SIGNAL–TO–NOISE RATIO IN A COHERENT LIDAR WITH AN INTRACAVITY DETECTION OF RETURN SIGNALS USING AN Nd^{3+}:YAG LASER. THEORY

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Sensitivity of a coherent lidar with an Nd^{3+}:YAG *laser used both as a source of sounding radiation and as a resonance nonlinear amplifier of a weak return signal modulation is investigated. It is shown that an essential (up to four orders of magnitude) increase in the sensitivity can be expected by using this method of return signal detection compared to a direct detection.*

INTRODUCTION

Along with coherent lidars with the heterodyne detection of return signals coherent lidars with an alternative technique of a highly sensitive detection based on resonance amplification of a return signal by a laser are studied. The latter are called the autodyne or LD–lidars. The autodyne lidars based on the CO_2 laser have been considered in Refs. 1–5.

Let us consider the principle possibilities of using an Nd:YAG laser in an autodyne lidar taking into account the fact that lidar systems based on solid–state lasers have small overall dimensions, long lifetime, they are eye–safe, and the effect of turbulence in the spectral range of the Nd:YAG laser generation is negligible. In this paper it is shown that an Nd:YAG laser used in an autodyne lidar is a resonance nonlinear amplifier of a weak modulation of a return signal prior to its detection by a photodetector. The return signal in the low–frequency region of the modulation spectrum can be amplified by a factor of tens. The maximum amplification of the return signal occurs at the modulation frequencies that are in resonance with the relaxation oscillations of the Nd:YAG laser, where the amplification can reach four orders of magnitude.

Thus, a considerable amplification of a recorded return signal prior to its detection by a photodetector makes it possible to essentially increase the level of the signal above the noise of the photorecording channel and above the background noise which is especially strong during the day. When determining the atmospheric optical characteristics the sensitivity of a lidar will be limited by the fluctuations of a sounding signal acquired during the propagation through the atmosphere in the direction to the reflecting object and backward, as well as by the intrinsic noise of a laser.

The expressions for the limiting values of the coefficient of modulation of the sounding signal, that could yet be measured are given in the paper when the laser noise is dominating.

1. THEORY

Block diagram of a coherent lidar functioning with the intracavity amplification of a return signal in an Nd:YAG laser can be described as follows. The Nd:YAG laser generates sounding radiation. A weak return signal from aerosol particles (or from an external mirror, topographic target, etc.) is collected by a telescope and then it is directed to the cavity of the transmitting laser. The resultant signal is finally directed to a photodetector. The boundary condition at the mirror R_1, which relates the fields inside the laser cavity $E^{(-)}(l)$ and $E^{(+)}(l)$ with the field of radiation $E^{(+)}(l + L)$ passed through the path L from the laser to the reflector and back, has the form

$$E^{(-)}(l) = \sqrt{R_1}E^{(+)}(l) + \left(1 - \sqrt{R_1}\right)^2 \sqrt{R_1(L,\, \mu t)} \times$$
$$\times \exp\left[-\alpha(\mu t)\, L\right] E^{(+)}(l + L) \exp\left[i(\varphi_1 - \varphi_2 + \theta)\right], \qquad (1)$$

where R_0 and R_1 are the reflection coefficients of the laser cavity mirrors, R_2 is the reflection coefficient of the external mirror or of the reflecting aerosol formation, $\alpha(\mu t)$ is the integral coefficient of power losses of the sounding signal propagating along the path to the external mirror R_2 and back, θ is the phase difference occurring in the sounding signal during the propagation along the path from the mirror R_1 to the reflector and back.

If the reflector moves with the velocity $v_{\|}$ then $\theta = 4\pi v_{\|}/\lambda$ ($v_{\|}$ is the projection of the reflector velocity onto the sounding beam). The quantities $\alpha(\mu t)$ and $R_2(\mu t)$ can have a slow ($\mu \ll 1$) dependence on time (compared to the lifetime of the sounding photons in the laser cavity R_0 and R_1).

Let the fields $E^{(+)}(l + L)$ and $E^{(+)}(l)$ be related to each other by the relation $E^{(+)}(l + L) = B{\cdot}E^{(+)}(l){\cdot}\exp(2\, i\, kL)$, where B is the coefficient of a spatial and temporal matching of a return signal with the laser cavity mode the sounding signal is formed at. Then Eq. (1) takes the form

$$E^{(-)}(l,\, t) = \sqrt{R_1}E^{(+)}(l,\, t)\, [1 + \mathrm{Re}\langle\varepsilon(\mu t)\rangle],$$

where

$$\mathrm{Re}\langle\varepsilon(\mu t)\rangle \equiv$$
$$\equiv \langle B \exp[-\alpha L]\cos(2\, kL + \theta)\left(1 - \sqrt{R_1}\right)^2 \sqrt{R_2}/\sqrt{R_1}\rangle, \qquad (2)$$

$\langle\ldots\rangle$ denotes the averaging since in the general case α, R_2, θ, and L are random values. By this the description of a laser detection of a return signal is reduced to the known problem[6] on a laser with modulated losses. Since we are interested in the detection of superweak return signals, we can at certain describe the problem within the framework of

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a linear regime of modulation, i.e., far from the regime when the laser could be forced to generate at a new frequency. In this connection, we will not include the equation for the phase variable in the further description. Taking into account Eqs. (1) and (2) the system of equations, which describes the dynamics of the number of photons of the generated mode n and the inverse population of the generating atoms N in the cavity, has the form[7–9]

$$\dot{n} = - C n + g\, n N ;$$

$$\dot{N} = \Lambda - N/T - g\, n N , \qquad (3)$$

where $C = c/l\left[2 - \sqrt{R_0} - \sqrt{R_1}(1 + \mathrm{Re}<\varepsilon>) + 2\alpha_0 l\right]$ is the rate of photons escape from the laser cavity, c is the speed of light, g depends, in a known way, on the parameters of the active medium and the cavity, Λ is the rate of pumping to the upper working level, and T is the relaxation time of the difference in populations.

Spectral component of the amplitude of the photocurrent response to changing in the output power of the laser emission P_{out} due to its modulation by a return signal has the form

$$(\delta\, i)^{\mathrm{LD}}_{\Omega} = \frac{qe}{h\nu} \frac{\mathrm{Re}<\varepsilon>_{\Omega}}{\mathrm{k}} P_{\mathrm{out}}\, F(\Omega) , \qquad (4)$$

where $\kappa \equiv \left(2 - \sqrt{R_0} - \sqrt{R_1} + 2\alpha_0 l\right)$ is the total value of losses inside the laser cavity without the return signal action, q is the quantum efficiency of the photodetector, $F(\Omega)$ is the spectrum of the response function of an Nd:YAG laser to the modulation action. In the case of direct detection one and the same modulation action on the sounding signals of the same output power P_{out} can lead to the following variation in the photocurrent:

$$(\delta\, i)^{\mathrm{dd}}_{\Omega} = \frac{qe}{h\nu} \mathrm{Re}<\varepsilon>_{\Omega} P_{\mathrm{out}} . \qquad (5)$$

We denote the ratio $(\delta\, i)^{\mathrm{LD}}_{\Omega}/(\delta i)^{\mathrm{dd}}_{\Omega}$ as Γ and it has the form

$$\Gamma = F(\Omega)/\kappa . \qquad (6)$$

Let us now consider two characteristic cases: 1) $\Omega \ll \Omega_{\mathrm{rel}}$ and 2) $\Omega \sim \Omega_{\mathrm{rel}}$, where Ω_{rel} is the relaxation frequency of an Nd:YAG laser. In the first case $F(\Omega) = m/(m - 1)$, while in the second one $F(\Omega_{\mathrm{rel}}) = \dfrac{c\mathrm{k}\, T}{2\, l\, m}$ (see Refs. 8 and 9), where m characterizes the excess of the pumping above the generation threshold. Let us assume that $\kappa = 0.1$, $m = 2$, $l = 0.5$ m, and it is known[10] that $T \sim 2.3 \cdot 10^{-4}$, then in the first case $\Gamma \simeq 20$ and in the second one $\Gamma \simeq 3.45 \cdot 10^{4}$. Thus, an Nd:YAG laser can serve as a resonance nonlinear amplifier of a weak modulation of return signals. The maximum amplification of the modulation occurs at the frequency of relaxation oscillations. In the case of such a considerable amplification the noise of laser emission is the basic source of noise. Based on relation (4) and a known expression[10,11] for the power spectrum of the noise of an Nd:YAG laser we can write the signal–to–noise ratio for two characteristic spectral ranges: $\Omega \ll \Omega_{\mathrm{rel}}$ and $\Omega \sim \Omega_{\mathrm{rel}}$. In both cases the signal–to–noise ratio has the form

$$(S/N)^{\mathrm{LD}}_{\Omega\approx 0} = \left(\frac{qP_{\mathrm{out}}}{2\, h\nu\Delta f}\right)^{1/2} \frac{\mathrm{Re}<\varepsilon>}{\mathrm{k}} \frac{m}{m - 1} ; \qquad (7)$$

$$(S/N)^{\mathrm{LD}}_{\Omega\approx\Omega_{\mathrm{rel}}} = \left(\frac{\mathrm{k}/2\tau P_{\mathrm{out}}}{2\, h\nu\Delta f}\right)^{1/2} \frac{\mathrm{Re}<\varepsilon>}{\mathrm{k}} . \qquad (8)$$

Let us estimate based on Eqs. (7) and (8), for example, the minimum coefficient of reflection R_2^{min} assuming the signal–to–noise ratio to be equal to unity. In the low–frequency region of the R_2 modulation spectrum we have

$$(R^{\mathrm{min}}_2)^{1/2}_{\Omega\sim 0} = \left(\frac{2\, h\nu\Delta f}{qP_{\mathrm{out}}}\right)^{1/2} \frac{m - 1}{m} \frac{\mathrm{k}\sqrt{R_1}\left(1 - \sqrt{R_1}\right)^{-2}}{B \exp(-\, \alpha L)} , \qquad (9)$$

in the region of relaxation oscillations

$$(R^{\mathrm{min}}_2)^{1/2}_{\Omega\sim\Omega_{\mathrm{rel}}} = \left(\frac{2\, h\nu\Delta f}{\mathrm{k}/2\tau P_{\mathrm{out}}}\right)^{1/2} \frac{\mathrm{k}\sqrt{R_1}\left(1 - \sqrt{R_1}\right)^{-2}}{B \exp(-\, \alpha L)} . \qquad (10)$$

CONCLUSIONS

Any measurements of atmospheric parameters or objects moving in the atmosphere and reflecting the sounding signal which are carried out using an Nd:YAG laser and the method of direct detection can be more effectively carried out based on a coherent lidar with the intracavity detection of return signals by the Nd:YAG laser. Considerable resonance amplification of return signals prior to detection by a photodetector is the main basic for higher efficiency. This makes it possible, in particular, to get rid of the noise caused by the sky background which usually limits the sensitivity of lidars during the day. The worsening of temporal resolution in return signals is a peculiar fee for a considerable growth of the sensitivity in the vicinity of the frequency of relaxation oscillations.

And finally I would like to note that the factor of resonance amplification of a return signal in a lidar system with the CO_2–laser intracavity detection has been experimentally studied in Ref. 12.

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