Algorithm for spatiotemporal prediction of weather parameters based on Kalman filtering using a second-order polynomial model

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An algorithm for spatiotemporal prediction of weather parameters based on Kalman filtering using a second-order polynomial model with the varying polynomial coefficients is considered. The experimental tests of the algorithm developed as applied to spatial prediction of mesoscale fields of temperature and zonal and meridional wind components are discussed.

Introduction

Methods based on the Kalman filtering theory are extensively used in recent time for spatiotemporal prediction of meteorological fields.[1–3] These methods fall in the category of dynamic-statistical methods and require a model of dynamic systems to be specified. The system of first-order differential stochastic equations determines the spatiotemporal variability of meteorological parameters on a given mesoscale. This system accounts for statistical characteristics of both the measurement errors and random processes entering into the model of the space of states. Algorithms developed based on the Kalman filtering can be easily implemented on modern microcomputers and do not impose strong requirements on the memory and processor speed. This is first explained by the fact that the algorithms have the recursion form and allow in current time a stepwise correction of the estimated model parameters to be done using discrete measurement data.

The synthesized Kalman filtering algorithms are traditionally divided into linear and nonlinear.[4,5] In this paper, we propose a linear algorithm for prediction of weather parameters at a point inaccessible for direct measurements.

The spatial variability of the weather parameters is determined by the regression model of the space of states. The polynomial coefficients enter into the state vector of the dynamic system and are random processes with preset statistical characteristics. In contrast to classical polynomial models,[6] which use the assumption on constant approximating coefficients all over the measurement range, the polynomial coefficients in the proposed algorithm may vary in time. Thus, the approach considered in this paper is a further development of the classical regression algorithm for prediction of weather parameters by accounting for the time variability of the polynomial coefficients. This paper continues investigations presented in Ref. 1 on the development of new methods and algorithms for spatiotemporal prediction of atmospheric parameters.

1. Statement of the problem

To solve the problem of spatiotemporal prediction of weather parameters at a point inaccessible for direct measurements, we use a mesoscale testing ground with s aerological stations that provide measurements of weather parameters in a given atmospheric layer. All measurements at a fixed time are presented in the form of a profile (vector), each component of which corresponds to a certain height. Therefore, it becomes possible to use the splitting method, in which the whole altitude range is divided into a certain number of independent Kalman filters. Every Kalman filter uses only those measurements of weather parameters that correspond to the given altitude level. The set of predicted estimates of weather parameters for every altitude level provides the estimate of the whole altitude profile inaccessible for direct observations.

The following reasoning is presented for the given altitude level and a fixed weather parameter.

2. Polynomial algorithm for spatial prediction based on Kalman filter

The algorithm for dynamic-stochastic spatial prediction proposed in this paper is based on the Kalman filtering technique proposed in Ref. 1. To synthesize the algorithm for prediction of weather parameters in terms of Kalman filtering, it is necessary to specify the vector in the space of states of the dynamic system and the model of measurements.

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One of the possible variants to construct the algorithm for prediction of weather parameters can be specified based on the Kalman filter estimating the values of the second-order polynomial. The value of the weather parameter $\xi_i(t)$ at the ith point at the time t is determined by the following equation:

$$\xi_i(t) = a_0(t) + a_1(t)\,x_i + a_2(t)\,y_i + a_3(t)\,x_i y_i + a_4(t)\,x_i^2 + a_5(t)\,y_i^2, \quad (1)$$

where x_i and y_i are the Cartesian coordinates of a measuring or forecasting station.

Thus, the coefficients $a_0(t)$, $a_1(t)$, $a_2(t)$, $a_3(t)$, $a_4(t)$, and $a_5(t)$ determine the value of the weather parameter at every time and any point within the mesoscale. Therefore, it seems to be possible to specify the column vector of states of the dynamic system in the following way:

$$\mathbf{X}(t) = |a_0(t),\ a_1(t),\ a_2(t),\ a_3(t),\ a_4(t),\ a_5(t)|^{\mathrm{T}}, \quad (2)$$

where T denotes transposition.

The space of the states of the dynamic system described by the vector (2) is continuous, but in practice it is convenient to pass from the continuous time to the discrete one with an arbitrary estimation step (for example, equal to the input period of weather parameter measurements).

In this case, at the corresponding change of variables, the state vector (2) acquires the following form:

$$\mathbf{X}(k) = |X_1(k),\ X_2(k),\ X_3(k),\ X_4(k),\ X_5(k),\ X_6(k)|^{\mathrm{T}}. \quad (3)$$

The variation dynamics of the state vector components can be described by the system of difference equations:

$$\begin{cases} X_1(k+1)=X_1(k)+\omega_1(k), \\ X_2(k+1)=X_2(k)+\omega_2(k), \\ \dots\dots\dots\dots\dots\dots \\ X_6(k+1)=X_6(k)+\omega_6(k), \end{cases} \quad (4)$$

where $\omega_1(k)$, $\omega_2(k)$, $\omega_3(k)$, $\omega_4(k)$, $\omega_5(k)$, and $\omega_6(k)$ are random perturbations of the system (generating noise, or state noise).

The system of equations (4) in the vector form is as follows:

$$\mathbf{X}(k+1) = \mathbf{X}(k) + \mathbf{\Omega}(k), \quad (5)$$

where $\mathbf{\Omega}(k)$ is the generating noise vector.

Consider then the model of measurement channels. Variations of the weather parameters $\tilde{Y}_i$ at the ith point and the kth instant in time is an additive mixture of its true value and the measurement error $\varepsilon_i(k)$:

$$\tilde{Y}_i = \xi_i(k) + \varepsilon_i(k). \quad (6)$$

The model of measurements can be expressed through the state variables. For this purpose, introduce the transient measurement matrix **H** and write, in the vector form, the relation between the measurement vector $\tilde{\mathbf{Y}}$ and the state vector $\mathbf{X}(k)$ as:

$$\tilde{\mathbf{Y}} = \mathbf{H}(k,\ x,\ y)\,\mathbf{X}(k) + \mathbf{E}(k). \quad (7)$$

The dimension of vectors $\tilde{\mathbf{Y}}$ and $\mathbf{E}(k)$ is determined by the number of the measurement stations s.

The $(6{\times}s)$ transient measurement matrix **H** is determined as:

$$\mathbf{H}(k+1,\ x,\ y) = \begin{vmatrix} 1 & x_1 & y_1 & x_1y_1 & x_1^2 & y_1^2 \\ 1 & x_2 & y_2 & x_2y_2 & x_2^2 & y_2^2 \\ & & & \dots & & \\ 1 & x_s & y_s & x_sy_{si} & x_s^2 & x_s^2 \end{vmatrix}. \quad (8)$$

The model of the dynamic system and the model of measurements are linear, therefore the estimation problem can be solved based on the Kalman–Bucy linear filter that provides estimation of the state vector with the minimum variance.

The traditional algorithm of the Kalman filter uses the following *a priori* information:

$\mathbf{M}[\mathbf{X}(0)] = \mathbf{X}_0$ is the mathematical expectation of the vector of estimated parameters at the initial time;

$\mathbf{M}[(\mathbf{X}(0) - \mathbf{X}_0)\ (\mathbf{X}(0) - \mathbf{X}_0)^{\mathrm{T}}] = \mathbf{P}_0$ is the covariance matrix of the initial estimate of the state vector;

$\mathbf{M}[\mathbf{\Omega}(k)\ \mathbf{\Omega}^{\mathrm{T}}(i)] = \mathbf{R}_{\Omega}\,\delta_{ki}$ is the covariance matrix of the estimated process;

$\mathbf{M}[\mathbf{E}(k)\ \mathbf{E}^{\mathrm{T}}(i)] = \mathbf{R}_{\mathrm{E}}\,\delta_{ki}$ is the covariance matrix of measurement errors;

$\mathbf{M}[\mathbf{\Omega}(k)\ \mathbf{E}(k)^{\mathrm{T}}] = 0$ because the random processes $\mathbf{\Omega}(k)$ and $\mathbf{E}(k)$ do not correlate;

$\mathbf{M}[\mathbf{X}_0\,\mathbf{\Omega}^{\mathrm{T}}(k)] = \mathrm{M}[\mathbf{X}_0\,\mathbf{E}(i)^{\mathrm{T}}] = 0$ because the initial state $\mathbf{X}_0$ does not correlate with the perturbations $\mathbf{\Omega}(k)$ and $\mathbf{E}(k)$.

The algorithm for prediction of the weather parameters has the following form:

$$\hat{\mathbf{X}}(k+1) = \hat{\mathbf{X}}(k+1|k) + \mathbf{G}(k+1)\cdot[\tilde{\mathbf{Y}}(k+1) - \mathbf{H}(k+1,\ \mathbf{x},\ \mathbf{y})\cdot\hat{\mathbf{X}}(k+1|k)], \quad (9)$$

where $\hat{\mathbf{X}}(k+1) = |\hat{X}_1,\ \hat{X}_2,\ \dots,\ \hat{X}_6|^{\mathrm{T}}$ is the estimate of the state vector at the time $(k+1)$; $\hat{\mathbf{X}}(k+1|k) = \hat{\mathbf{X}}(k)$ is the vector of predicted estimates at the time $(k+1)$ calculated from the data at the step k; $\mathbf{G}(k+1)$ is the $(6{\times}s)$ matrix of weighting coefficients.

In the classical Kalman–Bucy linear filter, calculation of the weighting coefficients is a recurrence procedure independent of Eq. (9) and connected with solution of matrix equations for covariance of estimation errors[5]:

$$\mathbf{G}(k+1) = \mathbf{P}(k+1\,|\,k)\,\mathbf{H}^{\mathrm{T}}(k+1,\ \mathbf{x},\ \mathbf{y}) \times [\mathbf{H}(k+1,\mathbf{x},\mathbf{y})\,\mathbf{P}(k+1\,|\,k)\,\mathbf{H}^{\mathrm{T}}(k+1,\mathbf{x},\mathbf{y}) + \mathbf{R}_{\mathbf{E}}(k+1)]^{-1}; \quad (10)$$

$$\mathbf{P}(k+1\,|\,k) = \mathbf{P}(k\,|\,k)\ \mathbf{R}_{\Omega}(k), \quad (11)$$

$$\mathbf{P}(k+1\,|\,k+1) = [\mathbf{I} - \mathbf{G}(k)\ \mathbf{H}(k,\ \mathbf{x},\ \mathbf{y})]\ \mathbf{P}(k+1\,|\,k), \quad (12)$$

where $\mathbf{P}(k+1|k)$ is the *a posteriori* (6×6) covariance matrix of prediction errors; $\mathbf{P}(k+1|k+1)$ is the *a priori* (6×6) covariance matrix of prediction errors; $\mathbf{R_E}(k+1)$ is the diagonal ($s \times s$) covariance matrix of observation noise; $\mathbf{R}_\Omega(k)$ is the diagonal (6×6) covariance matrix of state noise; $\mathbf{I}$ is the (6×6) unit matrix.

Final calculation of the predicted value of the weather parameter $\hat{\xi}_i(k+1)$ at the ith point at the $k+1$ instant is performed by the equation:

$$\hat{\xi}_i(k+1) = \hat{X}_1(k+1\,|\,k) + \hat{X}_2(k+1\,|\,k)x_i + \hat{X}_3(k+1\,|\,k)y_i + \hat{X}_4(k+1\,|\,k)x_i y_i + \hat{X}_5(k+1\,|\,k)x_i^2 + \hat{X}_6(k+1\,|\,k)y_i^2 . \quad (13)$$

For the filtering algorithm (9)–(12) to begin operating at the time $k = 0$ (the initiation time), it is necessary to specify the initial conditions: the initial estimation vector $\hat{\mathbf{X}}(0)$, the initial covariance matrix of estimation errors $\mathbf{P}(0)$, as well as the elements of the covariance noise matrices $\mathbf{R_E}(0)$ and $\mathbf{R}_\Omega(0)$. In practice, $\hat{\mathbf{X}}(0)$ and $\mathbf{P}(0)$ can be specified based on the minimum information about the actual properties of the system, and in the case of complete absence of the useful information it is set that $\hat{\mathbf{X}}(0) = 0$ and $\mathbf{P}(0) = \mathbf{I}$.

3. Results of investigating the quality of Kalman filtering algorithm based on the polynomial model

The above Kalman filtering algorithm based, unlike that in Ref. 1, on a polynomial model with varying polynomial parameters was studied to reveal its quality and efficiency when applied to the problem of spatial prediction (extrapolation) of the mesoscale temperature and wind fields.

Since the spatial extrapolation in this paper is considered as applied to prediction of a pollution cloud of industrial origin, we took the mean values of temperature and wind in some altitude interval $h_k - h_0$, rather than the measured values of these parameters at some levels (here $h_0 = 0$ coincides with the ground level, and h_k is the altitude of the top boundary of the studied kth atmospheric layer). In this case, the layer-mean values of temperature and the zonal and meridional wind velocity components were calculated by the equation

$$<\xi>_{h_k-h_0} = \sum_{i=1}^{k}\left[\left(\frac{\xi_{h_{i-1}} + \xi_{h_i}}{2}\right)\left(\frac{h_i - h_{i-1}}{h_k}\right)\right], \quad (14)$$

where $<\cdot>$ means averaging of the observation data in some atmospheric layer, and ξ is the measured value of a weather parameter at different atmospheric layers.

To estimate the quality of the Kalman filtering algorithm, we used the data of two-month observations (at 0 and 12 h GMT) at five aerological stations: Warsaw (52°10′N, 20°58′E), Kaunas (54°53′N, 23°50′E), Brest (52°07′N, 23°41′E), Minsk (53°56′N, 27°38′E), and Lvov (49°49′N, 23°57′E) that form a typical mesometeorological testing ground. All the temperature and wind observation data presented on standard isobaric surfaces and singular point levels were reduced, using the linear interpolation, to a single system of geometric heights taken as 0 (ground level), 0.2, 0.4, 0.8, 1.2, 1.6, 2.0, 2.4, 3.0, 4.0, 5.0, 6.0, and 8.0 km. This system of geographic heights allows us to describe almost the whole troposphere with high vertical resolution.

As the initial conditions, we took $\hat{\mathbf{X}}(0) = 0$ and $\mathbf{P}(0) = \mathbf{I}$, while the diagonal elements of the noise correlation matrices of observation $\mathbf{R_E}(0)$ and the state $\mathbf{R}_\Omega(0)$ were taken based on the errors of radiosonde measurements given in Ref. 7.

To estimate the accuracy of the Kalman filtering algorithm, as control points we used the Warsaw and Kaunas stations spaced by 185 and 250 km from the closest stations Brest and Minsk that have available measurement data. An important circumstance in this case is that the Warsaw station (under conditions of zonally mean west-to-east transport) is located at the territory to the west of the region, for which we have observations, that is, we consider the case, when the problem of spatial prediction cannot be solved based on the hydrodynamic approach.

As to the evaluation of the quality of the proposed algorithm in the procedure of spatial prediction, it is performed using the root-mean-square error of this prediction

$$\delta_\xi = \left[\frac{1}{n}\sum_{i=1}^{n}(\xi_i^* - \xi_i)^2\right]^{1/2} \quad (15)$$

(here ξ_i and ξ_i^* are the measured and extrapolated values of a weather parameter; n is the number of realizations), as well as the relative deviation $\theta = \delta_\xi/\sigma_\xi$, where σ_ξ is the root-mean-square deviation of this weather parameter.

As an example, Tables 1 and 2 present the root-mean-square (rms) δ and relative θ errors of spatial extrapolation of the layer-mean values of temperature and zonal and meridional wind velocity components up to the distances of 185 and 250 km using Kalman filtering algorithm. For a comparison, they also present the rms and relative errors obtained for the case, when spatial prediction was performed using the optimal extrapolation method based on the use of analytical functions of the form[8]:

for temperature

$$\mu_T(\rho) = \{\exp(-\alpha\rho)\}\cos(\beta\rho), \quad (16)$$

where $\alpha = 0.436$ and $\beta = 0.863$;

for wind velocity components

$$\mu_U(\rho) = \mu_V(\rho) = (1-\alpha\rho)\exp(-\rho)^2, \quad (17)$$

where $\alpha = 1.162$. In Eqs. (16) and (17) ρ is the distance in thousand km.

Table 1. RMS (δ) and relative (θ) errors of prediction of the layer-mean values of temperature and zonal and meridional components of the wind velocity up to the distance of 185 km using the Kalman filtering algorithm with the polynomial model (1) and the method of optimal extrapolation (2)

Layer, km	Winter				Summer			
	δ		θ, %		δ		θ, %	
	1	2	1	2	1	2	1	2
Temperature, °C								
0–0.2	1.9	2.1	49	54	1.9	1.9	44	44
0–0.4	1.8	2.2	47	58	1.9	2.1	49	54
0–0.8	1.8	2.3	49	59	1.8	2.1	50	58
0–1.2	1.8	2.3	51	66	1.7	2.1	49	60
0–2.0	1.7	2.3	45	61	1.4	2.1	41	62
0–4.0	1.6	2.9	37	67	1.1	2.7	33	82
0–6.0	1.7	3.3	39	77	1.1	3.0	31	86
0–8.0	1.7	3.5	40	83	1.1	3.2	30	89
Zonal component of the wind velocity, m/s								
0–0.2	2.1	3.2	54	82	2.1	2.8	64	85
0–0.4	2.8	3.3	64	75	2.3	2.8	67	80
0–0.8	3.2	3.3	59	61	2.5	2.7	66	71
0–1.2	3.2	3.3	52	54	2.5	2.7	62	64
0–2.0	3.0	3.1	45	46	2.5	2.7	58	60
0–4.0	3.0	3.2	42	50	2.3	2.6	47	53
0–6.0	3.5	3.6	40	46	2.4	2.6	45	49
0–8.0	3.8	3.9	38	41	2.6	2.6	46	46
Meridional component of the wind velocity, m/s								
0–0.2	2.3	2.7	66	79	2.1	3.0	66	94
0–0.4	2.7	3.0	66	75	2.3	3.1	66	89
0–0.8	3.1	3.2	66	68	2.1	3.1	57	84
0–1.2	3.2	3.3	64	66	1.9	3.1	49	79
0–2.0	3.1	3.3	60	63	1.7	3.0	40	71
0–4.0	3.0	3.5	47	55	2.1	2.9	49	67
0–6.0	3.6	3.6	46	46	2.4	3.0	50	62
0–8.0	3.9	3.9	41	41	2.8	3.2	52	59

Table 2. RMS (δ) and relative (θ) errors of prediction of the layer-mean values of temperature and zonal and meridional components of the wind velocity up to the distance of 250 km using the Kalman filtering algorithm with the polynomial model (1) and the method of optimal extrapolation (2)

Layer, km	Winter				Summer			
	δ		θ		δ		θ	
	1	2	1	2	1	2	1	2
Temperature, °C								
0–0.2	1.9	2.1	46	51	2.4	2.5	58	61
0–0.4	2.0	2.2	51	56	2.4	2.5	63	66
0–0.8	2.1	2.3	55	60	2.3	2.4	65	69
0–1.2	2.1	2.4	55	63	2.2	2.4	66	80
0–2.0	1.9	2.7	46	66	2.0	2.5	64	86
0–4.0	1.7	3.2	38	71	1.6	2.9	59	88
0–6.0	1.6	3.4	34	72	1.5	3.1	55	89
0–8.0	1.5	3.6	33	80	1.4	3.4	48	94
Zonal component of the wind velocity, m/s								
0–0.2	2.6	2.9	66	76	2.4	3.0	65	81
0–0.4	3.2	3.3	78	80	2.5	3.1	64	79
0–0.8	3.3	3.4	75	77	2.5	3.1	60	74
0–1.2	3.4	3.5	66	69	2.5	3.0	57	68
0–2.0	3.7	3.8	64	66	2.5	2.9	52	60
0–4.0	3.6	4.0	54	60	2.3	3.0	46	60
0–6.0	4.1	4.2	54	55	2.4	3.2	41	54
0–8.0	4.3	4.4	51	52	2.8	3.4	42	51
Meridional component of the wind velocity, m/s								
0–0.2	1.8	3.4	50	94	2.0	3.5	56	97
0–0.4	2.2	3.4	59	87	2.1	3.5	55	92
0–0.8	2.4	3.5	58	80	2.2	3.5	55	85
0–1.2	2.5	3.6	51	73	2.4	3.5	56	81
0–2.0	2.5	3.5	41	57	2.8	3.4	57	69
0–4.0	2.9	3.8	37	49	2.9	3.3	50	57
0–6.0	3.4	4.3	37	47	2.8	3.4	43	52
0–8.0	4.1	4.8	39	46	3.1	3.5	42	48

Analysis of data presented in Tables 1 and 2 shows that

– the Kalman filtering algorithm based on the use of the polynomial model with varying polynomial parameters gives quite acceptable accuracy of spatial extrapolation, especially, at a distance of 185 km. Actually, at the distance of 185 km, regardless of the weather parameter, season, and atmospheric layer, the relative error of such extrapolation varies mostly from 30 to 51% (for temperature) and from 38 to 66% (for the zonal and meridional wind velocity components);

– the Kalman filtering algorithm gives the best results at extrapolation of the layer-mean values of temperature, when, regardless of season, the rms errors of such extrapolation do not exceed 1.9°C and even 1.1°C in summer above 3 km;

– the quality of spatial extrapolation of the parameters $<T>_{h_0,h}$, $<U>_{h_0,h}$, and $<V>_{h_0,h}$, as would be expected, worsens markedly with the distance. Only in winter at extrapolation of the layer-mean values of the meridional wind up to the distance of 250 km, the results are somewhat better than at the same extrapolation up to 185 km, when it is performed against the zonally mean west-to-east transport;

– finally, the Kalman filtering algorithm gives the higher-quality results on spatial prediction than in the case of using optimal extrapolation method, and this algorithm provides for the highest gain when predicting the layer-averaged values of temperature in the free atmosphere (above 2 km).

Thus, the statistical estimation of the quality of Kalman filtering algorithm using the polynomial model with the varying polynomial parameters showed that this algorithm is rather efficient. It almost does to yield to the algorithm proposed in Ref. 1 in quality and can be successfully used for spatial extrapolation of the layer-mean values of temperature and wind velocity components for meteorological support of numerical prediction of the spread of technogenic pollutants over a mesoscale region.

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