THE EFFECT OF THE PHASE FLUCTUATION DISTRIBUTION LAW ON THE TYPE OF ALGORITHM OF OPTIMAL MEASURING OF AN ANGULAR COORDINATE

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The asymptotically optimal algorithm for measuring an angular coordinate of the light source with phase fluctuation distribution not following the Gaussian law with independent increments is synthesized.

The problem in measuring an angular coordinate of a source of coherent light based on the phase front of a receiving lens is considered in Ref. 1. It is assumed that the phase fluctuations caused by the effect of a great amount of inhomogeneities in the propagation medium are distributed following the normal distribution law. However, as shown in Ref. 2, this condition does not always hold. Therefore it is interesting to analyze the effect of the distribution law of phase fluctuations on the type of the optimal algorithm and the potential accuracy in measuring an angular coordinate.

Let us consider a meter whose input signal is a digitized sampling of counts of a phase difference over the aperture

$$\mathbf{Y} = \beta\,\mathbf{X} + \mathbf{N}\,, \tag{1}$$

where β is the amplitude factor related to the angle of a light–signal arrival in terms of the known dependence, $\mathbf{X} = \|x_1, x_2,\ldots, x_m\|^{\mathrm{T}}$ is the vector of the reference signal; $\mathbf{N} = \|n_1, n_2,\ldots, n_m\|^{\mathrm{T}}$ is the vector of noise phase fluctuations. Thus the problem is reduced to measuring the parameter β which is of energy nature.

To solve this problem the following assumptions should be made. Let us consider that the antenna is matched with the direction of the beam arrival, i.e., $\beta = 0$, the parameter β is measured against the background of asymptotically intense sampling of noise with independent increments, and the distribution of non–Gaussian increments of noise n_i follows the law

$$p(y) = a \exp\left[\,-\,b\,|y|^{2\nu}\right]. \tag{2}$$

Taking into account the aforementioned assumptions the logarithm of the likelihood ratio takes the form[3]

$$\ln l = \ln\left(\prod_{i=1}^{m} p_n(y_i - \beta x_i) \Big/ \prod_{i=1}^{m} p_n(y_i)\right) =$$

$$= \sum_{i=1}^{m} \left(\ln p_n(y_i - \beta x_i) - \ln p_n(y_i)\right). \tag{3}$$

Since the measured parameter is of energy nature, in the approximation of the probability density logarithm we must restrict our consideration to the first three terms of the Taylor series

$$\ln p_n(y - x) = \ln p_n(y) - \frac{\mathrm{d}\ln p_n(y)}{\mathrm{d}y}\,\beta\,x + \frac{1}{2!}\,\frac{\mathrm{d}^2\ln p_n(y)}{\mathrm{d}y^2}\,\beta^2 x^2. \tag{4}$$

Substituting expression (4) in Eq. (3) and using the likelihood equation $\mathrm{d}\ln l/\mathrm{d}\beta\,|_{\beta=\hat\beta}$ give the optimal estimate

$$\hat\beta = \sum_{i=1}^{m} \frac{\mathrm{d}\ln p_n(y_i)}{\mathrm{d}y_i}\,x_i \Big/ \sum_{i=1}^{m} \frac{\mathrm{d}^2\ln p_n(y_i)}{\mathrm{d}y_i^2}\,x_i^2\,. \tag{5}$$

As can be seen from Eq. (5), in constructing the optimal meter of energy parameters, in contrast to measuring the nonenergetic ones, knowledge of the second derivative of the probability density is also required.

We now write the asymptotically synthesized optimal algorithm for measuring the parameter β in more detail. To this end, in Eq. (5) we substitute the first and second derivatives of the probability density which, by virtue of Eq. (2), have the form

$$\frac{\mathrm{d}\ln p_n(y)}{\mathrm{d}y} = -\,2\,b\,\nu\,|y|^{2\nu}\,\frac{1}{y}\,,$$

$$\frac{\mathrm{d}^2\ln p_n(y)}{\mathrm{d}y^2} = -\,2\,b\,\nu\,\frac{|y|^{2\nu}\,(2\nu - 1)}{y^2}\,.$$

We finally obtain

$$\hat\beta = \sum_{i=1}^{m} |y_i|^{2\nu}\,\frac{1}{y_i}\,x_i \Big/ \sum_{i=1}^{m} \frac{|y_i|^{2\nu}}{y_i^2}\,(2\nu - 1)\,x_i^2\,. \tag{6}$$

It is easy to verify that if the increments y_i are distributed following the normal distribution, i.e., $\nu = 1$, then the algorithm of optimal measuring (6) is reduced to the known form[3]

$$\hat\beta = \sum_{i=1}^{m} y_i\,x_i \Big/ \sum_{i=1}^{m} x_i^2 = \xi/q^2\,. \tag{7}$$

Here ξ is the weight sum and q^2 is the signal-to-noise ratio.

Let the following designations be introduced by analogy with Ref. 3:

$$\varphi(y) = |y_i|^{2\nu}\,\frac{1}{y_i}\,;\quad \varphi'(y) = |y_i|^{2\nu}\,\frac{1}{y_i^2}\,(2\nu - 1)\,,$$

where $\varphi(y)$ and $\varphi'(y)$ are the characteristics of the nonlinear element[3] and its derivative, respectively. The figure depicts the plots of probability densities and the corresponding

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functions $\varphi(y)$ and $\varphi'(y)$. As can be seen in the figure, in measuring the amplitude multiplier the most important are those increments of the reference signal **Y** which are distorted with a nonlinear element with the characteristic $\varphi(y)$ to the lowest degree.

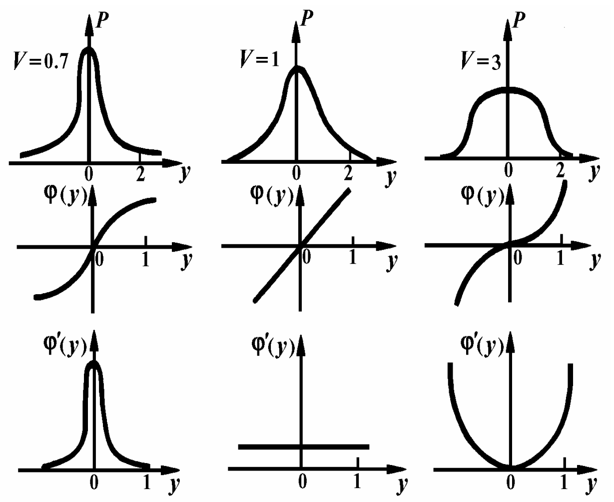

FIG. 1.

Let us now estimate the gain in accuracy in the measurement of the parameter β provided with the asymptotically optimal algorithm (6) as compared with the algorithm being optimal in the presence of Gaussian phase fluctuations.

The calculations reveal that the gain in the measurement accuracy takes place for the values $\nu \neq 1$. When $\nu = 1$ the measurement errors are the same. Thus for $\nu = 3$ the gain in m is equal to $\sigma_{\beta}^2/\sigma_{\beta\,\mathrm{opt}}^2 \approx 2.5$. The maximum gain in accuracy is observed for $\nu = 1$, i.e., with increase of the so–called "tails" of probability density and when $\nu = 0.4$ and $m = 5$.

It should be noted that all of the foregoing discussions are valid when the sampling increments of phase fluctuations are independent. However, the latter, by their nature, are the correlated ones. It is obvious that the gain in measurement accuracy becomes lower with increase of correlation between the counts.

REFERENCES

1. P.A. Bakut, V.A. Loginov and I.N. Troitskii, Radiotekh. Elektron. **2**, 286–291 (1977).

2. E.Gundze and Lyu Chzhaokhan', Proc. IEEE **4**, 5–45 (1982).

3. Ya.D. Shirman and V.N. Manzhos, *Theory and Technique of Processing of Noisy Radar Information* (Radio i Svyaz', 1988), 416 pp.