Optical vortices in inhomogeneous media

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The paper describes the results of investigations in a rapidly progressing area of modern physics — singular optics. Basic properties of optical vortices (wave front dislocations), which are the principal objects of the investigation, are presented. The vortices peculiarities connected with the coherent light propagation in an inhomogeneous medium are determined. Initiation of the vortices is caused by the occurrence of isolated intensity zeros in an optical beam cross section that is the necessary condition of their existence. New theoretical approaches to the problem of dislocations are proposed, which is especially topical due to the necessity of improvement of atmospheric optical systems. An efficiency of adaptive optics functioning is evaluated under conditions of strong intensity fluctuations in the turbulent atmosphere.

The subjects of investigation in the field of propagation of coherent optical radiation in random inhomogeneous media have been formed over last two decades owing to the demands for modern practice. With advances in guided optical systems, a problem arose for studying fine structure of optical fields under conditions of strong intensity fluctuations $I(x, y, z)$ of so-called speckle-fields, since in such fields the objects were revealed disturbing the regular character of surfaces of equal phase $S(x, y, z) = \text{const}$, which are not amenable to the traditional methods of compensation of distortions used in adaptive optics.

Even the first works on dislocations done by D. Nai, M. Berry,[1] N.B. Baranova, B. Ya. Zel'dovich[2] have revealed that the spatial distribution of dislocations reflects global structural peculiarities in the field configuration in heterogeneous media. In many cases the spatial distribution of dislocations is a basis of the field wave pattern.[3–4] Therefore it is necessary to develop theoretical approaches describing in detail the conditions of formation and the spatial dynamics of dislocations in heterogeneous media taking into account different physical aspects of this phenomenon found by native and foreign researchers.[5–12]

The distinction between these aspects is manifested in the variety of names used by researchers for denoting this phenomenon. Apart from the name "dislocations of wave front" resulting from the similarity to the defects of crystal lattice, the terms "phase singularities," "intensity zeros," "optical vortices" are used. The name "optical vortices" denoting the similarity of vector field of a phase gradient at points of intensity zeros to the vortex liquid flow has been widely used recently.

A characteristic property of speckle-fields is the availability of isolated points in the transverse plane where the intensity is reduced to zero, the phase is not determined, and the integration of $\nabla_\perp S(\rho, z)$ over a closed circuit surrounding this point gives nonzero circulation

$$\oint_\Gamma \nabla_\perp S(\rho, z)\, d\mathbf{l} = 2\pi m, \quad (1)$$

where $\rho = \{x, y\}$; m is a topological charge, a positive or negative integer. The behavior of intensity and the Poynting vector in the vicinity of zero point ($m = 1$) is represented in the form

$$I \cong a_x x^2 + a_y y^2 + a_{xy} xy, \quad \mathbf{P}_\perp \cong -\mathbf{e}_x ay + \mathbf{e}_y ax. \quad (2)$$

Whence it follows that at zero point and in its vicinity

$$|\mathbf{P}_\perp| \cong a|\rho|, \quad \text{rot}\, \mathbf{P}_\perp = 2a\, \mathbf{e}_z. \quad (3)$$

Equating the expansion (2) for the intensity to zero, we obtain the equation of degenerate curve of the second order. In this case the invariant D of this curve

$$D = \begin{vmatrix} a_x & a_{xy} \\ a_{xy} & a_y \end{vmatrix} = a^2$$

may be larger or equal to zero. The condition $D > 0$ corresponds to the requirement that the intensity tends to zero at an isolated point $\rho = 0$ and we deal with the screw dislocation. The condition $D = 0$ corresponds to the case of intensity vanishing on the line passing through the point $\rho = 0$. For this case $\text{rot}\mathbf{P}_\perp = 0$, and the phase distribution has no peculiarities typical for the screw dislocation. Such a situation is observed, for example, at radiation diffraction on round or rectangular holes when the intensity becomes zero on closed lines or lines becoming infinite.

As is seen from Ref. 4, the physical reason of occurrence of optical vortices is the field interference from different parts of the aberration wave front. This fact makes possible different approaches to the analysis of the speckle-field behavior. It is evident that the simplest is the numerical analysis of interference field from several (as minimum three) sources of plane and spherical waves.[13,14] A disadvantage of this approach is that it does not allow one to follow the processes of initiation and annihilation of vortices. For these

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purposes the most suitable is the solution of the problem of diffraction for a beam with an aberration (nonparabolic) wave front, Refs. 8, 13, and 15. Optical vortices appear also at nonlinear beam refraction with an originally parabolic front.[16] The most complicated problem is the investigation of a speckle-field behavior in a randomly heterogeneous medium.[17]

The propagation of optical waves through a heterogeneous medium will be considered in the scalar approximation. Note that the scalar formulation of the problem is sufficient to describe the interference and diffraction of waves. Let us assume that the heterogeneous medium occupies a half-space $z > 0$ and the direction of propagation of an incident wave coincides with the direction of the axis z. In this case the scattering by large angles is neglected. Then for a complex amplitude of a monochromatic wave field $U(\rho, z)$ we derive a parabolic wave equation

$$2ik\frac{\partial U}{\partial z} + \Delta_\perp U + k^2\,\tilde{\varepsilon}(\rho, z)\;U(\rho, z) = 0, \qquad (4)$$

where $\Delta_\perp = \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}$ is the Laplacian transverse operator; $k = 2\pi\lambda$ is the wave number corresponding to the wavelength λ; $\tilde{\varepsilon} = \dfrac{\varepsilon - \langle\varepsilon\rangle}{\langle\varepsilon\rangle}$, $\langle\varepsilon\rangle$ is the average over the realization ensemble value of dielectric constant of a medium. In the wave propagation theory Eq. (4) is widely used[1]; its applicability is connected with the requirement of inhomogeneities smoothness of a medium at the wavelength, the condition that the backscattering is small, and with transition to the Fresnel approximation of Green's function.

In particular, in vacuum ($\tilde{\varepsilon} = 0$) the solution of Eq. (4) can be written as

$$U(\rho, z) = \frac{k}{2\pi iz}\int U_0(\rho')\exp\left[\frac{ik(\rho-\rho')^2}{2z}\right]\mathrm{d}\rho', \qquad (5)$$

where $U_0(\rho') = U(\rho', z = 0)$ is the initial condition.

Representing the field as $U(\rho, z) = \{I(\rho, z)\}^{1/2} \times \exp\{iS(\rho, z)\}$, we can write an equivalent to Eq. (4) set of eikonal and transfer equations:

$$2kI^2\frac{\partial S}{\partial z} + I^2\{\nabla_\perp S\}^2 = k^2 I^2\,\tilde{\varepsilon}(\rho, z) + \frac{1}{2}\,I\,\Delta_\perp I(\rho, z) - \frac{1}{4}\{\nabla_\perp I(\rho, z)\}^2; \qquad (6)$$

$$\nabla_\perp\{I(\rho, z)\,\nabla_\perp S\} = -k\frac{\partial I}{\partial z}. \qquad (7)$$

The quantity

$$\mathbf{P}_\perp = I(\rho, z)\;\nabla_\perp S(\rho, z)$$

correct to the constant factor represents a mean over the period value of the Poynting vector transverse component. In this case the longitudinal component $P_z = kI$. For describing the optical wave propagation in a heterogeneous medium we use, as a rule, the numerical schemes of equation solution for a complex amplitude of a light field $U(\rho, z)$ or its statistical moments.[19] As a rule, when solving the propagation problems, Eqs. (6) and (7) are not used directly. The diffraction beam method is widely used, which is based on the following equation[20,21]:

$$\frac{\mathrm{d}^2\rho}{\mathrm{d}z^2} = \frac{1}{2}\nabla_\perp\tilde{\varepsilon}(\rho, z) + \frac{1}{4k^2}\nabla_\perp\left\{\frac{\Delta_\perp I(\rho, z)}{I(\rho, z)} - \frac{1}{2}\frac{[\nabla_\perp I(\rho, z)]^2}{I^2(\rho, z)}\right\}, \qquad (8)$$

where $\rho = \rho(z)$ is the running transverse coordinate of a diffraction beam derived from Eq. (6). Equation (8) relates the second derivative of a diffraction beam coordinate with the intensity. A diffraction beam (streamline of energy) is the integral curve of the field of directions of the Poynting vector. The direction of tangent at each point of the integral curve coincides with the direction of the vector of energy current density and simultaneously with the normal direction to the wave front at a given point. According to this determination, the differential equation of the energy flow trajectory (Fig. 1) is of the form

$$\frac{\mathrm{d}\rho}{\mathrm{d}z} = \frac{\mathbf{P}_\perp(\rho, z)}{P_z(\rho, z)} = \frac{1}{k}\nabla_\perp S(\rho, z). \qquad (9)$$

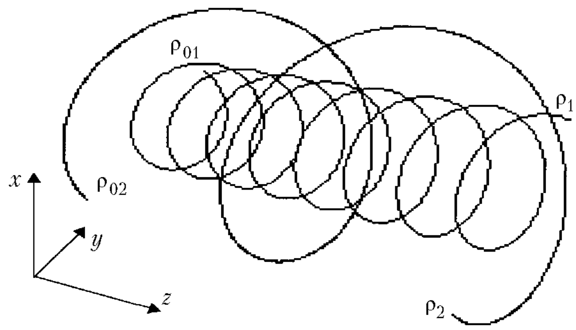

Fig. 1. Behavior of the energy flow lines in the vicinity of dislocation.

The phase sets natural parametrization on a given line[3] since it is related with the wavelength by a simple differential equality

$$\mathrm{d}S = k\,|\theta|\,\mathrm{d}l\,,$$

where l is the line length; θ is the unit vector, whose direction coincides with the direction of the Poynting vector. In the framework of paraxial approximation for changing the phase along the line of energy flow, the following equation

$$S = k\int_{z_0}^{z_1}\mathrm{d}z\left[1 + \frac{1}{2}\left(\frac{\mathrm{d}\rho}{\mathrm{d}z}\right)^2\right] = k\int_{z_0}^{z_1}\mathrm{d}z\left[1 + \frac{1}{2}\,(\mathbf{P}_\perp/kI)^2\right]$$

is derived.

The phase difference between the two arbitrary points lying in one plane, normal to the axis of radiation propagation, can be calculated as follows:

$$\Delta S_{\rm pl} = \int\limits_{\rho_1}^{\rho_2} d\mathbf{l}_\perp \nabla_\perp S(\rho, z_0) = \int\limits_{\rho_1}^{\rho_2} I^{-1}[P_x\, dl_x + P_y\, dl_y],$$

where $d\mathbf{l}_1 = \{dl_x, dl_y\}$ is the element of the line connecting the points ρ_1 and ρ_2; $\mathbf{P}_\perp = \{P_x, P_y\}$ is the transverse component of the Poynting vector. Note that for the speckle-field this phase difference depends on selection of the line connecting these points. For the two different lines the phase difference varies by even number of π, if the lines do not intersect the dislocations, and varies by odd number of π if one of the lines intersects one dislocation.

The following phase calculations have been made. In the plane $z = z_0$, the two points ρ_{01} and ρ_{02} were selected. The phase difference between the points calculated in this plane along straight line connecting the points equals to zero. We calculated the trajectories of streamlines coming from the above points and intersecting the plane $z = z_1$ at points ρ_1 and ρ_2 as well as the advances along these lines between the planes $z = z_0$ and $z = z_1$ (S_1 is the phase advance along the first line, S_2 is the phase advance along the second line). Next the phase difference $\Delta S_{\rm pl}$ was calculated between the points ρ_1 and ρ_2 in the plane $z = z_1$ along the direct line (not intersecting the dislocation). For any pairs of lines we obtained the following relationships:

$$S_1 - S_2 = \Delta S_{\rm pl} + 2\pi n,$$

where n is the integer.

When approaching the initial point ρ_{01} to the dislocation, the number of turns of energy streamline around the dislocation between the fixed planes increased. With increasing the number of such turns, n grows, and when ρ_{01} tends to the dislocation point, n tends to infinity.

In each plane, normal to the axis of propagation, the streamlines of the vector field $\mathbf{P}_\perp$ can be introduced, for which the equation is of the form

$$\frac{dx}{dy} = I(\rho, z)\,\frac{\partial S(\rho,z)}{\partial x} \Big/ \left[I(\rho, z)\,\frac{\partial S(\rho,z)}{\partial y} \right]. \quad (10)$$

At the points, where the intensity goes to zero, the conditions of unambiguity and continuity of the right–hand sides of Eqs. (9) and (10) and their first derivatives are not fulfilled, therefore these points in the plane (x, y) (or the lines in space (x, y, z)) are the particular points of the vector field $\mathbf{P}_\perp$.[22,23] When the number of such points is greatly increased, that is typical for the conditions of saturated intensity scintillations,[6] the description of the wave field with the use of diffraction beams is just problematic. This depends upon the fact that Eq. (8) is derived from Eq. (6) due to the effect of the operator $\nabla_\perp$ on Eq. (6). In this case in a rigorous expression

$$\nabla_\perp(\nabla_\perp S)^2 = 2(\nabla_\perp S \nabla_\perp)\nabla_\perp S + 2\nabla_\perp S \times \mathrm{rot}(\nabla_\perp S)$$

the second component is omitted. However, the second component equals zero only in the case when the phase gradient field is potential, i.e., in the absence of dislocations.

Therefore in Refs. 24 and 25 we propose a "hydrodynamic" approach for description of wave fields with wave front dislocations using a structural similarity of the vector field $\nabla_\perp S$ and the vector field of rate of potential plane flow of compressible fluid. In hydrodynamics[26] the concept of "circular flow from an isolated vortex" exists. The vortex vector of such a flow equals zero everywhere except for a point of singularity where it tends to infinity. In this case the total rate of fluid flow is the sum of the potential rate and the superposition of circular rates from the point vortices. The vector field of the wave phase gradient with dislocations possesses these properties and allows the introduction based on the property (1) of the vortex vector (rotor) of the phase gradient:

$$\mathbf{e}_z\Omega(\xi, \eta, z) = \mathrm{rot}\{\nabla_\perp S(\rho, z)\} = \mathbf{e}_z 2\pi m\delta(x - x_{\rm d}, y - y_{\rm d}),$$

where m is the positive or negative integer; $\delta(x, y)$ is the Dirac delta-function. In this case for a set of dislocations the following representation of phase gradient

$$\nabla_\perp S(\rho, z) = \nabla_\perp S_\partial(\rho, z) -$$
$$-\frac{1}{2}\iint\limits_{-\infty}^{\infty} d\xi\, d\eta\, \Omega(\xi, \eta, z)\,\frac{\mathbf{e}_x(y-\eta) - \mathbf{e}_y(x-\xi)}{(x-\xi)^2 + (y-\eta)^2}, \quad (11)$$

can be written. $\nabla_\perp S_\partial(\rho, z)$ is the divergent part of gradient satisfying the condition

$$\mathrm{rot}\{\nabla_\perp S_\partial(\rho, z)\} = 0 \,. \quad (12)$$

The relationships (6) and (7), characteristics of the vector field $\nabla_\perp S(\rho, z)$, as well as the rules of differential transforms of vector fields have made it possible to obtain instead of the eikonal equation (6) the evolution equation of the phase gradient rotor

$$\mathbf{e}_z\frac{\partial}{\partial z}\Omega(\rho, z) =$$
$$= \frac{1}{4k}\nabla_\perp \times \left\{ \nabla_\perp \left[\frac{\Delta_\perp I(\rho, z)}{I(\rho, z)} - \frac{1}{2}\frac{[\nabla_\perp I(\rho, z)]^2}{I^2(\rho, z)} \right] \right\}. \quad (13)$$

Having introduced into consideration the density of "phase sources"

$$Q(\rho, z) = \mathrm{div}\,\{\nabla_\perp S_\partial(\rho, z)\},$$

we can write the following equation:

$$k\frac{\partial}{\partial z}Q(\rho, z) + [\Omega(\rho, z)]^2 - \frac{\partial S}{\partial x}\frac{\partial}{\partial y}\Omega(\rho, z) + \frac{\partial S}{\partial y}\frac{\partial}{\partial x}\Omega(\rho, z) +$$

$$+\frac{\partial}{\partial x}\{\nabla_\perp S(\rho, z)\,\nabla_\perp\}\frac{\partial S}{\partial x}+\frac{\partial}{\partial y}\{\nabla_\perp S(\rho, z)\,\nabla_\perp\}\frac{\partial S}{\partial y}=$$

$$=\frac{1}{2}\Delta_\perp\left\{\tilde{\varepsilon}(\rho, z)+\frac{1}{2}\frac{\Delta_\perp I(\rho, z)}{I(\rho, z)}-\frac{1}{4}\frac{[\nabla_\perp I(\rho, z)]^2}{I^2(\rho, z)}\right\}. \quad (14)$$

The phase gradient curl differs from zero on the lines of singularity of the right-hand side of Eq. (13) corresponding to intensity zeros – zero lines. For a numerical description of wave fields by means of a set obtained, the regularization of the right sides of singular equations (13) and (14) should be provided by the rules of regularization of generalized functions.[27] In this case the unknown functions $\nabla_\perp S_\partial(\rho, z)$ and $\Omega(\rho, z)$ are regularized automatically as a result of calculations. The regularization of right sides can be made by their convolution as a "small cap"[27]:

$$\omega_\varepsilon(x)=\begin{cases} C_\varepsilon \exp\left\{-\dfrac{\varepsilon^2}{\varepsilon^2-|x|^2}\right\}, & |x|\le\varepsilon,\\ 0 & |x|>\varepsilon,\end{cases}$$

where C_ε is the normalizing constant. Figure 2 shows the form of the scalar function calculated for a singular phase of a Laguerre–Gaussian laser beam.[23] It is suggested that the procedure of regularization and calculation of the curl of a vector of phase gradient is fulfilled with 10% error. The regularization suggests that $\Omega(\rho, z)$ takes the form of smooth "hills" and "valleys" (corresponding to opposite directions of screw whirl) with the "height" or "depth" fitting the absolute value of the topological charge.

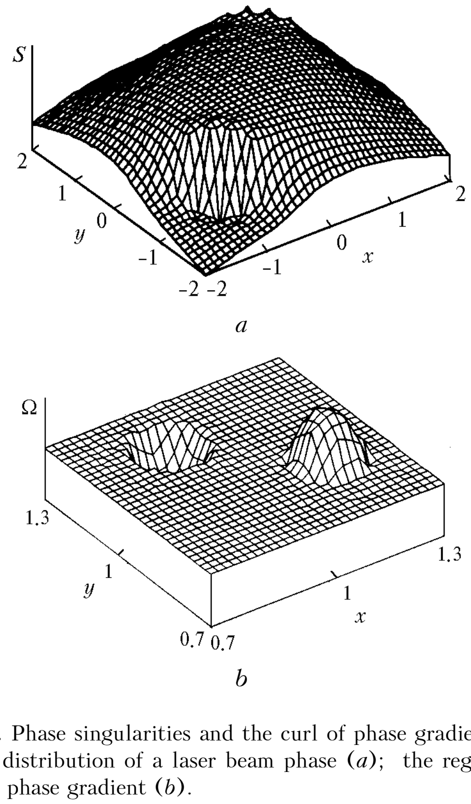

Fig. 2. Phase singularities and the curl of phase gradient: the initial distribution of a laser beam phase (*a*); the regularized curl of phase gradient (*b*).

As we have mentioned, the occurrence of optical vortices in the light wave is accompanied by formation of intensity zeros. Taken alone, zero is not an object that can be immediately recorded in the natural experiment. Its localization is also problematic in the numerical experiments simulating the optical wave propagation in the turbulent medium. This is associated with the fact that for such a simulation the net-point functions are used determined in the finite number of points. The position of zeros, as a rule, does not coincide with these points. Therefore, to record zeros, it is necessary to use the indirect criteria. One of those is a well-known dichotomy of interference fringes.[1,2] At points of real zeros we observe the bifurcation of maxima and minima of the interference pattern as well as the appearance and disappearance of interference fringes. This criterion is not localized. It can be observed and it indicates that the wave function has real zeros of the first order. Figure 3 shows the behavior of interference fringes at the speckle-field interference with a plane wave.

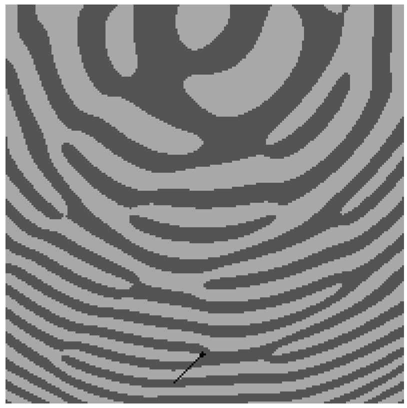

Fig. 3. The result of the speckle-field interference with a plane wave. There are 16 dislocations of wave front on a given fragment.

Branching of interference fringes at one point into three and more fringes is due to the presence of real zero of higher order at this point. These fringes occur when the wave has a wide range of spatial frequencies that is manifested in convergence and connection of lines of sign inversion.

The focal spot from the aperture with zero at the center differs essentially from the spot when zero is absent. When the lines of equal intensity are extended sufficiently and have the form of ellipse, the intensity along the main ellipse axis is close to zero. The maximum intensity values are observed in the fields removed from this axis. The Fourier transform from such a field is a doublet.[28] Figure 4 shows the form of this doublet in a focal plane.

In the case when the form of lines of equal intensity is close to a circular one, the intensity distribution in a focal plane is also circular. This intensity distribution, minimal at the center and growing to the edges, results in the increase of the second radial moments of inertia as compared with the Gaussian distribution.

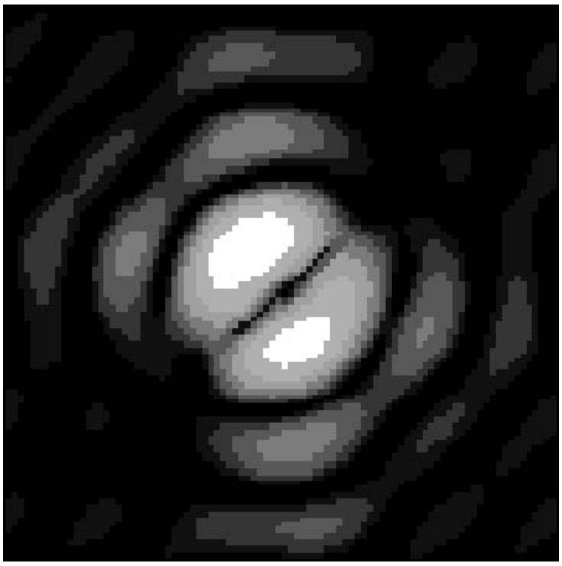

Fig. 4. Intensity distribution in the focal spot from the aperture that has a phase dislocation of the first order.

In the numerical experiment, the width of spots in the focal plane of subapertures of the Hartmann detector was estimated depending on the turbulence intensity for the situations when the intensity zeros are present or absent in the aperture. The width was calculated as the mean ratio of the second radial moment of inertia to zero moment of the power 1/2. The number of experiments varied from 20 to 300 depending on the presence of zero points in the aperture. The matrix order was equal to 96 and corresponded to one meter.

We observed the continuous increase of the focal spot width or spectral width of spatial wave frequencies with intensity zeros throughout the entire range of coherence radius variation from weak to strong fluctuations of intensity. There was also a tendency for saturation of the focal spot width with decreasing the coherence radius when the intensity zeros were missing from the subaperture. Really, with strengthening the fluctuations, the absence of zero in subaperture becomes less probable, and hence the increase of the size of focal spots under these conditions occurs mainly at the cost of the increase of the number of real zeros. This fact is consistent with the conclusion of the paper, Ref. 29, whereby the number of zeros is proportional to the width of the field angular spectrum if it has normal probability density.

Besides, it has been known that the experimental data support the conclusion about the normality of probability density of an amplitude (level) logarithm in the cases when the first approximation of SPM is applicable. But the normal law of level fluctuations allows no occurrence of intensity zero. As noted in Ref. 30, after the occurrence of zeros the random process of level fluctuations ceases to be normal.

The region where zeros began to occur might be of interest. The results of the numerical experiment on correlation of the behavior of the plane wave scintillation index and the probability of occurrence of intensity zeros with increasing the turbulence intensity were published in the literature.[6,31] In the region of large coherence radii corresponding to weak turbulence (i.e., the lack of zeros), the monotonic increase of the wave scintillation index is observed. When the index is 0.7, the probability of occurrence of zeros increases by a factor of 30 with varying the coherence radius from 20 to 14 cm.

We consider the propagation in a heterogeneous medium[32] along the coordinate axis z of a Gaussian beam with a multiplier $r^n\, e^{in\theta}$ in the initial plane $z = 0$:

$$W(r) = r^n\, e^{in\theta} \quad \exp\left\{\frac{r^2}{2c^2} - ik\,\frac{r^2}{2R^2}\right\},$$

where r and θ are the polar coordinates; c and R are the constants characterizing the initial beam width and the wave front curvature. The introduced factor creates zero of nth order at the origin of the coordinates and the corresponding optical vortex around this zero. Up to the definite distances, the vortex is retained at wave propagation in the heterogeneous medium. The turbulence increase will eventually disturb the initial monotonic phase variation around zero point and the vortex will disappear. To investigate this phenomenon, the numerical experiment was carried out. The order of counts matrix equals 100 that provided adequacy of discrete functions to their continuous prototypes. Two phase screens were used for simulating the heterogeneous medium with the spectral density of the refractive index corresponding to atmospheric turbulence in an inertial interval. The law of energy conservation was fulfilled in the model within the computer accuracy.

In the numerical experiment, the path length was determined in the heterogeneous medium where the optical vortex in the Gaussian beam remained. The presence of the vortex at wave propagation in the heterogeneous medium was defined visually based on the presence or absence of the monotonic phase function around the point of zero. The two versions were considered, namely, the presence of the monotonic phase in the ring area where the wave intensity is large and in the small vicinity of zero point where the intensity is low. The probability of the vortex conservation was determined by the ratio of favorable cases to the sample volume being equal to 10.

The analysis of the results indicates that the optical vortex with higher order keeps better, when propagating in the heterogeneous medium. We know that the higher is the carrier frequency, the wider the range of modulating frequency without loss of characteristics of analytical signal of a modulated wave. Greater stability of the vortex in the beam central part than in the ring area can be explained by a power-law spectrum of inhomogeneities of the medium refractive index. With such a spectrum the large-scale inhomogeneities affect more highly the wave phase fluctuations.

Because the concept of adaptive optics is based on the concepts of the type of "reference wave phase," "phase corrector," "wave front sensor", it is not surprising that the problem of phase dislocations is closely connected with the problems of adaptive optics. In this case as a practical matter, the most important are the following problems:

1) The degree of decrease in efficiency of existing methods and means of adaptive optics under the effect of phase dislocations;

2) How to construct the sensor and the corrector of phase distortions for efficient work under these conditions;

3) How the requirements must follow the basic technical characteristics of adaptive optical system intended for operation under conditions of strong intensity fluctuations.

The literature[33–35] provides answers to the first question. In Refs. 33 and 34 it is shown, that the correction of only "vortex-free" part of the phase results in a marked decrease of phase correction efficiency of turbulent distortions beginning with the path lengths corresponding to the diffraction length on the coherence radius, i.e., practically when passing to the region of strong intensity fluctuations.

At the same time, the amplitude-phase correction enables one to fully cancel the turbulent beam broadening, and an ideal phase correction remains nearly as efficient as the amplitude-phase one. This conclusion agrees with the experimental results of Livermore laboratory.[35]

Thus, the principles of construction of an adaptive system, intended for operation in the field of strong intensity fluctuations, with the availability of phase dislocations in the reference wave should differ from those intended for operation in the field of weak fluctuations. First of all, this is true for the construction of the wave front sensor and its algorithm.

The calculations made in Ref. 33 have shown that the occurrence of dislocations does not impose new requirements on the corrector design. The corrector must be composite and the size of the element should be comparable with the coherence radius. At the same time, the traditional approaches to the design of the phase sensor become inadequate. In this case the two reasons are available. First, almost all types of existing adaptive sensors are the sensors of local tilts but not the phase differences and, second, the existing algorithms of phase calculation over the entire aperture are intended for the "smooth" phase.

The basis for existing algorithms is an idea that the phase differences, summed over the closed contour, must give zero in the absence of errors and measurement noise.[5,36,37] Mathematically, this means that the circulation of the potential phase gradient equals zero.[38] Nonzero value is interpreted as the measurement error, i.e., some discrepancy of the sensor algorithm. Thus, the calculation algorithm of phase values attributed to subapertures must minimize the sum of squares of discrepancies or the related value.

If inside the contour the dislocation is found, the proper sum is not equal to zero but 2π plus the measurement error. In this case one can use an algorithm,[39,40] which drops $2\pi n$ in calculating the discrepancies. However, the problem occurs here associated with the fact that, when using the Hartmann sensor or a similar wave front sensor measuring the local tilts of the wave front, the estimation error of phase difference increases rapidly determined through the product of the local tilt and the subaperture size.[33] Indirectly it is also manifested in the relative decrease of the gain of the use of the compound adaptive mirror with the correction of local tilts and the piston phase as compared to the corrector, whose segments compensate only the piston phase.[33]

As it has already been mentioned, the size of segment of the compound adaptive mirror, required for correcting turbulent distortions, is not varied when passing to the range of the intensity strong fluctuations (Fig. 5). This is also valid as to the speed of operation of the adaptive system.[33] Figure 5 shows the dependence of the normalized intensity of the point source image in a focal point of an adaptive telescope on the length of the turbulent layer through which the observation is made. The aberration sensor is assumed to be ideal and the corrector – to be compound, with the segment size d. Here $r_0 = (0.49\ k^2\ C_n^2\ L)^{-3/5}$ is the Fried radius; C_n^2 is the structural constant of the refractive index.

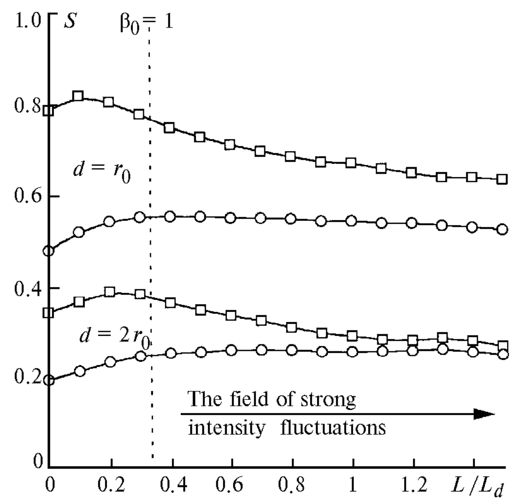

Fig. 5. The dependence of the Strehl parameter on the normalized path length $L/(kr_0^2)$ in the adaptive system with the compound corrector. The control of the corrector elements position is denoted by small circles, and the control of the above position and tilts is denoted by small squares; d is the size of the compound corrector element.

Thus, the problem lies in the design of the phase difference sensor. The sensor would be optimal if it measured the phase difference between the central and all the other apertures. In this case the wavelengths of the reference and corrected radiation must coincide. This problem can be solved using a certain interferometric sensor, which determines the phase difference based on the interference fringe position. It is unclear how the intensity fluctuation will influence upon the operation of this sensor. Probably, the equalizing of the reference wave intensity using a certain nonlinear amplifier could solve this problem.

As noted above, the occurrence of dislocations in the reference wave is possible not only as a result of random phase fluctuations but in some other cases, for example, when compensating thermal blooming in a regular medium. As was shown in Ref. 16, in the course of adaptive correction the phase dislocations may occur in the reference beam. Besides, just the occurrence of phase dislocation in the reference beam is a mechanism determining the iteration or auto-oscillation modes of operation of the phase-coupling adaptive optical system at correction of thermal blooming.

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