

Dependence of light intensity in the geometric shadow of an opaque screen on the angles of light deflection near screen edge

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Relations have been derived for the intensity of edge light (boundary wave) propagated from the screen's edge area toward its shadow. The relations were obtained based on the empirical expressions that characterize the deflection angles of edge rays depending on the distance between the initial ray trajectory and the straight edge of a thin screen. A comparison performed with the experimental data proves the validity of the obtained formulas.

According to Ref. 1 the source of an edge light (boundary wave) is not the screen edge, but the area (zone) above it in which the incident rays decline along the direction off the screen and towards its shadow, becoming the edge rays.

According to the experimental investigations in Ref. 2 the deflection of light rays with $\lambda = 0.53 \mu\text{m}$ in zone at the straight edge of a thin screen (razor blade) are described by the formula

$$\varepsilon = 259.5 / (h_z + 0.786), \quad (1)$$

where ε is the deflection angle in minutes of arc; h_z is the distance between the initial ray trajectory and the screen's edge, in microns.

As was established in Ref. 3, for other λ values the formula (1) takes the following form

$$\varepsilon = 259.5\lambda / 0.53(h_z + 0.786). \quad (2)$$

These relations allow one to establish the dependence of the edge light intensity on the ray deflection angles and on the distance between the scanning slit and the geometric shadow boundary (s.b.), as well as its connection with an incident light intensity and the parameters of diffraction optical layout.

Let's show these opportunities with reference to the light propagating toward the screen shadow in various optical arrangement of diffraction.

I. A convergent incident beam, $\lambda = 0.53 \mu\text{m}$. The corresponding optical arrangement is presented in Fig. 1, where point O is the slit image center illuminated by a parallel beam and being a linear light source; ob is the lens objective; ε_1 and ε_2 are deflection angles of the rays 1 and 2 in the screen deflection area at the distances h_{z1} , $h_{z2} = (h_{z1} + \Delta h_z)$, μm ; L is the distance from the screen to the scanning plane, mm; ΔH is the scanning slit width in μm ; R is the distance from the middle of ΔH to the s.b. (point O) in μm ; H_1 and H_2 are the distances from the rays 1 and 2 impact area to the shadow boundary, in μm .

The rays deflected within Δh_z arrive at ΔH :

$$H_1 = 10^3 L \tan(\varepsilon_1 + \alpha_1) - 10^3 L \tan \alpha_1.$$

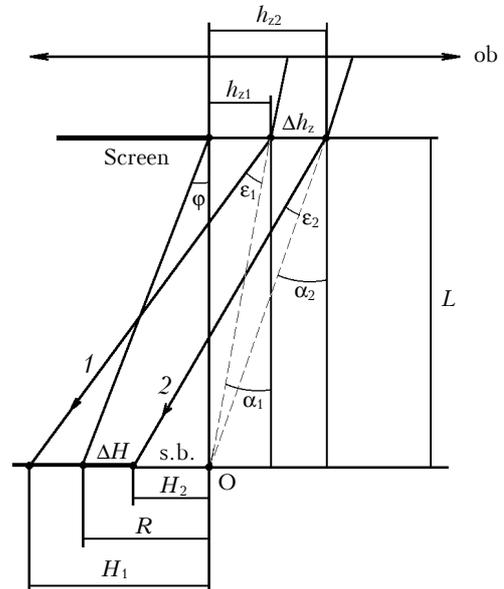


Fig. 1. Diagram explaining the formation of light intensity in the thin screen shadow area with a straight edge for a converging light beam.

As for angles $\leq 5^\circ$ the angle tangent is equal to the angle in radians $H_1 = 10^3 L \varepsilon_1$, rad.

Since

$$1 \text{ rad} = 57.3^\circ \cdot 60' = 3438', \text{ then } H_1 = \frac{L}{3.438} \varepsilon_1'. \quad (3)$$

Similarly $H_2 = \frac{L}{3.438} \varepsilon_2'$. From this it follows that

$$\Delta H = (H_1 - H_2) = \frac{L}{3.438} \Delta \varepsilon'.$$

On the basis of Eq. (1)

$$\varepsilon_1' = \frac{259.5}{h_{z1} + 0.786}, \quad \varepsilon_2' = \frac{259.5}{h_{z1} + \Delta h_z + 0.786}.$$

Therefore:

$$\Delta\varepsilon' = \frac{259.5\Delta h_z}{(h_z + 0.786)(h_z + \Delta h_z + 0.786)},$$

where $h_z = h_{z1}$.
Then

$$\Delta H = \frac{75.48L\Delta h_z}{(h_z + 0.786)(h_z + \Delta h_z + 0.786)}.$$

Hence

$$h_z = -\frac{(1.572 + \Delta h_z)}{2} + \sqrt{\frac{(1.572 + \Delta h_z)^2}{4} + \left(\frac{75.48L}{\Delta H} - 0.786\right)\Delta h_z - 0.6177}. \quad (4)$$

According to Eqs. (3) and (1)

$$H = H_1 = \frac{75.48L}{(h_z + 0.786)}; \quad (5)$$

$$h_z = \left(\frac{75.48L}{H} - 0.786\right). \quad (6)$$

According to Eqs. (4) and (6)

$$\left(\frac{75.48L}{H} - 0.786\right) = -0.786 - 0.5\Delta h_z + \sqrt{0.25(1.572 + \Delta h_z)^2 + \left(\frac{75.48L}{\Delta H} - 0.786\right)\Delta h_z - 0.6177}.$$

Having re-arranged the equality, we obtain

$$\Delta h_z = \frac{75.48L\Delta H}{H^2 - H\Delta H}; \quad (7)$$

$$H = (R + 0.5\Delta H),$$

therefore

$$\Delta h_z = \frac{75.48L\Delta H}{R^2 - 0.25\Delta H^2}. \quad (8)$$

Let φ denote the angle between the lines, passing through the screen edge in the center of ΔH and point O.

As

$$\varphi(\min) = \frac{R}{10^3 L} 3438 = \frac{3.438R}{L},$$

then $R^2 = \frac{\varphi^2 L^2}{3.438^2}$ and

$$\Delta h_z = \frac{75.48\Delta H}{0.0845\varphi^2 L - 0.25\Delta H^2 / L}. \quad (9)$$

In experiments with a convergent beam⁴ at $L = 21.9$ mm, $\Delta H = 36$ μ m have been established, that the edge fluxes propagating toward the shadow area and to the opposite side have approximately equal values, and the intensity in them at $\varphi \geq 15'$ varies proportional to φ^{-2} and R^{-2} . (At smaller φ the intensity variation was not investigated because direct rays penetrate the scanning slit at small R).

Let $\Delta\Phi_i$ denote the flux incident on Δh_z , its intensity is $I_{i.s}$, and the flux propagating from Δh_z toward the screen shadow as $\Delta\Phi_{sh}$, its intensity is $I_{sh.s}$. As $\Delta\Phi_{sh} = 0.5\Phi_i$, then $I_{sh.s} = 0.5I_{i.s}$. In the scanning plane $\Delta\Phi_{sh}$ will be distributed over ΔH .

Hence, the edge light intensity in a shadow area at the distance L from the screen is $I_{sh.1} = 0.5I_{i.s}\Delta h_z / \Delta H$. Substituting instead of Δh_z its values determined by formulas (8) or (9), we shall obtain, respectively,

$$I_{sh.1} = 0.5 \frac{75.48LI_{i.s}}{R^2 - 0.25\Delta H^2}, \quad (10)$$

$$I_{sh.1} = \frac{75.48I_{i.s}}{0.0845\varphi^2 L - 0.25\Delta H^2 / L}. \quad (11)$$

In Table 1 the calculated results on $I_{sh.1}/I_{i.s}$ (Eq. (10)), Δh_z (Eq. (7)), H (Eq. (5)), R , ε (Eq. (1)), φ , $I_{sh.1.i}/I_{sh.1.0}$, $(R_0/R_i)^2$ for $L = 21.9$ mm and $\Delta H = 20$ μ m are presented for various h_z . From the comparison of $I_{sh.1.i}/I_{sh.1.0}$ with $(R_0/R_i)^2$ it is evident, that for $\varphi \geq 15'$ $I_{sh.1}$ is inversely proportional to R^2 , as well as in the above-mentioned experiments. On the basis of Eq. (10) the disturbance of the given dependence occurs and it becomes stronger as R^2 approaches $0.25\Delta H^2$.

Table 1. The dependence of light intensity in the screen shadow area on R at various values of h_z

#	h_z , μ m	H , μ m	R , μ m	ε , min	Δh_z , μ m	$\frac{I_{sh.1} \cdot 10^3}{I_{i.s}}$	$\frac{I_{sh.1.i}}{I_{sh.1.0}}$	$\left(\frac{R_0}{R_i}\right)^2$	φ , min	$\frac{I_{sh.1.i}}{I_{sh.1.0}} / \left(\frac{R_0}{R_i}\right)^2$
0	2	593.33	583.33	93.14	0.0971	2.425	1	1	91.57	1
1	5	285.69	275.69	44.85	0.4355	10.885	4.4886	4.477	43.28	1.0025
2	10	153.26	143.26	24.06	1.6188	40.47	16.6886	16.58	22.5	1.0065
3	15	104.71	94.71	16.44	3.727	93.17	38.4206	37.931	14.87	1.0129
4	20	79.53	69.53	12.48	6.984	174.595	71.998	70.3937	10.91	1.0227
5	25	64.11	54.11	10.06	11.693	292.325	120.546	116.239	8.49	1.037
6	30	53.69	43.69	8.43	18.274	456.85	188.392	178.233	6.86	1.057
7	35	46.19	36.19	7.25	27.326	683.1	281.69	259.783	5.68	1.0843
8	40	40.53	30.53	6.36	39.735	993.38	409.641	365.09	4.79	1.122
9	45	36.1	26.1	5.67	56.866	1421.66	586.25	499.393	4.1	1.174
10	55	29.63	19.63	4.65	115.843	2896.1	1194.26	882.93	3.08	1.353

In contrast to R , the H parameter has no general reference point. To pass from H to R , we shall replace in Eq. (12) $h_{z1} = h_z$ by its value from Eq. (6). As a result we shall obtain the relation

$$R = \left[H - 0.5\Delta H - \left(\frac{75.48L}{H} - 0.786 \right) \frac{L+l}{l} \right]. \quad (17)$$

From formula (17) we have

$$H = \left[0.5 \left(R + 0.5\Delta H - 0.786 \frac{L+l}{l} \right) + \sqrt{B^2 + 75.48L \frac{L+l}{l}} \right], \quad (18)$$

where

$$B = 0.5 \left[R + 0.5\Delta H - 0.786 \frac{L+l}{l} \right].$$

From Eqs. (6) and (12), in the case of $R = 0$ we have:

$$h_z = -0.5 \left(\frac{0.5\Delta H l}{L+l} + 0.786 \right) + \sqrt{C^2 + \frac{(75.48L - 0.393\Delta H)l}{L+l}}, \quad (19)$$

where

$$C = 0.5 \left(\frac{0.5\Delta H l}{L+l} + 0.786 \right).$$

In Table 2 the values of H , Δh_z , $I_{sh.l}/I_{i.l}$, R are given, being calculated by Eqs. (5), (14), (16), and (17) at $l = 12$ mm; $L = 100$ mm; $\Delta H = 20$ μ m for various values of h_z , and also the values of the φ angle, determined from the relation of $\tan\varphi = R/L$. It follows, that the actual light intensity at shadow boundary ($R = 0$) is equal to 0.25 of the incident light intensity ($I_{sh.l}/I_{i.l} = 0.2534$). According to the

quantities close to unity $\frac{I_{sh.l.i}}{I_{sh.l.o}} \left(\frac{H_0}{H_i} \right)^2$, the light intensity in the screen shadow in the case of a divergent incident beam is inversely proportional to H^2 at $\varepsilon \geq 45'$, i.e. at greater threshold value of ε , than for the case of a convergent beam. The inversely proportional dependence of $I_{sh.l}$ on R^2 occurs even at a larger ε .

In Table 3 the experimental and calculated by Eq. (16) values of the relative light intensity in the screen shadow area $I_{sh.l.s}/I_{s.b.s}$, $I_{sh.l}/I_{i.l}$ at $R = H_{max1}$ are given (s' is the slit image width being a linear light source, when a beam is convergent, or the slit width in the focus of a lens making beam parallel; t_s is the scanning slit width; $I_{s.b.s} = I_{i.l}$ at s.b., H_{max1} is the distance from max_1 to s.b. in the diffraction pattern from the screen with a straight edge, determined according to Ref. 5 by the formula

$$H_{max1} = \frac{h_z(L+l)}{l} + \sqrt{\frac{0.5\lambda L(L+l)}{l}}.$$

Table 2. The variation of light intensity in the screen shadow for the case of a divergent incident beam

#	h_z , μ m	H , μ m	R , μ m	ε , min	Δh_z , μ m	$\frac{I_{sh.l.i}}{I_{i.l}} \cdot 10^3$	$\frac{I_{sh.l.i}}{I_{sh.l.o}}$	$\left(\frac{H_0}{H_i} \right)^2$	$\left(\frac{R_0}{R_i} \right)^2$	φ , min	$\frac{I_{sh.l.i}}{I_{sh.l.o}} \left(\frac{H_0}{H_i} \right)^2$
0	2	2709.26	2680.6	93.14	0.0205	4.783	1	1	1	92	1
1	5	1304.53	1247.86	44.85	0.0863	20.134	4.2094	4.311	4.614	42.9	0.976
2	10	699.8	596.46	24.06	0.2754	64.25	13.433	14.99	20.18	20.51	0.896
3	15	478.15	328.15	16.44	0.5172	120.66	25.227	32.08	66.73	11.28	0.786
4	20	363.13	166.46	12.48	0.7637	177.94	37.2	55.66	259.3	5.72	0.668
5	25	292.72	49.33	10.06	0.9868	230.22	48.133	85.66	2945.7	1.7	0.562
6	27.51	266.76	0	9.171	1.086	253.4	52.98	103.15		0	0.514

Table 3. The relative intensity of shadow light at $R = H_{max1}$ under various conditions

#	Screen	s' , μ m	l , mm	L , mm	R , mm	$\frac{I_{sh.l.s}}{I_{s.b.s}}$	$\frac{I_{sh.l}}{I_{i.l}}$	t_s , μ m				
1	Razor blade "Ladas"	35	12	99.5	0.629	0.0344	0.0594	50				
2	Razor blade "Sputnik"					0.0353						
3						52.5			99.5	0.3527	0.0362	0.0582
4	Razor blade "Neva"					161			99.5	0.2641	0.0355	0.0584
5						8.63			198.5	1.4248	0.034	0.0597
6	Razor blade "Ladas"	20				0.0425						
7	Razor blade "Ladas"	35	12	99.5	0.629	0.0316	0.0594	25				
8	Cu-foil, thickness 20 μ m					0.0334						
9	Pb-foil, thickness 30 μ m					0.0314						
10	Opaque layer of soot, thickness 51 μ m on a glass, thickness of 1 mm					0.0442			0.0594			
11	Razor blade "Neva"	20	∞			114.2	0.223	0.032	0.0582	20		
						148.8	0.2543	0.0323	0.0582			
						279.5	0.3488	0.03	0.058			

Apparently according to the rows 1 to 5 of the Table 3, calculated values of the relative intensity in the screen shadow similarly to the experimental ones have constant value at various $l, L, R = l, L, R = H_{\max 1}$.

Conformity to the experiment in such a peculiarity, and also the equality $I_{sh,l}/I_{i,l} = 0.25$ at s.b. demonstrate the validity of formula (16) convincingly enough.

A smaller value of the experimental intensity compared with the calculated one is caused, in particular, by that the actual width s' of a linear light source is nonzero (Fig. 3).

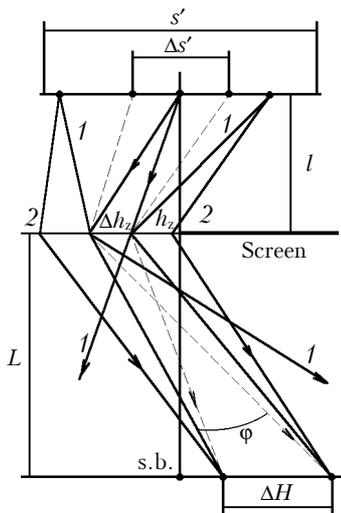


Fig. 3. Diagram explaining the reason of dependence of light intensity in the screen shadow on the linear light source width.

Under these conditions the rays incident on ΔH through Δh_z are incident on Δh_z with $\Delta s' = 2\Delta Hl/L$. The rays 1 incident on Δh_z from more distant parts of s' after the deflection on a screen are incident on the scanning plane external to ΔH . Therefore the light flux, which is incident on Δh_z and then coming then through ΔH , turns out to be weaker than the light flux which is incident on Δh_z . In addition to the rays deflected toward the screen containing Δh_z the rays 2, deflected in the zone outside Δh_z , also reach ΔH thus increasing the flux incident on ΔH . All this yields the relation which is different from the above-described between the light fluxes incident on Δh_z and falling on ΔH , being a cause of the discrepancy between calculation and the experiment.

From all the above-considered it follows, that reduction of s' should be accompanied by the growth of the experimental relative intensity, while the reduction of ΔH by its decrease. Really, according to row 6 of the Table 3 the reduction of s' from 35 to 20 μm has caused the relative intensity increase in the screen shadow from 0.0344 to 0.0425. The reduction of t_s from 50 up to 25 μm , on the contrary, has decreased it a little (see rows 7 to 9).

The other reason of smaller value of the experimental intensity as compared with the calculated one consists in superposition on an edge light, deflected directly to the shadow and characterized by Eq. (16), light reflected from the screen edge after the incidence on it during the deflection toward the screen from smaller h_z (Fig. 4). As it was noted in Ref. 1 owing to the half-wave losses at reflection the second component appears in antiphase with the first and consequently attenuates it.

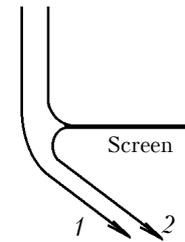


Fig. 4. The diagram of formation of the resulting flux of an edge light propagating in the screen shadow.

According to Ref. 6, the application of soot on the screen, reducing the reflected light, causes a significant light amplification in the screen shadow. For the same reason at the use of an opaque soot layer as a screen (row 10) the light intensity in a shadow area grows up to 0.0442.

Recurrence of the performed operations with the use of relation (2) instead of Eq. (1) leads to formulas, valid for different λ , with the factor $\lambda/0.53$ at a constant 75.48, where λ is in μm .

III. A plane incident wave, $\lambda = 0.53 \mu\text{m}$.

The diagram corresponding to this case is depicted in Fig. 5. In it $R = (H_1 - h_z - 0.5\Delta H)$. Below we take $H_1 = H$.

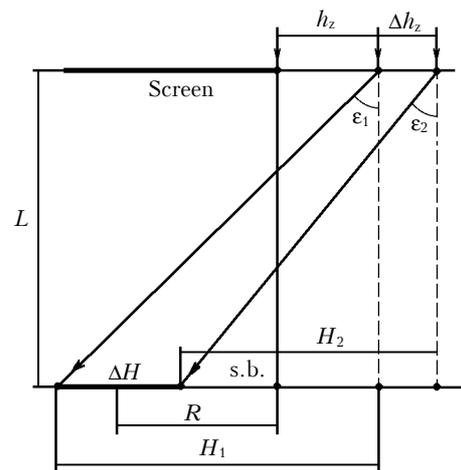


Fig. 5. Diagram explaining the formation of light intensity in the screen shadow in the case of a plane incident wave.

It is easy to understand, that relations necessary in the considered case can be obtained without a preliminary derivation, equating the l parameter to infinity in formulas for a diverging beam. They have a form of

$$\Delta h_z = \left[-0.5 \left(\frac{75.48L}{H} + H - \Delta H \right) + \sqrt{A^2 + \frac{75.48L\Delta H}{H}} \right], \tag{20}$$

where

$$A = 0.5 \left(\frac{75.48L}{H} + H - \Delta H \right);$$

$$I_{sh.1}/I_i = 0.5 \left[-0.5 \left(\frac{75.48L}{H\Delta H} + \frac{H}{\Delta H} - 1 \right) + \sqrt{0.25 \left(\frac{75.48L}{H\Delta H} + \frac{H}{\Delta H} - 1 \right)^2 + \frac{75.48L}{H\Delta H}} \right], \tag{21}$$

where $I_i = I_{i.s} = I_{i.l}$;

$$R = \left(H - \frac{75.48L}{H} - 0.5H + 0.786 \right); \tag{22}$$

$$H = 0.5(R + 0.5\Delta H - 0.786) + \sqrt{0.25(R + 0.5\Delta H - 0.786)^2 + 75.48L}. \tag{23}$$

On the basis of formula for the approximate calculations ($\sqrt{1+x} = (1 + 0.5x)$ at $x \leq 0.09$), expression (20) will be transformed to the following form

$$\Delta h_z = \frac{75.48L\Delta H}{75.48L - H\Delta H + H^2}. \tag{24}$$

In that case $H^2 \gg (75.48L - H\Delta H)$

$$\Delta h_z \approx \frac{75.48L\Delta H}{H^2} \text{ and } I_{sh.1} = I'_{sh.1} = \frac{75.48L}{2H^2} I_i. \tag{25}$$

In Table 4 the values of H , R , Δh_z , $I_{sh.1}/I_i$, ϵ , and $I'_{sh.1}/I_i$ calculated by Eqs. (5), (20), (21), (1), and (25) are given for $L = 100$ mm, $\Delta H = 20$ μ m, and various values of h_z .

According to the values of $\frac{I_{sh.1.i}}{I_{sh.1.0}} \left(\frac{H_0}{H_i} \right)^2 \approx 1$, $I_{sh.1}$ at the given L and $\epsilon \geq 12.5' = \epsilon_{crit}$ is inversely proportional to H^2 . The inverse proportionality of $I_{sh.1}$ to R^2 is established at $\epsilon \geq 45' = \epsilon'_{crit}$ as in the case of a diverging beam. With the increase of L the value ϵ_{crit} essentially decreases; ϵ'_{crit} varies insignificantly; disturbance of the $I_{sh.1}$ dependence on R^2 at $\epsilon < \epsilon'_{crit}$ becomes slower. So, at the increase of L up to 300 mm ϵ_{crit} has reduced down to 6.36', and ϵ'_{crit} has remained almost the same; at $h_z = 40$ μ m $\frac{I_{sh.1.i}}{I_{sh.1.0}} \left(\frac{R_0}{R_i} \right)^2$ instead of 0.473 have increased to 0.805.

From Table 4 it is evident, that the relative light intensity on s.b. is equal to 0.25; the results calculated by Eq. (25) are close to the results obtained on the basis of Eq. (21) while $I_{sh.1}$ is inversely proportional to H^2 .

According to data from row 11 of the Table 3, in the case of a plane incident wave at $R = H_{max1}$ the experimental ($I_{sh.1.s}/I_{s.b.s}$) and calculated ($I_{sh.1}/I_{i.l}$) values of the relative intensity in the shadow area also do not depend on L , have the same value and are related in the same way, as in the case of a diverging incident beam.

The comparison of data from Tables 2 and 4 shows, that ϵ_{crit} for a parallel incident beam is much less, than under conditions of a diverging beam.

Just as in case of a diverging beam, at $\lambda \neq 0.53$ μ m a constant 75.48 in the obtained expressions is necessary to be multiplied by $\lambda/0.53$.

The considered relations show the relation of light intensity propagating in the screen shadow, to the ray deflection angles, consisting in dependence of a light flux incident on the scanning slit on the corresponding to it value of Δh_z and the reduction of Δh_z with the increase of ϵ .

The comparison of calculated results with the experimental ones enables one to find out a degree of influence on the intensity of the reflectance and absorptance of the screen edge, and the width of a light source.

Table 4. The variation of light intensity in the screen shadow for the case of a plane incident wave

#	h_z , μ m	H , μ m	R , μ m	ϵ , min	Δh_z , μ m	$\frac{I_{sh.1}}{I_{i.l}} \cdot 10^3$	$\frac{I'_{sh.1}}{I_{i.l}} \cdot 10^3$	$\frac{I_{sh.1.i}}{I_{sh.1.0}}$	$\left(\frac{H_0}{H_i} \right)^2$	$\left(\frac{R_0}{R_i} \right)^2$	$\frac{I_{sh.1.i}}{I_{sh.1.0}} / \left(\frac{H_0}{H_i} \right)^2$	$\frac{I_{sh.1.i}}{I_{sh.1.0}} / \left(\frac{R_0}{R_i} \right)^2$
0	2	2709.26	2697.26	93.14	0.0206	0.515	0.514	1	1	1	1	1
1	5	1304.53	1289.53	44.85	0.0896	2.24	2.22	4.35	4.31	4.37	1.008	0.994
2	10	699.8	679.8	24.06	0.3122	7.805	7.707	15.16	15	15.74	1.011	0.963
3	20	363.1	333.1	12.48	1.1387	28.47	28.26	55.28	55.66	65.56	0.993	0.843
4	30	245.18	205.18	8.43	2.3833	59.58	62.78	115.7	122.1	172.82	0.947	0.67
5	40	185.06	135.06	6.36	3.89	97.23	110.2	188.8	214.32	398.8	0.881	0.473
6	50	148.62	88.62	5.11	5.493	137.33	170.85	266.66	332.3	—	0.802	—
7	60	124.17	54.73	4.27	7.067	176.68	—	343.06	476	—	0.721	—
8	81.61	91.608	0	3.15	10.045	251	450	487.5	874.7	—	0.557	—

Equality of the calculated light intensity to the experimental one on the s.b., a conformity of the obtained expressions to the experiment in the above-mentioned representative conditions, and the inverse proportionality of light intensity to $\tan^2 \epsilon$ at $\epsilon \geq \epsilon_{\text{crit}}$ confirmed by experimental researches are the new facts confirming the occurrence of an edge light due to the deflection of light rays in the zone located at the screen edge, and not being a result of the secondary waves' due to the currents, induced by the incident wave in the screen edge.

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