ON THE POWER OF RETURN SIGNALS FROM A RANDOMLY ROUGH SURFACE WITH A COMBINED LOCAL REFLECTANCE BEING SOUNDED ALONG A SLANT PATH IN THE ATMOSPHERE

M.L. Belov

All–Russian Scientific–Research Institute of Marine Fisheries and Oceanography, Moscow the Firm "EKKOS", Moscow

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Power of return signals from a randomly rough surface with a combined reflectance of locally plane areas at a bistatic sounding in the atmosphere is investigated. An expression for a mean power of a return signal is derived for the case of sounding through an optically dense aerosol atmosphere and the surface reflectance being composed of a diffuse and quasispecular components. It is shown that the return signal shape strongly depends on the ratio of diffuse and quasispecular components of the surface reflectance as well as on the surface roughness and the state of the atmosphere.

In Ref. 1 we have already considered the power of the echo signal from a plane surface with a combined scattering phase function involving quasispecular and diffuse components when the surface is sounded along a slant path. Below we study the energy characteristics of the echo signal in the case of a pulsed laser sounding of a randomly rough surface with a combined scattering phase function of local areas through the atmosphere.

Let us assume that the combined scattering phase function comprising quasispecular and diffuse components can be calculated for each locally plane element of a randomly rough surface S. The surface S is sounded by a pulse of radiation through the atmosphere. Thus, the integral expression for the power recorded by the receiver has the form[1] (we use the small–angle approximation for the source and receiver and assume that shading of surface elements is negligible)

$$P(t) = \int_S \mathrm{d}\mathbf{R} \int \mathrm{d}\Omega(\mathbf{m}) \cos\theta_s\, J(\mathbf{R},\, \mathbf{m})\, J_\mathrm{r}(\mathbf{R},\, \mathbf{m}) \times \times f\left(t - \frac{|\mathbf{R} - \mathbf{r}_\mathrm{s}|}{c} - \frac{|\mathbf{R} - \mathbf{r}_\mathrm{r}|}{c}\right), \quad (1)$$

where

$$J(\mathbf{R},\, \mathbf{m}) = \frac{E(\mathbf{R})}{\alpha \dfrac{2\pi}{n+2} + \beta\pi\Delta^2} \times \times \left[\alpha \cos^n\theta + \beta \exp\left\{-\frac{(\theta - \theta_0)^2 \cos^2\theta_0 + (\varphi - \varphi_0)^2 \sin^2\theta_0}{\Delta^2}\right\}\right], \quad (2)$$

$E(\mathbf{R}) = E_\mathrm{s}(\mathbf{R})\, A$; $J(\mathbf{R}, \mathbf{m})$ is the brightness of radiation reflected from an elementary locally plane area at the point $\mathbf{R}$ of the surface S, $J_\mathrm{r}(\mathbf{R}, \mathbf{m})$ is the brightness of radiation from a virtual source (with the parameters of the receiver)[2] at the point $\mathbf{R}$ of the surface S, A is the reflection coefficient of an elementary area, $E_\mathrm{s}(\mathbf{R})$ is the irradiance of the surface produced by radiation from the source, $\mathbf{R}$ is the spatial coordinate of the elementary scattering area, θ_S is the angle between the normal to the surface S at the point $\mathbf{R}$ and the direction towards the receiver, $\mathbf{r}_\mathrm{s}$ and $\mathbf{r}_\mathrm{r}$ are the vectors of the source and the receiver, positions $f(t)$ describes the shape of the sounding pulse, α and β are fractions of the diffuse and quasispecular reflection, (θ, θ_0) and (φ, φ_0) are zenith and azimuthal angles of observation and of the direction of the reflected radiation maximum (for the quasispecular component of reflection) in the local system of coordinates related to the elementary reflecting area. The angles θ_0 and φ_0 are related to the corresponding angles θ_s and φ_s which describe the direction of the incident radiation propagation by the laws of geometric optics. The parameters n and Δ characterize the angular width of the diffuse and quasispecular components of reflection, respectively. Formula (2) is derived for the case of $\Delta \ll 1$.

Let us assume that the atmosphere is a homogeneous scattering medium with a strongly forward–peaked scattering phase function.[3,4] Then, if the angle, at which the receiving aperture can be observed from the points of the scattering surface, is much smaller than the angular width of the scattering phase function of radiation reflected from the surface, the characteristic scale of the variation in the surface slopes, and the field–of–view angle of the receiver, the expression for the power recorded by the receiver takes the form (assuming that the source, receiver, and their optical axes lie in the XOZ plane, passing from integration over the rough surface S to the integration over its projection S_0 on the plane $z = 0$, and using the results obtained in Refs. 1–5)

$$P(t) \simeq \frac{A}{\pi} \frac{1}{\alpha \dfrac{2}{n+2} + \beta\Delta^2} \left[\alpha \int_{S_0} \frac{\mathrm{d}\mathbf{R}_0}{n_z} \cos^n\theta_\mathrm{s}\, E_\mathrm{s}(\mathbf{R}'_{0\xi})\, E_\mathrm{r}(\mathbf{R}''_{0\xi}) \times \times F(t', \mathbf{R}_0, \zeta) + \beta \int_{S_0} \frac{\mathrm{d}\mathbf{R}_0}{n_z} E_\mathrm{s}(\mathbf{R}'_{0\xi})\, E_\mathrm{r}(\mathbf{R}''_{0\xi})\, F(t', \mathbf{R}_0, \xi) \times \times \exp\left\{-\frac{k_y^2}{\Delta^2}(C - \gamma_x D)^2 - \frac{k_x^2}{\Delta^2}\left[R_{0y} s_2 - \gamma_y k_y (C\gamma_x + D)\right]^2\right\}\right], \quad (3)$$

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where

$$F(t', \mathbf{R}_0, \zeta) = f\left[t' + \frac{R_{0x}(\sin\theta_s - \sin\theta_r)}{c} + \frac{\zeta(\cos\theta_s - \cos\theta_r)}{c} - \frac{R_0^2 + \zeta^2}{2c}\left(\frac{1}{L_s} + \frac{1}{L_r}\right)\right];\ t' = t - \frac{L_s + L_r}{c};$$

$$\mathbf{R}'_{0\xi} = \{(R_{0x}\cot\theta_s - \zeta(\mathbf{R}_0))\sin\theta_s,\ R_{0y}\};$$

$$\mathbf{R}''_{0\xi} = \{(R_{0x}\cot\theta_r - \zeta(\mathbf{R}_0))\sin\theta_r,\ R_{0y}\};$$

$$k_y = \frac{n_z}{\sqrt{1 - n_z^2\gamma_y^2}};\ k_x = \frac{n_z}{\sqrt{1 - n_z^2\gamma_x^2}};\ s_2 = \frac{A_r}{B_r} + \frac{A_s}{B_s};$$

$$C = (\sin\theta_s - \sin\theta_r) + R_{0x}s_1;\ s_1 = \frac{A_r\cos^2\theta_r}{B_r} + \frac{A_s\cos^2\theta_s}{B_s};$$

$$D = \left(1 - \left(\sin\theta_r - \frac{R_{0x}A_r\cos^2\theta_r}{B_r}\right)^2 - \left(\frac{A_r R_{0y}}{B_r}\right)^2\right)^{1/2} + \left[1 - \left(\sin\theta_s + \frac{R_{0x}A_s\cos^2\theta_s}{B_s}\right)^2 - \left(\frac{A_s R_{0y}}{B_s}\right)^2\right]^{1/2};$$

$$A_{s,r} = 0.5\,(\alpha_{s,r}^2 + \sigma L_{s,r}\langle\gamma^2\rangle)^{1/2};$$

$$B_{s,r} = \frac{0.5L_{s,r}(\alpha_{s,r}^2 + 0.5\sigma L_{s,r}\langle\gamma^2\rangle)}{\alpha_{s,r}^2 + \sigma L_{s,r}\langle\gamma^2\rangle};$$

$E_s(\mathbf{R})$ and $E_r(\mathbf{R})$ are the irradiances of the surface S produced by radiation from the real and virtual sources, respectively,[2,3] L_s and L_r are the slant distances from the source and the receiver to the surface, $2\alpha_s$ and $2\alpha_r$ are the angular divergence of the source and the field–of–view angle of the receiver, σ is the scattering coefficient of the atmosphere, $\langle\gamma^2\rangle$ is the variance of the angle of deflection due to the elementary scattering event in the atmosphere, ζ and $\gamma = \{\gamma_x, \gamma_y\}$ are the height and the vector of the slopes of a rough surface S, $\mathbf{n} = \{n_x, n_y, n_z\}$ is the unit vector of the normal to the elementary area, θ_s and θ_r are the angles between the normal to the surface S_0 and the direction towards the source and receiver, respectively.

Assuming the distribution of heights and slopes of the surface S to be normal and averaging expression (3) over ζ and γ we obtain the following relation for the average (over the ensemble of surfaces) power of the echo signal recorded by the receiver from a randomly rough surface with a combined scattering phase function of the local areas sounded along a slant path (assuming that the shape of the sounding pulse is Gaussian, i.e., $f(t) = \frac{2}{\sqrt{\pi}}\exp\{-4t^2/\tau_p^2\}$, while the surface is smoothly rough, i.e., $\gamma_{x,y}^2 \ll 1$, and $R_0\sin\gamma_{s,r} \gg \frac{R_0^2}{2L_{s,r}}$

$$\bar{P}(t) \simeq \frac{2}{\sqrt{\pi}}\frac{A\bar{\nu}^{-1/2}r_r^2\alpha_r^2 m_{sz}m_{rz}P_0\exp\{-(\varepsilon-\sigma)(L_s+L_r)\}}{\left(\alpha\frac{2}{n+2} + \beta\Delta^2\right)16B_r^2B_s^2} \times \left[\alpha F(\gamma_0)p_1^{-1/2}q_1^{-1/2}\exp\left\{-\frac{4(t')^2a_1}{\tau_p^2}\right\} + \beta\frac{G}{\mu}p_2^{-1/2}q_2^{-1/2}\exp\left\{-\frac{4(t')^2a_2}{\tau_p^2} - \frac{8t'b_2}{\tau_p^2 c} - c_2\right\}\right], \quad (4)$$

where

$$p_1 = \frac{1}{4B_s^2} + \frac{1}{4B_r^2};\ p_2 = p_1 + \frac{s_2^2}{\Delta^2\mu};$$

$$q_2 = q_1 + \frac{s_1^2}{\Delta^2\mu};\ \mu = 1 + \frac{2\gamma_0^2\,2q_z^2}{\Delta^2};$$

$$q_1 = \frac{\cos^2\theta_r}{4B_r^2} + \frac{\cos^2\theta_s}{4B_s^2} + \frac{4q_x^2}{c^2\tau_p^2} - \frac{2s_0^2q_{11}^2}{\bar{\nu}};$$

$$q_{11} = \frac{\sin\theta_s\cos\theta_s}{4B_s^2} + \frac{\sin\theta_r\cos\theta_r}{4B_r^2} + \frac{4q_zq_x}{\tau_p^2c^2};$$

$$\bar{\nu} = 1 + 2\sigma_0^2\left(\frac{\sin^2\theta_s}{4B_s^2} + \frac{\sin^2\theta_r}{4B_r^2} + \frac{4q_z^2}{\tau_p^2c^2}\right);$$

$$G = \frac{(m_{sz} - \gamma_{mx}m_{sx})(m_{rz} - \gamma_{mx}m_{rx})}{m_{sz}m_{rz}};$$

$$\gamma_{mx} = -\frac{q_xq_z}{\frac{\Delta^2}{2\gamma_0^2} + q_z^2};\ q_z = -(\cos\theta_s + \cos\theta_r);$$

$$q_x = (\sin\theta_s - \sin\theta_r);$$

$$F(\gamma_0) = (m_{rz})^n(2\gamma_0^2)^{-n/4}\exp\left(\frac{1}{4\gamma_0^2}\right) \times \left[(2\gamma_0^2)^{-1/4}W_{-\frac{(n+1)}{4},-\frac{(n-1)}{4}}\left(\frac{1}{2\gamma_0^2}\right) + \frac{m_{sx}m_{rx}(n+1)}{2m_{rz}m_{sz}}(2\gamma_0^2)^{1/4}W_{-\frac{(n+3)}{4},-\frac{(n-3)}{4}}\left(\frac{1}{2\gamma_0^2}\right)\right];$$

$$a_1 = a_0 - \frac{4}{\tau_p^2c^2}\frac{q_3^2}{q_1};\ q_3 = q_x - \frac{2\sigma_0^2q_zq_{11}}{\bar{\nu}};$$

$$a_0 = 1 - \frac{8\sigma_0^2q_z^2}{\bar{\nu}c^2\tau_p^2};\ a_2 = a_0 - \frac{4}{\tau_p^2c^2}\frac{q_3^2}{q_2};$$

$$b_2 = \frac{q_3q_xs_1}{\Delta^2\mu q_2};\ c_2 = \frac{q_x^2}{\Delta^2\mu}\left[1 - \frac{s_1^2}{\Delta^2\mu q_2}\right];$$

σ_0^2 and γ_0^2 are the variances of the heights and slopes of the randomly rough surface S, $\mathbf{m}_s = \{m_{sx}, m_{sz}\}$ and $\mathbf{m}_r = \{m_{rx}, m_{rz}\}$ are the unit vectors of incident radiation propagation and of the direction towards the receiver, $W_{n,m}(x)$ is the Whittaker function, r_r is the effective radius of the receiving aperture, P_0 is the power of the source, ε is the atmospheric extinction coefficient, and τ_p is the width of the sounding pulse.

When σ_0 and as $\gamma_0 \to 0$, formula (4) coincides with the formula for the echo–signal power recorded by the receiver from a plane surface with a combined scattering phase function sounded through the atmosphere.[1] For $\beta = 0$, $n = 0$, and $\sigma = 0$ formula (4) becomes the expression for average power of the echo signal received from a randomly rough locally Lambertian surface being sounded through the transparent atmosphere.[6] For $\alpha = 0$, $\Delta \to 0$, and $\sigma = 0$ formula (4) becomes the expression for the average power of the echo signal received from a randomly rough locally specular surface sounded through the transparent atmosphere.[7]

Figures 1 and 2 show the results of calculations of the shape of the echo pulse received from a randomly rough surface with a combined scattering phase function of local surface areas at different values of the parameter α/β (the ratio of diffuse to quasispecular components of the reflection).

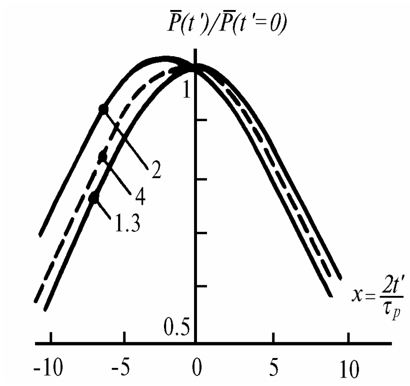

FIG. 1 The echo pulse received from a randomly rough surface with a combined scattering phase function sounded through the transparent atmosphere.

The values $\bar{P}(t')/\bar{P}(t' = 0)$ were calculated using formula (4) for the following values of the parameters: $\theta_s = 70°$, $\theta_r = 65°$, $L_s = 10^4$ m, $L_r = 10^3$ m, $n = 0$, $\alpha_s = 10^{-2}$, $\alpha_r = 10^{-1}$, $\Delta = 3\cdot10^{-2}$, $\tau_p = 3\cdot10^{-9}$ s, $\sigma\langle\gamma^2\rangle = 0$ (Fig. 1), $\sigma\langle\gamma^2\rangle = 10^{-4}$ m^{-1} (Fig. 2), $\beta/\alpha = 0$ (curves *1* and *3*), $\beta/\alpha = 1$ (curves *2* and *4*), $\sigma_0 = 0$ and $\gamma_0 = 0$ (curves *1* and *2*), and $\sigma_0^2 = 2$ m^2 and $\gamma_0^2 = 10^{-3}$ (curves *3* and *4*).

It can be seen from the figures that the echo–signal shape depends on the relative contributions of the diffuse and quasispecular components of the scattering phase function of the surface.

Random roughness of the surface results in a spread of directions of normals to the local reflecting areas and, thus, in the increase in the effective angular width of the quasispecular component of the scattering phase function. (In the case of a perfectly reflecting surface these problems were studied in detail in Ref. 5). An increase in the effective angular width of the quasispecular component of the scattering phase function results, in its turn, in a weakening of the dependence of the echo–signal shape on the relative contributions coming from the diffuse and quasispecular components of the scattering phase function. The turbidity of the atmosphere results in the same effect that can be physically explained by the spreading of the quasispecular component of the scattering phase function in the optically dense atmosphere.

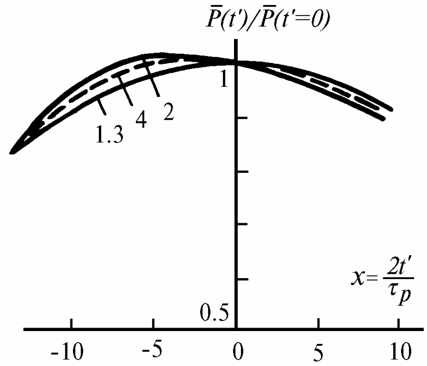

FIG. 2. The echo pulse received from a randomly rough surface with a combined scattering phase function sounded through the optically dense atmosphere.

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