SPECTRAL DISTORTIONS OF SIGNALS ON LARGE APERTURES IN SENSING OF TURBULENT MEDIA

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The distorting effects of the low–frequency filtration and signal normalization on the frequency spectra of signals received by large–aperture systems in sensing of turbulent media are analyzed by asymptotic and numerical methods.

It is well known that in the receiving large–aperture systems the fluctuations of the signal parameters are averaged.[1–4] This effect is accompanied by a number of physical manifestations in sensing of turbulent media. Thus the antenna gain, image brightness pulsations, and variations of the directions of wave arrival due to the propagation media decrease. A physical aspect of these averaging effects has been sufficiently studied. However, one more effect occurring on large apertures, i.e., the transformation of the fluctuation distribution law, remains to be investigated. If a sufficiently large number of inhomogeneity scales fall on the aperture, in view of the central limit theorem, independently of the initial signal parameter distribution, the signal is normalized at the receiver output. The transformation of the distribution law is in essential for small rather than large field fluctuations.

The signal transformation on large apertures is described theoretically in the simplest way by the analytical perturbation methods for small fluctuations.[1,3] For large fluctuations the infinite series are summed, and the solution is obtained numerically.[1] Particular attention is given to the energy signal parameters such as intensity and its moments. The behavior of nonenergy parameters such as polarization, phase, and frequency of the wave fields is less well understood, in particular, for large fluctuations.[3] The effect of normalization is not pinpointed, though, as is shown below, it is this effect rather than averaging which causes the spectral distortions of signals.

The goal of this paper is to provide an asymptotic and numerical analysis of the distorting effect of the low–frequency filtration and normalization on the frequency fluctuation spectrum of signals received by large–aperture systems in sensing of the turbulent media.

1. PROBLEM FORMULATION

In many cases a wave field at the receiving aperture can be written as[3,5]

$$E(\mathbf{r},\, t) = E_0 \exp [\, \imath\, \varphi_0 (\mathbf{r},\, t)], \tag{1}$$

where the function $\varphi_0(\mathbf{r}, t)$ specifies the phase fluctuations due to inhomogeneities of a propagation medium whose temporal and spatial variations are correlated in the context of the Taylor hypothesis of "frozen" turbulence.

After field summation over the receiving aperture, at its output the signal

$$E_s(t) = \frac{1}{s} \int\int E(\mathbf{r},\, t)\, A(\mathbf{r})\, (d^2\, \mathbf{r}).$$

is obtained. Here $A(\mathbf{r})$ describes weight processing of the field (current distribution) over the receiving aperture s. In general the instantaneous frequency of the initial field $\Omega_0 = d\varphi_0/dt$ and the output signal frequency $\Omega = d\varphi/dt$ ($\varphi = \arg E_s$) are nonlinearly related, and only for small phase fluctuations this relation is linearized and takes the simplest form

$$\Omega(t) = \int\int \Omega_0(\mathbf{r},\, t)\, (d^2\, \mathbf{r})\, /\, s,\ A = \text{const}. \tag{2}$$

The majority of the familiar theoretical results have been obtained for small perturbations.[1,3] For large phase fluctuations the problem of estimating the output signal frequency fluctuations is nonlinear and to solve it one use summation of infinite series.[1] However, one circumstance, which allows simplification of the solution to be made, must be taken into account for large apertures. The case is that within the aperture the summation over a sufficiently large number of contributions from independent inhomogeneities of various scales results in normalization of the output signal E, and for normal processes the moments, including those for phase and frequency, can be written in a comparatively compact form. In particular, the correlation function of frequency fluctuations is represented as[6–8]

$$\begin{aligned} B_\Omega(\tau) &= \{1 - 2\exp\, (\, -\gamma\, /\, 2)\, /\, (1 - R)\, + \\ &+ \exp\, [\, -\gamma\, /\, (1 - R)]\, (1 + R)\, /\, (1 - R)\} \times (\ln R)'^2\, /\, 2\, - \\ &- \{\mathrm{Ei}(\gamma\, /\, 2R) - 2\mathrm{Ei} \times (\gamma\, (1 - R)\, /\, 2R)\, + \\ &+ \mathrm{Ei}(\gamma\, (1 - R)\, /\, 2R \times (1 + R)]\}\, [(\ln R)''\, + \\ &+ (\ln R)'^2\, \gamma\, /\, 2R\,] \times \exp\, (\, -\, \gamma\, /\, 2R)\, /\, 2, \end{aligned} \tag{3}$$

where R is the correlation coefficient of the normally distributed field E_s, γ is the signal–to–noise ratio for power of the output process E_s which for model (1) has the form $\gamma = 1/[\exp(\sigma_\varphi^2) - 1]$ and $\mathrm{Ei}(x)$ is the integral exponent.

For small phase fluctuations ($\sigma_\varphi^2 \ll 1$), $\gamma \sim \sigma_\varphi^{-2}$ and the correlation function B_Ω takes the familiar form[1,3,8]

$$B_\Omega(\tau) = -\, \sigma_\varphi^2\, d^2\, R\, /\, d\, t^2, \tag{4}$$

where R coincides with the phase correlation coefficient R_φ. For large fluctuations $\sigma_\varphi^2 \gg 1$ the coherent component vanishes, and expression (3) allows simplification[8]

$$B_\Omega(\tau) = \ln(1 - R^2)\,(\ln R)''/\,2. \quad (5)$$

It should be noted that whereas for $\sigma_\varphi^2 \ll 1$ the variance of the frequency fluctuations of the output signal $\sigma_\Omega^2 = B_\Omega(0)$ is a finite value, for $\gamma \to \infty$ the value σ_Ω^2 increases unboundedly. The energy spectrum of frequency fluctuations

$$W_\Omega(\omega) = \frac{1}{\pi}\int\limits_\infty^0 \cos \omega\tau\, B_\Omega(\tau)\, d\,\tau, \quad (6)$$

in both cases remains a finite value. Let us compare the effects of averaging and normalization of signals on large apertures on E_s.

2. AVERAGING OF FLUCTUATIONS

In accordance with Eq. (2) it is possible to write for small fluctuations in the context of the hypothesis of frozen turbulence

$$B_\Omega(\tau) = -\,B_\varphi''(\tau),$$

$$B_\varphi(\tau) = \frac{1}{s^2}\int\int (d^2\,\mathbf{r}_1)\int\int (d^2\,\mathbf{r}_2) B_\varphi^{(0)}\,(\mathbf{r}_1 - \mathbf{r}_2 + \mathbf{v}\,\tau), \quad (7)$$

where $B_\varphi^{(0)}(\mathbf{r})$ is the spatial phase correlation function of the incoming wave in the aperture plane and $\mathbf{v}$ is the drift velocity of the inhomogeneities across the aperture. The energy spectrum of frequency fluctuations is calculated by taking the Fourier transform of Eq. (7) and has the form

$$W_\Omega(\omega) = \omega^2 \int\limits_{\omega/v}^{\infty} |I(\kappa)|^2\, \frac{W_\varphi^{(0)}\,(\mathrm{k})\;\mathrm{k}\; d\;\mathrm{k}}{2\pi\sqrt{(\mathrm{k}\;\mathrm{v})^2 - \omega^2}}. \quad (8)$$

Here $W_\varphi^{(0)}(\kappa)$ is the spatial spectrum of isotropic phase fluctuations of the wave field. The function $I(\kappa)$ describes a directional pattern which is expressed in terms of the Bessel function $I(\kappa) = 2J_1(\kappa a)/\kappa a$ for a round aperture of radius a. For convenience, the approximation[2]

$$I(\kappa) = \exp\,[\,-\,(\kappa\; a)^2\,/\,4],$$

is usually employed, and the error in this case remains small.

A large body of theoretical and experimental data on wave propagation in turbulent media show that the energy spectrum of phase fluctuations in its principal portion can be described by the power–law function[9,10] $W_\varphi^{(0)}(\kappa) = W_0\,\kappa^{-\alpha}$, where the spectral power α is close to 11/3. On the basis of these simplifications, integral (8) assumes the form[5,10]

$$W_\Omega(\omega) = W_\Omega^{(0)}(\omega)\;\Psi\;[1/2,\,(3-\alpha)/2,\,(a\;\omega/2\;\nu)^2]\times$$

$$\times\exp\,[\,-\,(a\;\omega/2\;\nu)^2],$$

where

$$W_\Omega^{(0)}(\omega) = W_0\; c_\alpha\; \nu^{\alpha-2}\;\omega^{\alpha-3}/4,$$

$$(c_\alpha \equiv \Gamma((\alpha-1)/2)\,/\,\sqrt{\pi}\;\Gamma\,(\alpha/2))$$

represents the energy spectrum of frequency fluctuations of the initial wave field after passage of the turbulent medium, and the factor $\Psi(a,\,b,\,z^2)$ is the confluent hypergeometric function.[10] As could be expected, when $(a\omega/2\nu) \to 0$, $W_\Omega(\omega) = W_\Omega^{(0)}(\omega)$. When $(a\omega/2\nu) \gg 1$, $\Psi(a,\,b,\,z^2) \simeq z^{-1}$ and

$$W_\Omega(\omega) = W_\Omega^{(0)}(\omega)\;H(\omega), \quad (9)$$

where $H(\omega) = \exp\,[\,-\,(a\omega/2\nu)^2]\,/\,(a\omega/2\nu)$ represents an averaging effect of the finite aperture and illustrates the effect of the spatial aperture filtration, which in the case of frozen turbulence transfers into the domain of temporal frequencies. The form of the function $H(\omega)$ is determined by the type of the aperture. Finite formula (9) describes the essence of the effect of large–aperture averaging.

3. NORMALIZATION OF FLUCTUATIONS

To study the effect of normalization, let us start from representation of the phase fluctuation spectrum by the von Karman model

$$W_\Omega^{(0)}(\omega) = W_\alpha\;(\omega_0^2 + \omega^2)^{-(\alpha\,-\,1)/2}, \quad (10)$$

where $W_\alpha = \sigma_\varphi^2\;\omega_0^{\alpha-2} c_\alpha(\alpha-2)/2$ and σ_φ^2 is the variance of the phase fluctuations. The phase correlation coefficient corresponding to expression (10) has the form[10]

$$R_\varphi^{(0)}(\tau) = K_\nu\;(\omega_0\;\tau)\;(\omega_0\;\tau)^\nu\,/\,\Gamma\;(\nu)2^{\nu-1}\;,\;\; \nu = \alpha\,/\,2 - 1.$$

Assuming that the phase fluctuations φ are distributed by the normal law, it is possible write for the correlation coefficient of field quadratures

$$R = B(\tau)\,/\,B(0),$$

$$B(\tau) = \exp\,(\,-\,D(\tau)\,/\,2) - \exp\,(\,-\,D(\infty)\,/\,2). \quad (11)$$

where $D(\tau)$ is the phase structure function and $D(\tau) = 2\sigma_\varphi^2\,[1 - R_\varphi^{(0)}(\tau)]$. When $\sigma_\varphi^2 \gg 1$, everything is simplified: $R(\tau) = \exp\,[-\,D(\tau)/2]$.

The wave field E is subject to the low–frequency filtration as a result of summation over the aperture and is normalized. The effect of filtration has been described in the previous section; therefore, here we concentrate on the effect of normalization and assume that $R(\tau)$ is invariable by averaging. Formula (5) describing the normalization of large wave–field fluctuations can be directly used to estimate $W_\Omega(\omega)$. In accordance with Eq. (6), after integration by parts, there is

$$W_\Omega(\omega) = -\,\frac{1}{\pi}\int\limits_0^\infty d\;[B_\Omega(\tau)]\;\sin\,\omega\tau/\omega,$$

where the function $B_\Omega(\tau)$ is given by expressions (5) and (11). Here the analytical integration can be performed only in an asymptotic case $\omega \to \infty$ in which the main contribution to the integral comes from the integrand in the small vicinity of the point $\tau = 0$. Taking into account the fact that[10]

$$D(\tau) = 2\sigma_\varphi^2(\omega_0\,\tau\,/\,2)^{2\nu}\,\Gamma\,(1-\nu)\,/\,\Gamma\,(1+\nu),$$

after integration the expression[11]

$$W_\Omega(\omega) = W_\varphi^{(0)}(\omega)\,\ln\,(\omega/\omega_0)^\nu \qquad (12)$$

is obtained and this is valid for $\omega \gg \omega_0$.

Thus the signal normalization at the output from the receiving aperture for large phase fluctuations of the wave field results in an unbounded increase of the weight of the correlation for the signal frequency fluctuations for small separation (fluctuation variance) and as a consequence, in the decay of the initial spectrum slowed down by the logarithmic factor in the high–frequency region. The wider is the fluctuation spectrum of the initial wave field, the more pronounced is this effect.

When the normalization effect is added to the averaging effect of the receiving aperture, the factor $H(\omega)$, describing the low–frequency filtration, appears in Eq. (12).

4. THE GENERAL CASE AND NUMERICAL RESULTS

The energy spectrum $W_\Omega(\omega)$ can be numerically analyzed with successive discrete Fourier transform (DFT) of $W_\varphi^{(0)}(\omega)$ in the form of Eq. (10) for calculating $R_\varphi^{(0)}(\tau)$, substitution of $D(\tau)$ obtained in Eq. (11), calculation of $B_\Omega(\tau)$ using formula (3), and finally, the inverse DFT for calculating $W_\Omega(\omega)$. The numerical differentiation in Eq. (3) with minimum errors was made using spline approximations. Generalized formula (3) is preferable to asymptotes (4) and (5), since it allows one to analyze a continuous transition from small to large fluctuations.

Figure 1 depicts the results of numerical calculation of transformation of the spectrum $W_\Omega(\omega)$ as functions of the frequency $f = \omega/2\pi$ for different phase variances (curves *1–4*) at the predetermined frequency of the outer scale of turbulence $f_0 = \omega_0/2\pi = 10^{-3}$ Hz. For comparison, a dashed line shows the rate of spectrum decay which corresponds to the Kolmogorov model with $\alpha = 11/3$. It is seen that the effect of normalization on the frequency fluctuation spectrum is insignificant for small fluctuations $\sigma_\varphi^2 \ll 1$ and gradually intensifies with increase of σ_φ^2. This is in agreement with asymptote (12).

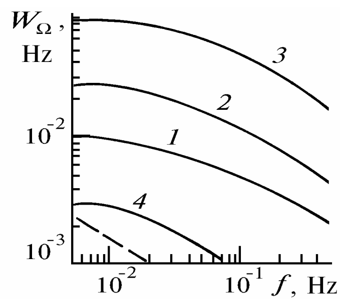

FIG. 1. Transformation of the energy spectrum of frequency fluctuations across the receiving aperture for different strengths of phase fluctuations $\sigma_\varphi = 1$ (*1*), 2 (*2*), 5 (*3*), *and* 0.2 (*4*).

The effect of the increased frequency of the outer scale of turbulence f_0 on the shape of $W_\Omega(\omega)$ is shown in Fig. 2 (curves *1–3*). The increase in the frequency f_0 results first, in increase of the frequency fluctuation strength and second, in less pronounced normalization effect. Estimate (12) and relation (10) support this conclusion.

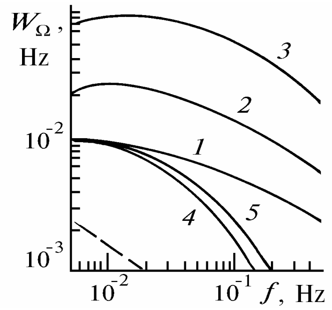

FIG. 2 Transformation of the energy spectrum of frequency fluctuations across the receiving aperture with the increase of the outer scale of turbulence (1–3) and due to averaging over the phase fluctuation spectrum (5) and over the aperture (4): $f_0 = 0.001$ (*1*), 0.002 (*2*), 0.005 (*3*), *and* $f_m = 0.1$ Hz (*4* and *5*).

To study the averaging effect of the large aperture, the wave–field correlation function was subject to low–frequency filtration with the transfer function

$$H(\omega) = \exp\,(\,-\,(\omega/\omega_m)^2).$$

The result of calculation of the spectrum $W_\Omega(\omega)$ at $f_m = \omega_m/2\pi = 0.1$ Hz is shown by curve *4* in Fig. 2. Here for comparison curve *5* shows the result of $W_\Omega(\omega)$ calculation when the low–frequency filtration with the same $H(\omega)$ was performed in the initial phase fluctuation spectrum of the wave field. As can be seen, the averaging effect of the aperture filters mainly the high–frequency components with $f > f_m$. The effects of lowering the high–frequency fluctuations in the phase spectrum of the initial wave field and averaging over the receiving aperture are approximately equivalent when $f \le f_m$, and when $f > f_m$ the effect of the aperture is somewhat stronger.

The obtained results and numerical estimates of possible spectral distortions of monochromatic emitted radiation in sensing of the turbulent media allow one to separate with confidence the effects caused by the propagation media and the distorting effect of the large receiving apertures when relatively weak signals are received.

Thus depicted in Fig. 3 are the energy spectra of frequency fluctuations observed on March 14, 1986 in the experiments with the "Vega–1" spacecraft. Monochromatic radio–frequency radiation of spaceborne transmitters at two wavelengths of 32 and 5 cm was used. The radiation propagating to the Earth was subject to the disturbing effects of the interplanetary medium and the Earth's atmosphere. It was received in Ussuriisk using a parabolic antenna 72 m in diameter capable of circular scanning. In the experiments the radiation frequency was averaged over a period of 1 s. The dispersion method was employed to separate the effect of ionized (upper plot) and neutral (lower plot) components of the propagation media which must be associated with the ionosphere and troposphere. These dependences, normalized to the corresponding dependence at a wavelength of 32 cm, were characterized by the levels of frequency fluctuations $\sigma_f = 0.057 \pm 0.011$ Hz and $\sigma_f = 0.017 \pm 0.002$ Hz, respectively.

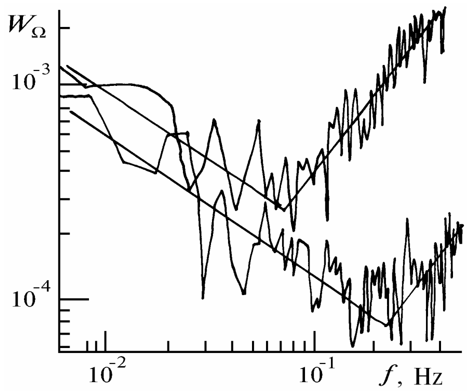

FIG. 3. Energy spectra of frequency fluctuations for sensing in the microwave range.

The energy spectra being Kolmogorov ones, on the average, in the low–frequency region begin to increase in the high–frequency region. The equivalent spectral power, decreasing approximately by a factor of two, approaches 1.67. In the light of the aforementioned analysis such a transformation cannot be attributed to averaging or normalizing effect of the large aperture. The Earth's atmosphere is responsible for the low–frequency Kolmogorov spectral region, and the high–frequency spectral region can then be attributed to the interplanetary medium. The versatile experiments on sensing of the solar–wind inhomogeneities over a wide range of distances to the Sun are indicative of closeness of its spectrum to the Kolmogorov model and of the spectral power to $\alpha = 11/3$ (see Ref. 5). Thus the observed spectral region must be unambiguously attributed to the Halley comet trail which was intersected by the radio–wave propagation path in radar sensing. The geometry of the experiment is shown in Fig. 4 where the dot *1* indicates the closest approach of the space craft to the comet (about 7000 km) on March 6, 1986.

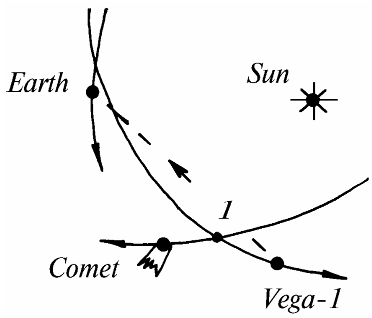

FIG. 4. Geometry of the experiment.

It is interesting to note that the magnitude of the neutral component of the comet trail inhomogeneities is an order of the magnitude less than that of the ionized component. The inhomogeneity spectra of both components are of the same physical origin and are described by the spectral power $\alpha = 1.67$. Such a situation is shown to be typical of anisotropic inhomogeneities.[12]

CONCLUSION

The analysis of the distorting effect of large receiving apertures in sensing of the turbulent media has shown that being reduced to averaging and normalization of output signal, it results in low–frequency filtration and logarithmically delayed power–law decay of high–frequency spectral components of the signal frequency fluctuations. This fact must be taken into account when interpreting the results of sensing in optical and microwave ranges and diagnostics of media. Taking into account the existing direct correlation between the frequency fluctuations of the field and the direction of radiation arrival, the obtained results can be generalized for describing the fluctuations of the angles of arrival of radiation in the turbulent media in the spatial spectral analysis of the wave fields received by large–aperture systems.

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