatiotemporal correlation function

$$\Psi_\varepsilon(\rho,\ \Delta z,\ \tau;\ z) = \overline{\tilde{\varepsilon}(x_1,\ y_1,\ z_1,\ t_1)\ \tilde{\varepsilon}(x_2,\ y_2,\ z_2,\ t_2)} = \Psi_{\varepsilon 0}(\rho - v(z)\tau,\ \Delta z,\ \tau;\ z), \tag{1}$$

where

$$\rho = (\Delta x,\ \Delta y) = (x_1 - x_2,\ y_1 - y_2);$$

$$\tau = t_1 - t_2, \quad z = (z_1 + z_2)/2$$

the vector $v(z)$ is the component of the wind where velocity that is perpendicular to the propagation path; $\Psi_{\varepsilon 0} = (\rho,\ \Delta z,\ \tau;\ z)$ is the spatiotemporal correlation function of the fluctuations of the field ε in the absence of wind and is obtained by averaging over an ensemble.

Assuming that along the path the nonuniformities are much smaller than the path length $2L$, we employ the Markovian approximation[1] and we write the normalized spatiotemporal coherence function of the random Green's function $G(\rho_2,\ z_2;\ \rho_1,\ z_1;\ t)$ as a function of the spatial separation across the path as

$$\Gamma(\Delta\rho_2,\ \Delta\rho_1,\ \tau) = \overline{G(\rho_2,\ L;\ \rho_1,\ -L;\ t)} \times \overline{G^*(\rho_2 - \Delta\rho_2,\ L;\ \rho_1 - \Delta\rho_1,\ -L;\ t-\tau)} = \exp\left\{ - \int_{-L}^{L} \sigma_0(z) H\left[\left(1 - \frac{z}{L}\right)\frac{\Delta\rho_1}{2} + \left(1 + \frac{z}{L}\right)\frac{\Delta\rho_2}{2} - v(z)\tau,\ \tau;\ z\right] dz + i\varphi\right\}, \tag{2}$$

where

$$\varphi = \frac{k}{2L}\left(|\rho_1 - \rho_2|^2 - |\rho_1 - \rho_2 - \Delta\rho_1 + \Delta\rho_2|^2\right);$$

$$\sigma_0(z) = \frac{k^2}{4}\int_{-\infty}^{\infty} \Psi_\varepsilon(0,\ \Delta z,\ 0;\ z)\ d\Delta z$$

is the extinction coefficient; and,

$$H(\rho,\ \tau;\ z) = \frac{k}{4\sigma_0(z)} \times \int_{-\infty}^{\infty}\left[\Psi_\varepsilon(0,\ \Delta z,\ 0;\ z) - \Psi_\varepsilon(\rho,\ \Delta z,\ \tau;\ z)\right] dz$$

is the normalized structure function of the phase, i.e.,

$$H(0,\ 0;\ z) = 0, \quad H(\infty,\ \tau,\ z) = H(\rho,\ \infty;\ z) = 1.$$

We shall study the limiting case when the optical thickness of the layer

$$\nu = \int_{-L}^{L} \sigma_0(z)\ dz \gg 1, \tag{3}$$

so that the forward multiple scattering components predominate, the nth order scattering phase function is the n-fold convolution of the cross section of the spatial spectrum of the fluctuations $\tilde{\varepsilon}$ (Ref. 2) and therefore the region of small values of ρ is important.

For small values of ρ and τ and any real spectra of nonuniformities the parabolic approximation is applicable for the function $H(\rho,\ \tau;\ z)$:

$$H(\rho,\ \tau;\ z) = \frac{1}{2}\left[\rho B(z)\tilde{\rho} + B_\tau(z)\tau^2\right] + \dots, \tag{4}$$

0235-6880/90/10 969-3 $02.00

where $\tilde{\rho}$ is a column vector; $B(z)$ is a matrix of second derivatives with respect to Δx and Δy at $\rho = 0$; and, for convenience, the ox and oy axes can always be chosen so that the matrix B is diagonal.

In particular, for homogeneous and isotropic turbulence the Kolmogorov-Obukhov 2/3 law is widely used. In this case, we have[1]

$$\sigma_0(z) \approx 0.0091\, k^2 C_\varepsilon^2 L_0^{5/3},$$

$$H(\rho,\ 0,\ z) \approx \begin{cases} l_0^{-1/3} L_0^{-5/3} \rho^2, & \rho \le l_0, \\ (\rho/L_0)^{5/3}, & l_0 < \rho \le L_0, \\ 1, & \rho > L_0, \end{cases}$$

$$B(z)_{xx} = B(z)_{yy} = 2 l_0^{-1/3} L_0^{-5/3}, \quad B(z)_{xy} = 0.$$

After substituting Eq. (4) we obtain for the coherence function (2) the Gaussian form

$$\Gamma(\Delta\rho_2,\ \Delta\rho_1,\ \tau) = \exp\Big[p W_0 \tilde{p} + c W_2 \tilde{c} + \tau^2 (q + s) - 2\tau (p \tilde{v}_0 - c \tilde{v}_1) + i\varphi\Big], \tag{5}$$

where

$$2p = \Delta\rho_1 (I - \bar{L}^{-1} Z_0) + \Delta\rho_2 (I + \bar{L}^{-1} Z_0),$$

$$2c = \Delta\rho_1 - \Delta\rho_2,$$

$$W_n = L^{-1} \int_{-L}^{L} \sigma_0(z) (zI - Z_0)^n B(z)\, dz,$$

$$v_n = L^{-n} \int_{-L}^{L} \sigma_0(z) v(z) (zI - Z_0)^n B(z)\, dz,$$

$$q = \int_{-L}^{L} \sigma_0(z) B_\tau(z)\, dz, \quad s = \int_{-L}^{L} \sigma_0(z) v(z) B(z) \tilde{v}(z)\, dz,$$

I is the unit matrix; Z_0 is a diagonal matrix, whose elements are fixed by the condition $W_1 = 0$ (in the case $B(z)_{xx} = \alpha B(z)_{yy}$ they are both equal to the coordinate z_0 of the center of the layer).

Thus in the limiting case the coherence function of the field is completely determined by the following parameters of the layer (if the separation vectors $\Delta\rho_1$ and $\Delta\rho_2$ are collinear, then the quantities appearing in the formula (5) can be assumed to be scalars); the coordinate of the center of the layer z_0 and the relative mean square thickness $\gamma^2 = W_0^{-1} W_2$ along the oz axis; the integrated values of the structure constants in space W_0 and in time q; the mean $\bar{v} = v_0 W_0^{-1}$ and the mean square $\bar{v}^2 = s W_0^{-1}$ of the component of the wind velocity $v(z)$ normal to the propagation path, as well as the average value of its linear part $\bar{v}_1 = v_1 W_0^{-1}$ (the last parameter takes into account the fact that the wind velocity can vary along the propagation path; this is manifested primarily in the fact that it is a linear function of the coordinate z).

With the help of the formula (5) the appropriate methods for estimating different parameters of the layer by remote sensing can be easily analyzed and the most accurate methods can be chosen.

1. To estimate the coordinate of the center of the layer z_0 the correlation coefficient at coincident times ($\tau = 0$) on all possible pairs of crossing propagation paths with $p = 0$ and $c =$ const are measured, and the particular pair on which the coefficient is maximum is chosen; then z_0 is the coordinate of the crossing point of these paths.[3,4]

2. The relative mean-square thickness of the layer is numerically equal to the ratio of the intervals of spatial correlation of the signals on parallel ($c = 0$) and crossing ($p = 0$) paths.[4]

3. The quantities z_0 and γ can also be estimated simultaneously after the correlation intervals are measured simultaneously on parallel, crossing, and converging ($\Delta\rho_2 = 0$) (or diverging ($\Delta\rho_1 = 0$)) paths.

4. If the position z_0 and the thickness γ of the layer are not known, then the mean value of the transverse component of the wind velocity $\bar{v}$ is best measured on parallel paths, in contrast to the traditional method of measurement on diverging paths.[5]

5. The most reliable method for estimating the characteristics of the wind $\bar{v}$ and $\bar{v}_1$ is the method of measurement based on the slope of the coherence function $\partial\Gamma(\Delta\rho_1,\ \Delta\rho_2,\ \tau)/\partial\tau$ at the zero point ($\tau = 0$) on parallel and crossing paths, respectively.[6]

In addition, it is very convenient to use the coherence function in the form (5) to estimate the distortions of the field in a medium in the presence of large or spatially separated receiving and transmitting apertures, especially if the Gaussian approximation is employed for them also.[7,8]

In conclusion we shall estimate for the case of homogeneous isotropic turbulence the error in the estimate of some parameters of the layer owing to the fact that the real coherence function (2) is different from its approximation (5). For example, the ratio of the correlation intervals on parallel paths, determined at the 1/e level from the formulas (2) and (5), is equal to $\nu^{-0,1}(L_0/l_0)^{1/6}$ and can reach several units for large values of L_0/l_0.

Significantly better results can be expected when measuring the relative characteristics, in particular, the ratio of the correlation intervals on parallel and crossing paths γ_0. For $L_0 \gg l_0$ from the formulas (2) and (5) we have $\gamma_0 = (3/8)^{3/5} \approx 0.5552$ and $\gamma_0 = \gamma = (1/3)^{1/2} \approx 0.5773$, respectively. The difference is only 3.8%, which is fully acceptable for most practical problems.

REFERENCES

1. S.M. Rutov, Yu.A. Kravtsov, and V.I. Tatarskii, *Introduction to Statistical Radiophysics* (Nauka, Moscow, 1978), Pt. II, 464 pp.
2. A.P. Yakimovich, Izv. Vuz. Radiofiz. **32**, No. 3, 307 (1989).
3. T.S. Wang, S.F. Clifford and G.R. Ochs, Appl. Opt. **13**, 2602 (1974).
4. Yu.P. Akulinichev and A.M. Golikov, Radiotekh.Electron. **32**, No. 8, 2602 (1974).
5. A.A. Semenov and T.I. Arsen'yan, *Fluctuations of Electromagnetic Waves on Paths Near the Ground* (Nauka, Moscow, 1978), 272 pp.
6. E.A Monastyrnyĭ and G.Ya. Patrushev, *"On the correlation analysis of fluctuations of radiation in a randomly inhomogeneous medium,"* Gorkii (1984); VINITI, No. 5927-84.
7. Yu.P. Akulinichev, V.A. But'ko, and A.M. Golikov, in: *Image Transfer in the Earth's Atmosphere*, Tomsk Affiliate of the Siberian Branch of the USSR Academy of Sciences, Tomsk (1988), pp. 87–90.
8. V.B. Fortes, Radiotekh., No. 10, 53–56 (1987).