Measurements of radial and tangential components of the horizontal wind velocity with a sodar

V.A. Fedorov

Institute of Atmospheric Optics, Siberian Branch of the Russian Academy of Sciences, Tomsk

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We describe algorithms for determination of the mean values, standard deviations, asymmetry, and excess coefficients of the distributions of radial and tangential components of the horizontal wind velocity from the data obtained with a VOLNA-3 sodar using any of the three-channel sensing schemes. Two possible approaches to construction of measurement algorithms and estimation of their errors are considered. Sodar measurements of the vertical profiles of the parameters under study are exemplified.

Introduction

Balser and Netterville[1] were among the first to report on sodar measurements of the vertical profiles of the radial u and tangential v components of the horizontal wind velocity and their standard deviations $\sigma(u)$ and $\sigma(v)$. The most detailed description of determining these characteristics for a particular sensing arrangement can be found in the materials (Ref. 2) that were kindly placed at our disposal by the REMTECH Inc., whose sodars are well known all over the world. These materials also present some vertical profiles of these parameters. However, these profiles are not accompanied by the corresponding estimates of the measurement errors: neither interval nor point estimates needed for more objective characterization of the reliability of sodar data are available. Such a situation is quite typical for acoustic sounding of the atmosphere.[3]

Often, (see, for example, Ref. 4) some theoretical reasons or results of test simulations are presented as justification for error estimates. Such considerations are usually restricted to only studying the estimates of the mean vertical component, as well as the mean speed and direction of the horizontal wind. However, even if these arguments are justified enough and valid, on the average, the actual accuracy characteristics are still determined by the particular state of the atmosphere and the noise situation during measurements. These factors determine the form of the statistical ensembles of the instantaneous radial components V_r measured with a sodar, the number of their significant readouts for a given observation time, and, finally, the achievable sounding heights and the uncertainty of estimates of the parameters sought.

The aim of this paper is to obtain the standard errors at 90% confidence intervals for the parameters of the uv wind components directly from the experimental data obtained using any of the three-channel sensing schemes. Along with the estimates of the means $M(\cdot)$ and standard deviations $\sigma(\cdot)$, we analyze the estimates of asymmetry $\gamma(\cdot)$ and excess $\varepsilon(\cdot)$, which provide for a more complete characterization of the atmosphere as a random medium.[5] We consider two alternative approaches to construction of both the measurement algorithms themselves and the estimates of their errors that are realized in the processing system of Volna-3 sodars.[6] The results obtained are illustrated with measurement data on the parameters under consideration.

1. Basic equations

These approaches are based on the corresponding functional dependences between the u and v-components of the wind velocity vector measured with a sodar and the sought ones. In the initially chosen Cartesian coordinate system, the relation between the instantaneous radial $V_r(i)$ and usual orthogonal components $V_x(i)$, $V_y(i)$ in some horizontal plane can be written as

$$V_x(i) = \sum_{r=1}^{3} a_r V_r(i),\ \ V_y(i) = \sum_{r=1}^{3} b_r V_r(i), \qquad (1)$$

where the coefficients a_r and b_r are determined by the geometry of sounding used. Then we turn the old Cartesian coordinate system by an angle θ agreed with the direction of the mean vector of the horizontal wind velocity,[1,2] that is,

$$\theta = \arctan[M(V_y)/M(V_x)] + \theta_0, \qquad (2)$$

where θ_0 is some constant multiple of $\pi/2$ that is determined by the position of the mean values $M(V_x)$ and $M(V_y)$ on the coordinate plane. Then the relation between the components $u(i)$, $v(i)$ and $V_x(i)$, $V_y(i)$, $V_r(i)$ sought in this horizontal plane can be presented as

$$\begin{cases} u(i) = V_x(i)\cos(\theta) + V_y(i)\sin(\theta) = \sum_{r=1}^{3} u_r V_r(i), \\ v(i) = V_y(i)\cos(\theta) - V_x(i)\sin(\theta) = \sum_{r=1}^{3} v_r V_r(i), \end{cases} \qquad (3)$$

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where

$u_r = a_r \cos(\theta) + b_r \sin(\theta)$, and $v_r = b_r \cos(\theta) - a_r \sin(\theta)$

are the coefficients of transition from the radial components of the vector **V** to its transverse and longitudinal components. From Eqs. (2) and (3) it follows that

$$M(u) = \sqrt{M^2(V_x) + M^2(V_y)},\ M(v) = 0, \qquad (4)$$

that is, the mean radial component $M(u)$ is equal to the absolute value of the mean vector of the horizontal wind, and the mean tangential component $M(v)$ is always zero.

The problem formulated is solved using the methods of mathematical statistics.[7,8] The field of the wind velocity is assumed horizontally homogeneous and stationary, which is the commonly accepted condition in sodar measurements.[2] It is also assumed that $V_r(i)$ obtained by the rth sodar channel at any fixed height form a set of independently sampled values corresponding to some continuous distribution $W_r(V_r)$.

2. Direct method for processing the *u* and *v*-components

If this method is used for processing, the current values of the u and v-components obtained using functional equations (3) are considered as results of direct measurements. For better functioning of this method, it is necessary to provide the matching in space and time of the $V_r(i)$ measurements in every ith sounding cycle.[2,7] Rigorous fulfillment of these conditions requires parallel operation of the sodar channels and realization of very complicated tristatic sensing schemes providing for simultaneous measurement of all the three radial wind velocity components in the same scattering volume. It is most likely that this method *a priori* does not find wide utility in acoustic sounding of the atmosphere. Nevertheless, to check its efficiency, keeping in mind its further simplicity in processing of data, this method was realized in sounding with Volna-3 sodars.

First, $V_x(i)$ and $V_y(i)$ are calculated from $V_r(i)$ by Eqs. (1), then, after the necessary averaging, the direction θ [Eq. (2)] and then the sought values of u and v-components are determined by Eqs. (3). In fact, this means that the case of indirect measurements (3) is reduced to multiple direct measurements. Therefore, the further processing of the orthogonal components can be carried out similarly to estimation of the parameters of the radial components, which is described in detail in Ref. 3.

The disadvantages of this method are the probability that in some measurement cycles the instantaneous values of $u(i)$ and $v(i)$ remain uncalculated in case of missing signal or insufficient signal-to-noise ratio at least in one of the sodar radial channels.[6] As a result, poor operation of only one measuring channel makes the number of significant readouts N of the u and v-components much lower than the maximum possible one. Finally, this leads to the low accuracy of measurements (large confidence intervals) and the low sounding height as compared to the potentially possible one, especially, at a short averaging time.

3. Indirect method for processing of *u* and *v*-components

An alternative, purely indirect method of processing is based on obtaining the functional relations between statistical moments of the u and v-components and the moments of the radial components V_r (Refs. 1 and 2). Then the estimates of the needed parameters are formulated and the equations for measurement errors are determined.[7] Note that matching in space and time of the measuring channels is desirable for realization of this approach too. However, if compared with the direct method, the lack of matching at the horizontally homogeneous field of the wind velocity much weaker influences the final results because of the initial statistical averaging. Nevertheless, in Ref. 2 in determination of $\sigma(u)$ and $\sigma(v)$ a method was proposed to ensure this matching. However, in our opinion, it calls for more careful tests, especially, when estimating higher statistical moments.

In obtaining these functional relations depending on the order of the considered central moment μ_k, let us use the following approximations. For $k = 2$ it is sufficient to restrict the consideration to pairwise lack of correlation between the channels for V_r measurements, and at $k > 2$ it is necessary to require their statistical independence. In fact, at $k = 2$ we neglect the values of interchannel second mixed central moments of the radial components $\mathrm{cov}(V_r, V_l)$ as compared to the channel variances $D(V_r)$. For $k = 3$ we neglect the values of the third mixed central moments with respect to $\mu_3(V_r)$, and for $k = 4$ the fourth mixed central moments are neglected with respect to $\mu_4(V_r)$. That means that we assume that for the most common sounding geometries with the space–time separation of measuring channels these probabilistic dependences should be so weak, that they can be neglected. Note that in Ref. 1 the standard deviations $\sigma(u)$ and $\sigma(v)$ are calculated neglecting the statistical relations between the u and v-components as well. It should also be noted that for Eqs. (4) to be valid, the conditions of the lack of correlation and dependence between V_r are not obligatory.

Taking into account the linear character of Eqs. (3) with respect to V_r and the results of Refs. 8 and 9, the sought relations for the moments of the tangential and radial components take the form (presented only for u, because for v they are similar):

$$\begin{cases} M(u) = \sum_{r=1}^{3} u_r M(V_r), \\ \sigma(u) = \sqrt{\mu_2(u)} = \sqrt{D(u)} = \sqrt{\sum_{r=1}^{3} u_r^2 D(V_r)}, \\ \mu_3(u) = \sum_{r=1}^{3} u_r^3 \mu_3(V_r), \\ \mu_4(u) = \sum_{r=1}^{3} u_r^4 \mu_4(V_r) + 6\sum_{r<k} u_r^2 u_k^2 D(V_r) D(V_k). \end{cases} \quad (5)$$

Effect of randomness of θ on the estimates of the moments of the *u* and *v*-components

Equations (4) and (5) were derived neglecting the random character of the angle θ. In fact, θ is always an *a priori* estimate of the unknown direction of the mean vector of the horizontal wind, namely, the function $\hat{\theta} = \arctan[\hat{M}(V_y)/\hat{M}(V_x)] + \theta_0$ of the estimates of the mean values of the Cartesian and radial components for the needed averaging time:

$$\hat{M}(V_x) = \sum_{r=1}^{3} a_r \hat{M}(V_r) \text{ and } \hat{M}(V_y) = \sum_{r=1}^{3} b_r \hat{M}(V_r).$$

In this case, even initially independent $V_r(i)$, the corresponding current values of the $u(i)$ and $v(i)$ calculated in different sounding cycles become dependent parameters. The initial value of cov(u, v) also changes, and extra terms appear in Eqs. (4) and (5).

To estimate quantitatively the effect of this distorting factor, expand the functions (3) of six random variables V_r, $\hat{M}(V_r)$ into a Taylor series in the vicinity of their mean values up to the square terms inclusive.[7–9] For certainty, assume that the corresponding samples of mean values[3] are used as $\hat{M}(V_r)$ and the number of significant readouts of $V_r(i)$ in each sodar measurement channel is the same and equal to N. In the further averaging, we take into account that the estimates $\hat{M}(V_r)$ are unbiased and correlate with V_r in each channel. Finally, we obtain

$$M_s(u) = M(u)[1 + D(v)/2M^2(u)N] = M(u) + O(N^{-1}).$$

At the same time, the equation for the mean of the tangential component keeps unchanged, that is, $M_s(v) = M(v) = 0$.

Restricting the consideration to linear terms in the series for v, after some transformations, we obtain the following equations:

$$D_s(v) = D(v)(1 - N^{-1}) = D(v) + O(N^{-1});$$

$$\mu_{3s}(v) = \mu_3(v)(1 - 3N^{-1} + 2N^{-2}) = \mu_3(v) + O(N^{-1}),$$

$$\mu_{4s}(v) = \mu_4(v)(1 - 4N^{-1} + 6N^{-2} - 3N^{-3}) +$$

$$+ D^2(v)(6N^{-1} - 15N^{-2} + 9N^{-3}) = \mu_4(v) + O(N^{-1}).$$

The square terms taken into account do not change the pattern. Similar equations accurate to the terms $O(N^{-1})$ are also valid for the moments of the u-component.

Let us now pass on to estimates of correlations between the u and v-components in different ith and jth sounding cycles. In this case, the corresponding parameters to be averaged are the functions of already nine random variables: $V_r(i)$, $V_r(j)$, and $\hat{M}(V_r)$, where $r = 1, 2, 3$. Among them, $V_r(i)$ is correlated with $\hat{M}(V_r)$ and $V_r(j)$ is correlated with $\hat{M}(V_r)$ in each channel. Then, taking into account the square terms:

$$\text{cov}_s[u(i), u(j)] = M_s[u(i)u(j)] - M_s^2(u) =$$

$$= -D^2(v)/[4M^2(u)N^2] = O(N^{-2}),$$

$$\text{cov}_s[v(i), v(j)] = -D(v)/N = O(N^{-1}),$$

$$\text{cov}_s[u(i), v(j)] = -\text{cov}[u(i), v(i)]/N = O(N^{-1}),$$

where $\text{cov}(u, v) = \sum_{r=1}^{3} u_r v_r D(V_r)$ is the initial value of the correlation moment between the u and v-components. For $i = j$ the following equation is valid:

$$\text{cov}_s(u, v) = \text{cov}(u, v)(1 - N^{-1}) = \text{cov}(u, v) + O(N^{-1}).$$

Thus, the neglect of the random character of θ leads, in the worst case, to the neglect of the terms of the order $O(N^{-1})$ in the equations for the moments of the u and v-components. However, in practice, the values of N usually achieve several tens and higher. Therefore, from this point on we neglect the influence of this factor in solving the problem formulated. (We plan to analyze in our future papers actually important systematic errors that may occur and which can be caused by inaccurate setting of the elevation and azimuth angles of the sodar antennas, as well as other, in particular, random factors.) Then, substituting the true moments by their estimates in Eq. (5), we obtain the equations for calculation of the parameters of the u-component of the horizontal wind velocity (the equations for the v-component are quite similar):

$$\hat{M}(u) = \sum_{r=1}^{3} u_r \hat{M}(V_r); \quad (6)$$

$$\hat{\sigma}(u) = \sqrt{\hat{\mu}_2(u)} = \sqrt{\hat{D}(u)} = \sqrt{\sum_{r=1}^{3} u_r^2 \hat{D}(V_r)}; \quad (7)$$

$$\hat{\mu}_3(u) = \sum_{r=1}^{3} u_r^3 \hat{\mu}_3(V_r), \quad (8)$$

$$\hat{\mu}_4(u) = \sum_{r=1}^{3} u_r^4 \hat{\mu}_4(V_r) + 6 \sum_{r<k} u_r^2 u_k^2 \hat{D}(V_r) \hat{D}(V_k); \quad (9)$$

$$\hat{\gamma}(u) = \hat{\mu}_3(u)/\hat{D}^{3/2}(u); \quad (10)$$

$$\hat{\varepsilon}(u) = \hat{\mu}_4(u)/\hat{D}^2(u), \quad (11)$$

where the unbiased channel by channel estimates of the V_r parameters are determined by the equations from Ref. 3.

Standard errors in estimates of the parameters of the *u* and *v*-components

The standard errors of the estimates (6)–(11) were obtained taking into account the earlier accepted assumptions. In the simplest case of the estimate of the mean value (6), the equation for its standard error has the form

$$\sigma[\hat{M}(u)] = \sqrt{\sum_{r=1}^{3} u_r^2\, D[\hat{M}(V_r)]}, \qquad (12)$$

where the variance of the estimate of the mean value $D[\hat{M}(V_r)] = \sigma^2[\hat{M}(V_r)]$ is determined from Eqs. (1) or (2) of Ref. 3.

To find the standard measurement errors for other parameters of the *u*-component, let us use the method of linearization, that is, consider only linear terms in the corresponding Taylor series.[7–9] Accounting for nonlinear term is inexpedient in this case from the practical point of view because of the further need to use sampled high-order moments, which are estimated with large errors at the limited number of observations N (Refs. 3, 7–9). Then, after the needed averaging taking into account that the applied estimates of $\hat{D}(V_r)$ are unbiased,[3] we obtain the equation for the standard error of the estimated standard deviation of the *u*-component of the horizontal wind velocity:

$$\sigma[\hat{\sigma}(u)] = \frac{1}{2\sigma(u)}\sqrt{\sum_{r=1}^{3} u_r^4\, D[\hat{D}(V_r)]}, \qquad (13)$$

where the variance of the estimate of the variance V_r is determined by Eq. (4) from Ref. 3.

To estimate the asymmetry coefficient of the *u*-component (10), we similarly come to the equation

$$\sigma[\hat{\gamma}(u)] = \frac{1}{D(u)}\left\{\frac{D[\hat{\mu}_3(u)]}{D(u)} + \frac{9}{4}\gamma^2(u)\, D[\hat{D}(u)] - \right.$$
$$\left. - \frac{3\gamma(u)}{\sigma(u)}\operatorname{cov}[\hat{\mu}_3(u),\, \hat{\mu}_2(u)]\right\}^{1/2}, \qquad (14)$$

where $\gamma(u) = \mu_3(u)/D^{3/2}(u)$ is the true value of the calculated parameter, and from Eq. (7) it follows that

$$D[\hat{D}(u)] = \sum_{r=1}^{3} u_r^4 D[\hat{D}(V_r)]. \qquad (15)$$

The needed equation for covariance of the estimates of the second and third central moments of the *u*-component can be found from Eqs. (7) and (8) of this paper and Eqs. (3) and (5) of Ref. 3:

$$\operatorname{cov}[\hat{\mu}_3(u),\, \hat{\mu}_2(u)] = \sum_{r=1}^{3} u_r^5 \operatorname{cov}[\hat{\mu}_3(V_r),\, \hat{\mu}_2(V_r)] =$$

$$= \sum_{r=1}^{3} \frac{u_r^5 N_r^3}{(N_r-1)^2\,(N_r-2)} \operatorname{cov}[m_3(V_r),\, m_2(V_r)],$$

where $m_k(V_r)$ is the central sampled kth-order moment of the radial wind velocity in the rth channel at the sample size N_r. According to Ref. 8, the following equation is valid accurate to the $O(N_r^{-2})$ terms

$$\operatorname{cov}[m_3(V_r), m_2(V_r)] = [\mu_5(V_r) - 4\mu_2(V_r)\mu_3(V_r)]/N_r.$$

Finally, from Eqs. (8) of this paper and Eq. (5) of Ref. 3 we obtain

$$D[\hat{\mu}_3(u)] = \sum_{r=1}^{3} \frac{u_r^6 N_r^4}{[(N_r-1)(N_r-2)]^2} D[m_3(V_r)],$$

where from Ref. 8 accurate to the $O(N_r^{-2})$ terms

$$D[m_3(V_r)] =$$
$$= [\mu_6(V_r) - 6\mu_2(V_r)\mu_4(V_r) - \mu_3^2(V_r) + 9\mu_2^3(V_r)]/N_r. \quad (16)$$

From Eq. (16) it follows that the variance of the V_r third central moment, of the sample along with the sample size N_r is determined, to a significant degree, by the sixth-order central moment of V_r. In this case, using Eq. (16) in practice, one always has to substitute the true value of $\mu_6(V_r)$ by the sampled value $\hat{\mu}_6(V_r)$. However, as was mentioned in Ref. 3, this necessary substitution, because of the low accuracy of $\hat{\mu}_6(V_r)$ determination at limited N_r, may lead to very large errors in calculation of $D[m_3(V_r)]$. Therefore, using Eq. (16) and applying the method realized when deriving Eqs. (9) and (10) of Ref. 3, we obtain the approximation dependence of $D[m_3(V_r)]$ for the excess ranging within $1 < \varepsilon \le 25.2$ (for simplicity we omit the corresponding arguments):

$$D[m_3] \cong \mu_2^3\varepsilon^2 \log\varepsilon\,(8.27 - 37.58\log\varepsilon + 70.8\log^2\varepsilon -$$
$$-52.57\log^3\varepsilon + 13.98\log^4\varepsilon)/N_r.$$

As in Ref. 3, for the testing purpose, we calculated $D[m_3(V_r)]$ by Eq. (16) and the approximate equation for the strongly asymmetric single-sided exponential distribution ($\gamma = 2$, $\varepsilon = 9$). The resulting relative error of approximation was as low as –0.1%. Thus, with the accuracy sufficient for practical needs, $D[m_3(V_r)]$ can be determined through preliminary estimation of only the variance and excess of the initial distribution $W_r(V_r)$.

The general equation for the standard error of the excess measurements (11) has the form

$$\sigma[\hat{\varepsilon}(u)] = \frac{1}{D(u)}\left\{\frac{D[\hat{\mu}_4(u)]}{D^2(u)} + 4\varepsilon^2(u)\, D[\hat{D}(u)] - \right.$$
$$\left. - \frac{4\varepsilon(u)}{D(u)}\operatorname{cov}[\hat{\mu}_4(u),\, \hat{\mu}_2(u)]\right\}^{1/2}, \qquad (17)$$

where $\varepsilon(u) = \mu_4(u)/D^2(u)$ is its true value. To determine $D[\hat{\mu}_4(u)]$, let us linearize the estimate (9) as a function of six random variables. In this case, $\hat{\mu}_4(V_r)$ and $\hat{\mu}_2(V_k)$ at $r = k$ are pairwise dependent. After some transformations we obtain

$$D[\hat{\mu}_4(u)] = \sum_{r=1}^{3} u_r^4 \{u_r^4 D[\hat{\mu}_4(V_r)] + 36\, d_r^2 D[\hat{D}(V_r)] + 12\, u_r^2 d_r \operatorname{cov}[\hat{\mu}_4(V_r),\, \hat{\mu}_2(V_r)]\},$$

where $d_r = D(u) - u_r^2 D(V_r)$. From Eqs. (7) and (9) it follows that

$$\operatorname{cov}[\hat{\mu}_4(u),\, \hat{\mu}_2(u)] = 6D(u)D[\hat{D}(u)] + \sum_{r=1}^{3} u_r^6 \{\operatorname{cov}[\hat{\mu}_4(V_r),\, \hat{\mu}_2(V_r)] - 6D(V_r)D[\hat{D}(V_r)]\}.$$

To find $\operatorname{cov}[\hat{\mu}_4(V_r),\, \hat{\mu}_2(V_r)]$, let us use Eqs. (3) and (6) from Ref. 3 for the estimates $\hat{\mu}_2(V_r)$ and $\hat{\mu}_4(V_r)$. After linearization of the statistics of $\hat{\mu}_4(V_r)$ and averaging accurate to the $O(N_r^{-2})$ terms taking into account that at $N \geq 6$ $N^2 - 2N + 3 \approx (N-1)^2$ is valid with the relative error less than 10%, we obtain the relation between the covariance of unbiased and biased estimates of the second-order and fourth-order central moments of the radial wind velocity components:

$$\operatorname{cov}[\hat{\mu}_4(V_r),\, \hat{\mu}_2(V_r)] = \frac{N_r^2}{(N_r-2)(N_r-3)} \operatorname{cov}[m_4(V_r),\, m_2(V_r)].$$

Similarly, we can obtain the equation for the variance of the estimate of the fourth central moment of V_r:

$$D[\hat{\mu}_4(V_r)] = \frac{N_r^2(N_r-1)^2}{[(N_r-2)(N_r-3)]^2} D[m_4(V_r)].$$

Then, from Ref. 8 it follows that

$$\operatorname{cov}[m_4(V_r),\, m_2(V_r)] = [\mu_6(V_r) - 4\mu_3^2(V_r) - \mu_2(V_r)\mu_4(V_r)]/N_r;$$

$$D[m_4(V_r)] = [\mu_8(V_r) - \mu_4^2(V_r) - 8\mu_3(V_r)\mu_5(V_r) + 16\mu_2(V_r)\mu_3^2(V_r)]/N_r.$$

Then, according to the above-said, we obtain the approximation dependences for $1 < \varepsilon \leq 25.2$:

$$D[m_4] = \mu_2^4\varepsilon^2 \log^2\varepsilon(5.33 - 11.03\log\varepsilon + 55.45\log^2\varepsilon - 65.69\log^3\varepsilon + 30.88\log^4\varepsilon)^2/N_r;$$

$$\operatorname{cov}(m_4, m_2) = \mu_2^3\varepsilon^2\log\varepsilon \times (1.68 + 4.29\log\varepsilon - 5.18\log^2\varepsilon + 2.36\log^3\varepsilon)/N_r.$$

In this case, for the single-sided exponential distribution the relative error of approximation was −0.2% for $D[m_4]$ and less than 0.1% for $\operatorname{cov}(m_4,\, m_2)$.

Thus, all the components needed for obtaining the standard errors (12)–(14), (17) in sodar measurements of the parameters of the orthogonal components of the horizontal wind velocity are determined with the accuracy sufficient for practical needs.

4. Comparison of the accuracy characteristics

First, assume that the sample-mean values[3] are used as $\hat{M}(V_r)$ and $N_r = N$ for all the three sodar channels. The estimates of the parameters for the first, direct, method of measurements are marked by the subscript dir. Then, if the above assumptions are valid, it is possible to show the identity of the estimates of the mean values, that is, $\hat{M}_{\text{dir}}(u) = \hat{M}(u)$ and $D[\hat{M}_{\text{dir}}(u)] = D[\hat{M}(u)]$. And for the estimates of the variances, which determine the standard deviations of the u and v-components, the following is valid:

$$\hat{D}_{\text{dir}}(u) = \hat{D}(u) + \frac{2}{N-1}\sum_{i=1}^{N}\sum_{r<k} u_r u_k [\dot{V}_r(i)\dot{V}_k(i) - \hat{M}(\dot{V}_r)\hat{M}(\dot{V}_k)], \quad (18)$$

where $\dot{V}_r$ is the centered random value. It follows from Eq. (18) that $M[\hat{D}_{\text{dir}}(u)] = M[\hat{D}(u)]$, that is, the results of $D(u)$ estimation by the two methods coincide on the average. However,

$$D[\hat{D}_{\text{dir}}(u)] = D[\hat{D}(u)] + \frac{4}{N-1}\sum_{r<k} u_r^2 u_k^2 D(V_r)D(V_k),$$

where $D[\hat{D}(u)]$ is determined by Eq. (15). Consequently, the standard errors of the estimates of standard deviations of the u and v-components for the first method are greater than those for the second method. This is also valid for the estimates of asymmetry and excess, and the difference in the error values increases with the increasing order of the used sampled moment, that is, it reaches maximum at $\hat{\varepsilon}$. When different median estimates[3] are used as $\hat{M}(V_r)$, these conclusions almost do not change. (The latter two statements are based on the results of simulation of the measurement algorithms for the direct and indirect methods at different distributions of the radial components $W_r(V_r)$ [Ref. 3].)

5. Interval estimates of the parameters of *u* and *v*-components

To get the idea of the accuracy and reliability of the above point estimates $\hat{g}$ [Eqs. (6), (7), (10), and (11)] of the parameters of the u and v-components of the horizontal wind velocity, it is necessary to pass to the corresponding interval characteristics. Taking into account that these measured parameters are similar to those considered in Ref. 3, apply again the approach that uses the properties of 90% confidence intervals $I_{0.9}$ [Ref. 10]. Following Ref. 3, for all the estimates $\hat{g}$ we determined the corresponding minimum sample size $N_{\min}$, starting from which it becomes possible to use this approach in practice. The simulation showed the consistency of $N_{\min}$ obtained and those given in Ref. 3. Finally, if $N_r \geq N_{\min}$ for all the sodar channels, then any of the measured result on parameter g of the u and v-components can be represented as

$$g_{0.9} = \hat{g} \pm 1.6\sigma(\hat{g}) \qquad (19)$$

with 90% confidence probability (the corresponding values of $N_{\min}$ are given in Ref. 3).

6. Experimental results

The above-said is illustrated with the vertical profiles of the parameters of the radial wind velocity component measured with a Volna-3 sodar by the two methods considered above for different averaging time: T_{av} = 18 min (Fig. 1) and T_{av} = 60 min (Figs. 2–5). The measurement results for the mean value $\hat{M}(u)$ are denoted as $M(u)$ in Figs. 1 and 2, for the standard deviation $\hat{\sigma}(u)$ as $\sigma(u)$ (see Fig. 3), for the asymmetry $\hat{\gamma}(u)$ (see Fig. 4) and excess $\hat{\varepsilon}(u)$ (see Fig. 5) as $\gamma(u)$ and $\varepsilon(u)$. Following Eq. (19), the corresponding 90% confidence intervals are plotted as well.

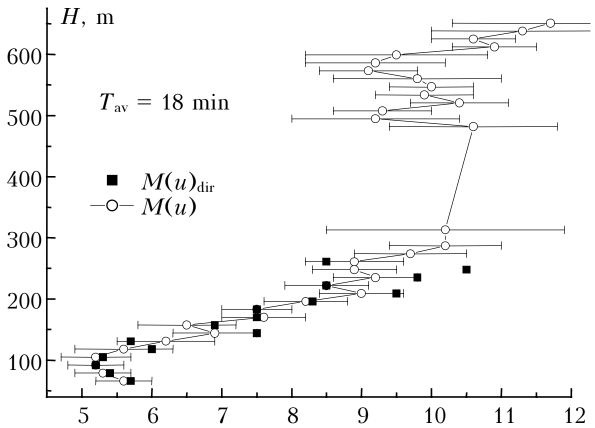

Fig. 1. Mean value of the radial u component, m/s.

The vertical profiles shown in Fig. 1 were obtained in Tomsk suburbs on January 26, 2000 at 18:35 L.T., and those depicted in Figs. 2 to 5 were obtained on November 20, 1999 at 19:34 L.T. We do not describe the physical states of the atmosphere during these measurements, which caused these quite unusual profiles. Our task is to compare the two methods for estimation of the parameters of the u and v-components and to demonstrate the achievable accuracy. Note only that, according to the data of sodar facsimile records, an elevated temperature inversion was observed above 500 m on January 26, 2000 during the whole day. This caused the presence of an echo signal from the corresponding heights (up to $H_{\max} \approx 650$ m), and the power of this signal was sufficient for measurement of the mean u values by the indirect method, though with rather large $I_{0.9}$ (see Fig. 1). At the same time, the value of the echo signal was insufficient for measurements by the direct method, according to which the instantaneous values of the radial components $V_r(i)$ are not accumulated by channels. The common region, where the characteristics of u could be measured by both the direct and indirect methods, was the height region of 60 to 260 m.

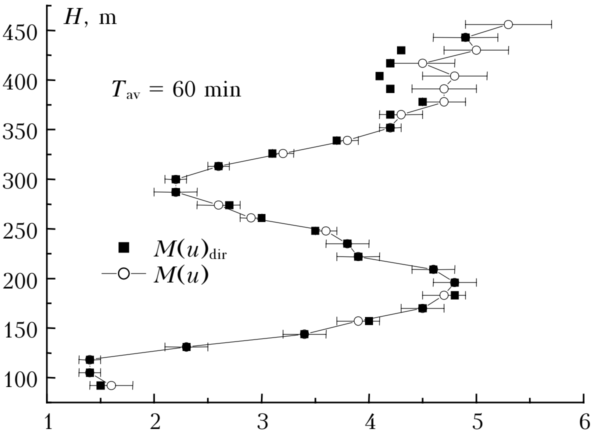

Fig. 2. Mean value of the radial u component, m/s.

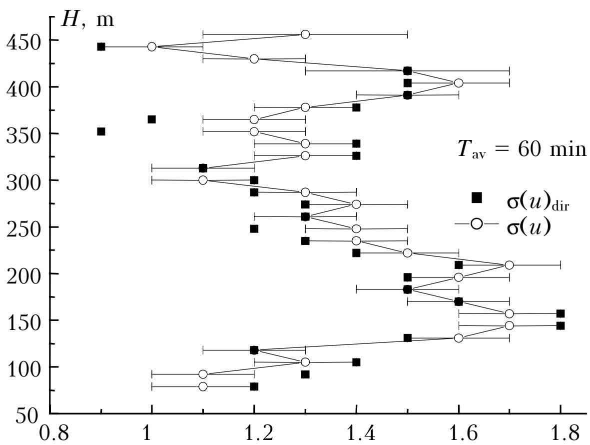

Fig. 3. Standard deviation of radial u component, m/s.

It is worth noting quite a close agreement between the data obtained using both of the methods. Thus, $I_{0.9}$ for $\hat{M}(u)$, except at the height of 248 m, cover the values of $M(u)_{dir}$. For this H, the confidence intervals in both of the methods overlap. (Not to overload Figs. 1–3 with information, $I_{0.9}$ for the direct method is omitted.) The central part of the considered height range was characterized by very weak echo signals that, finally, made the measurements of the wind velocity impossible. These boundaries of intermittent reception of the acoustic signals are clearly seen in Fig. 1. They are characterized by a sharp increase of the confidence intervals: thus, $I_{0.9} \approx 3.4$ m/s for $H = 313$ m.

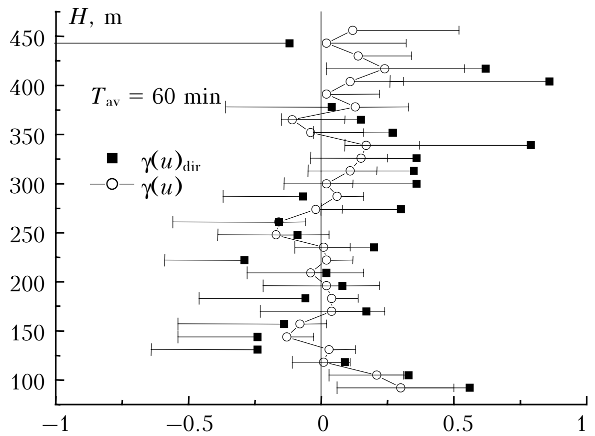

Fig. 4. Asymmetry coefficient of radial component.

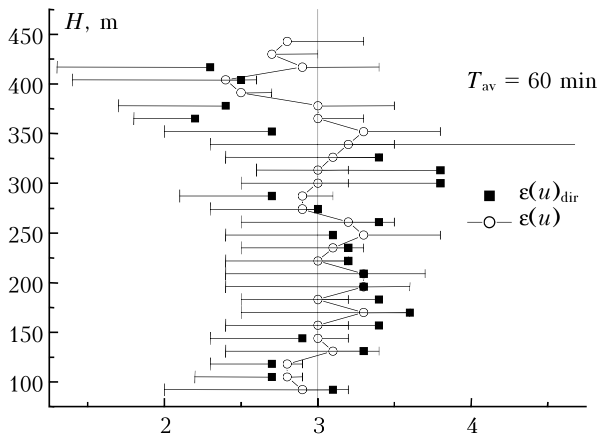

Fig. 5. Excess coefficient of the radial component.

With the increase of the averaging time, the maximum achievable height of measurement of the parameters of the u and v-components of the horizontal wind by the direct method also increases, as a rule, approaching H_{max} of the indirect method (see Figs. 2–5). In the case of a stable echoes in all the three radial channels, the deviation of $M(u)_{dir}$ from $M(u)$ (see Fig. 2) is small. It increases only starting from $H \approx 390$ m. This can be explained by the fact that as the echo signal power decreases, $M(u)_{dir}$ is determined from only a part of $V_r(i)$ measured by sodar because of the above reasons. At the same time, the estimate of $M(u)$ uses the entire statistical ensemble of the data obtained. However, the confidence intervals of both methods still overlap. Analogous conclusions are also valid, though to a lesser degree, for estimation of the vertical profiles of $\sigma(u)$ (see Fig. 3). In this case again, either the deviations of $\sigma(u)$ from $\sigma(u)_{dir}$ are insignificant or their confidence intervals overlap. This fails only for $H \approx 352$ m. However, the difference between $\sigma(u)$ and $\sigma(u)_{dir}$ is not large: it does not exceed 0.3 m/s.

The results of measurement of the asymmetry and excess coefficients $\gamma(u)_{dir}$, $\varepsilon(u)_{dir}$ oscillate about rather smooth vertical profiles $\gamma(u)$ and $\varepsilon(u)$ (see Figs. 4 and 5). In the upper part of the height range, the spread of the deviations may be large enough. In this case several point estimates $\gamma(u)_{dir}$, $\varepsilon(u)_{dir}$ can be thought unreliable. They are characterized by a sharp increase in the value of the confidence intervals: thus, $I_{0.9}[\varepsilon(u)_{dir}] \approx 4.8$ at $H \approx 340$ m. However, in this situation $I_{0.9}$ also covers the corresponding point value of $\varepsilon(u)$. (Note that only the left part of $I_{0.9}$ is shown for the direct method and only the right part for the indirect one.) Therefore, in this case it is also possible, in principle, to state the consistency of the data obtained by both of the processing methods, if the interval form (19) is taken as the corresponding measurement.

Conclusions

In general, our experience of using the two considered methods for estimating the parameters of the radial and tangential components of the horizontal wind velocity with a Volna-3 sodar suggests the following. The use of the direct and indirect methods for measurement of the mean values of the u-component in the presence of rather strong echo signals gives practically identical results. In this case, the mean value of the v-component is always zero. Somewhat larger differences are observed when measuring the standard deviations. However, this difference is, in general, statistically insignificant, which is not always true when estimating the asymmetry and excess coefficients.

The direct measurements of these parameters are characterized by far higher uncertainty in the data obtained as compared to indirect measurements. This manifests itself in the higher irregularity of the corresponding vertical profiles and significantly wider confidence intervals. As a result, it becomes difficult to use this method for measuring the asymmetry and the excess.

It should be also noted that correct estimation of the excesses of the u and v-components is difficult, if their probabilistic distributions are characterized by the presence of "heavy tails." The values $\varepsilon > 3.8$ roughly correspond to this situation. To determine ε with the acceptable statistical accuracy, the samples of the initial data in each radial sodar channel should have rather a large size N_r, and the larger is the excess to be estimated, the larger should be this size. Approximately, N_r should be equal to several hundreds of significant homogeneous readouts of V_r, which is not always achievable in the case of acoustic sounding.

An obvious advantage of the indirect method is seen when observing weak echo signals and during short averaging time, which, finally, allows the sounding height to be increased (sometimes by 100–250 m with respect to the direct method).

Using the equations presented in this paper, it is possible to evaluate the degree of uncertainty in the sodar measurements of the parameters of the *u* and *v*-components of the horizontal wind velocity, which opens the way for more correct interpretation of the results of acoustic sounding of the atmosphere.

Acknowledgments

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