Dynamics of cataphoresis in the repetitively pulsed discharge

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Evolution of the longitudinal distribution of metal vapor in repetitively pulsed discharge with cataphoretic delivery of vapor is investigated. In particular, the solution of the nonstationary diffusion equation for the metal vapor density, averaged over sufficiently large number of pulses, is derived. Axial distributions of metal vapor at different points in time are calculated. The criteria of homogeneity of such distributions are determined. The time needed for establishment of the homogeneous distribution under conditions typical of repetitively pulsed MVL turns to be sufficiently short (about a second). The obtained results demonstrate much promise for the use of cataphoresis in repetitively pulsed MVL to form the homogeneous active media. They also allow the purposeful selection of the excitation modes providing for a high degree of homogeneity of the media and, consequently, high output characteristics of the lasers.

Introduction

For a long time, cataphoresis is successfully used in continuous-wave ionic metal vapor lasers (MVL) for obtaining the homogeneous distribution of vapor along the active length.[1–4] The most widely used laser of this type is the He–Cd laser (λ = 441.6 and 325 nm). It has the highest commercial success among all MVLs and is used in various applications thanks to its simple and reliable design (a little more complex than that of the He–Ne laser) and convenience in operation.

We pioneered in applying the cataphoretic delivery of vapor into the active media of the repetitively pulsed MVL, in particular, He–Cd (λ = 533.7 and 537.8 nm) and He–Sr (λ = 430.5 nm) lasers.[5–7] Earlier this method was not applied to repetitively pulsed lasers, although various versions of the forced delivery of a mixture were used. It was shown that in the pulsed mode cataphoresis copes with the task of homogeneous vapor distribution in the active zone rather efficiently. In this case, the discharge channel is not blocked by metal pieces, so the arc discharge with uncontrolled evaporation does not arise. So, a possibility appears of independent controlling over the energy and pressure of the metal vapor; and the pumps for forced flowing of the active mixture become unnecessary.

The results of Refs. 5–7 have shown the use of cataphoresis to be not only possible, but promising for the repetitively pulsed lasers. Thus, for a small-size He–Sr laser with the discharge channel of 3 mm in diameter and the active length of 26 cm, the record value of the specific mean output power ($P_{\rm av}^{\rm sp}$ = 277 mW/cm^3) was obtained at cataphoretic delivery of the Sr vapor. The estimates have shown that the vapor flow rates in the repetitively pulsed lasers under certain conditions may be comparable with the rates in cw MVL. However, for creation of the efficient cataphoretic lasers, the homogeneous distribution of the metal vapor concentration along the gas-discharge channel is a necessary condition along with a high rate of pumping.

In this paper, we study theoretically the process of establishment in time of the metal vapor axial distribution due to cataphoresis in the repetitively pulsed discharge and determine the conditions for homogeneity of such distribution.

Theory

The formulated problem will be solved for the active medium of MVL excited by the repetitively pulsed discharge. Denote the concentrations of atoms and metal ions as $N_M(r, z, t)$ and $N_{M^+}(r, z, t)$. Assuming, for simplicity, that the diffusion coefficients of atoms and metal ions are equal, for their concentrations we can write the following equations[1]:

$$\frac{\partial N_M}{\partial t} = D\frac{\partial^2 N_M}{\partial z^2} + D\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial N_M}{\partial r}\right) - R_{\rm i}, \qquad (1)$$

$$\frac{\partial N_{M^+}}{\partial t} = D\frac{\partial^2 N_{M^+}}{\partial z^2} + D_{\rm a}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial N_{M^+}}{\partial r}\right) - \mu E_z\frac{\partial N_{M^+}}{\partial z} + R_{\rm i}, \qquad (2)$$

where D and D_a are the coefficients of diffusion of atoms and ambipolar diffusion of metal ions in helium, respectively; μ is the mobility of metal ions in helium; $R_{\rm i}$ is the term accounting for ionization of atoms and recombination of metal ions.

Then, we transform Eqs. (1) and (2) similarly to Ref. 1, but on the assumption that the problem is nonstationary (in Ref. 1 the similar problem was solved for the established mode of the cw He–Cd laser).

Multiplying the both sides of Eqs. (1) and (2) by r and integrating them over r from the center of the tube (r = 0) to the wall (r = R), we obtain

$$\frac{\partial}{\partial t}\langle N_M\rangle = D\frac{\partial^2}{\partial z^2}\langle N_M\rangle + DR\left(r\frac{\partial N_M}{\partial r}\right)_{r=R} - \langle R_{\rm i}\rangle, \qquad (3)$$

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$$\frac{\partial}{\partial t}\left\langle N_{M^+}\right\rangle = D\frac{\partial^2}{\partial z^2}\left\langle N_{M^+}\right\rangle + D_{\rm a} R\left(r\frac{\partial N_{M^+}}{\partial r}\right)_{r=R} - \\ -\mu E_z \frac{\partial}{\partial z}\left\langle N_{M^+}\right\rangle + \langle R_{\rm i}\rangle, \quad (4)$$

where the angle brackets denote averaging over the radius

$$\left\langle N_j\right\rangle = 2/R^2 \int_0^R rN_j {\rm d}r. \quad (5)$$

Sum up Eqs. (3) and (4); in this case, the second terms in the right-hand side of each equation, corresponding to the diffusion flows of ions to the tube walls and atoms in the opposite direction, will compensate each other. Thus, we obtain the equation

$$\frac{1}{D}\frac{\partial N}{\partial t} = \frac{\partial^2 N}{\partial z^2} - \frac{\beta}{L}\frac{\partial N}{\partial z}, \quad (6)$$

where $N = \langle N_M\rangle + \left\langle N_{M^+}\right\rangle$ is the metal vapor concentration averaged over the radius;

$$\beta = \mu E_z \theta L / D; \quad (7)$$

$\theta = \left\langle N_{M^+}\right\rangle / N$ is the degree of ionization of metal vapor; L is the active length.

To make Eq. (6) applicable to the repetitively pulsed mode, the field strength should be averaged over sufficiently large number of pulses. Schemes with complete discharge of a reservoir capacitor are commonly used for excitation of the pulsed discharge, and in this case the dependence of E_z on t can be approximately presented as

$$E_z = E_0 \exp(-t/\tau_{\rm c}), \quad (8)$$

where E_0 is the initial strength of the electric field in the pulsed discharge; $\tau_{\rm c}$ is the duration of the current pulse. The mean field for a pulse is equal to

$$\langle E_z\rangle = \frac{1}{\tau_{\rm c}}\int_0^\infty E_z(t){\rm d}t = E_0. \quad (9)$$

Averaging this field over a series of pulses, we obtain

$$E_f = \langle E_z\rangle f\tau_{\rm i} = E_0 f\tau_{\rm i}, \quad (10)$$

where f is the pulse repetition frequency.

Thus, for the repetitively pulsed mode, the parameter β (7) takes the form

$$\beta = \frac{\mu E_f \theta L}{D} = \frac{\mu E_0 \theta L}{D} f\tau_{\rm c} = \frac{\theta e E_0 L}{kT} f\tau_{\rm c}, \quad (11)$$

where k is the Boltzmann constant; T is the gas temperature; in Eq. (11) the Einstein relation $\mu/D = e/kT$ is taken into account.

Equation (6) was solved analytically by the Fourier method. If the anode is at the point $z = -L$, the evaporator is at the point $z = 0$, and the cathode is at the point $z = L$, then under the initial condition

$$N(z,0) = f(z) \quad (12)$$

and boundary conditions

$$\begin{cases} N(0,t) = N_0, \\ N(\pm L, t) = 0 \end{cases} \quad (13)$$

the solution of Eq. (6) describing the process of establishment of the axial distribution of metal vapor in the repetitively pulsed mode has the form

$$N(z,t) = \exp\left[-D\left(\frac{\beta}{2L}\right)^2 t\right] \exp\left(\frac{\beta}{2L}z\right)\times \\ \times\sum_{k=1}^{\infty}\alpha_k \exp\left[-D\left(\frac{\pi k}{L}\right)^2 t\right]\sin\frac{\pi k}{L}z + N(z,\infty), \quad (14)$$

where $N(z, \infty)$ is the solution of Eq. (6) for the established mode ($t \to \infty$):

$$N(z,\infty) = \begin{cases} N_0 \dfrac{1 - {\rm e}^{\beta(1+z/L)}}{1 - {\rm e}^{\beta}} & \text{at } -L \le z \le 0, \\ N_0 \dfrac{1 - {\rm e}^{-\beta(1-z/L)}}{1 - {\rm e}^{-\beta}} & \text{at } 0 \le z \le L. \end{cases} \quad (15)$$

In the general case, the coefficients α_k are determined as

$$\alpha_k = \begin{cases} \dfrac{2}{L}\displaystyle\int_{-L}^{0} [f(z) - N(z,\infty)]\,{\rm e}^{-\beta z/2L} \sin\frac{\pi k}{L}z{\rm d}z & \text{at } -L \le z \le 0, \\ \dfrac{2}{L}\displaystyle\int_{0}^{L} [f(z) - N(z,\infty)]\,{\rm e}^{-\beta z/2L} \sin\frac{\pi k}{L}z{\rm d}z & \text{at } 0 \le z \le L. \end{cases} \quad (16)$$

If the initial axial distribution of the metal vapor concentration is described by the fast decreasing function $f(z)$ having the form $f(z) = N_0 \exp(-\xi|z|)$, where N_0 is the concentration of metal vapor in the evaporator, and $\xi|z| \gg 1$, then from Eq. (16) we obtain the following equations for α_k:

$$\alpha_k = \begin{cases} \dfrac{2N_0}{\pi k}\left[\dfrac{(-1)^k {\rm e}^{\beta - 2L\xi/2} - 1}{1 + \left(\dfrac{\beta - 2L\xi}{2\pi k}\right)^2} + \dfrac{1}{1 + \left(\dfrac{\beta}{2\pi k}\right)^2}\right] & \text{at } -L \le z \le 0, \\ \dfrac{2N_0}{\pi k}\left[\dfrac{1 - (-1)^k {\rm e}^{-\beta + 2L\xi/2}}{1 + \left(\dfrac{\beta + 2L\xi}{2\pi k}\right)^2} - \dfrac{1}{1 + \left(\dfrac{\beta}{2\pi k}\right)^2}\right] & \text{at } 0 \le z \le L. \end{cases} \quad (17)$$

In the ideal case, the initial distribution should be infinitely narrow, therefore, let ξ tend to infinity, then the equation for the coefficients α_k takes the simple form

$$\alpha_k = \pm\frac{2N_0}{\pi k}\frac{1}{1 + (\beta/2\pi k)^2}, \quad (18)$$

where the sign "+" is for the range $-L \le z \le 0$, and the sign "−" is for the range $0 \le z \le L$.

Let us introduce new dimensionless variables: the reduced coordinate and time

$$x = z/L,\ \tau = (D/L^2)t \tag{19}$$

and a new function − relative concentration of metal vapor

$$n = N/N_0. \tag{20}$$

Then the solution of Eq. (14) can be written in the generalized form, in which the parameters, characterizing the properties of the medium, do not explicitly enter in it:

$$n(x,\tau) = \exp - \left[\left(\frac{\beta}{2}\right)^2 \tau\right] \exp\left(\frac{\beta}{2} x\right) \times \sum_{k=1}^{\infty} \alpha'_k \exp[-(\pi k)^2 \tau] \sin \pi k x + n(x, \infty), \tag{21}$$

where

$$n(x,\infty) = \begin{cases} \dfrac{1-e^{\beta(1+x)}}{1-e^{\beta}} & \text{at } -1 \le x \le 0, \\ \dfrac{1-e^{-\beta(1-x)}}{1-e^{-\beta}} & \text{at } 0 \le x \le 1 \end{cases} \tag{22}$$

and $\alpha'_k = \alpha_k/N_0$.

Note that the obtained solution (14) can be also used for description of cataphoresis dynamics in cw MVL. In this case, to calculate β, Eq. (11) should be replaced by Eq. (7).

Calculated results

Figure 1 depicts the calculated (by Eq. (21)) distributions of the metal vapor relative concentration in repetitively pulsed discharge over the tube length at different moments τ for two values of β. It can be seen from the plots that the larger is the parameter β, the more homogeneous is the established distribution of metal vapor and the shorter is the time needed for establishment of this distribution. It follows from Fig. 1 and Refs. 5–7 that sufficiently homogeneous distribution of metal vapor along the gas discharge channel in the gap between the vapor source and cathode and, at the same time, reliable blocking of the vapor from the anode direction can be achieved at such MVL operation modes, when

$$\beta \ge 10. \tag{23}$$

Just this is the condition for homogeneous axial distribution of metal vapor.

To estimate the time needed for establishment of the homogeneous vapor distribution at the arbitrary β (β > 10), we assume that the time when the condition

$$\sqrt{\int_0^1 [1 - n(x,\tau)]^2 \mathrm{d}x} = \sqrt{\int_0^1 [1 - n(x,\infty)|_{\beta=10}]^2 \mathrm{d}x} \tag{24}$$

is fulfilled is just the time of achievement of the acceptable degree of homogeneity. The left-hand part of this equality is the "distance" between the function describing the ideal homogeneous distribution of metal vapor (n = 1 at $0 \le x \le 1$ and 0 in all other cases) and the function describing the distribution of the metal vapor concentration at the time τ at the given value of the parameter β (β > 10). The right-hand part is the "distance" between the ideal distribution and that established ($\tau \to \infty$) at the minimal of all acceptable values of β (β = 10).

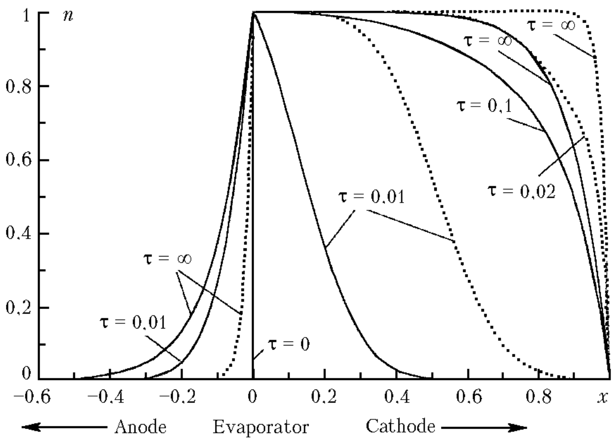

Fig. 1. Axial distribution of the relative concentration of metal vapor at different moments τ for β = 10 (solid curves) and β = 50 (dashed curves).

Figure 2 depicts the β dependence of the reduced time τ_h needed for achievement of the homogeneous vapor distribution. This dependence was found with the use of the condition (24). It can be seen from the plot that the time τ_h fast decreases up to $\beta \approx 50$ and then relatively slowly at β > 50. The curve in Fig. 2 is well approximated by the analytical function

$$\tau_h = a\beta/(b + \beta^2), \tag{25}$$

where a = 0.89 and b = −46.15.

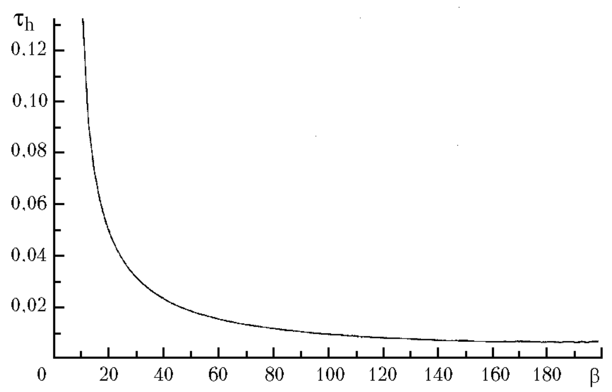

Fig. 2. The β dependence of the reduced time τ_h needed for achievement of the homogeneous axial distribution of metal vapor.

The Table gives the values of β and the vapor flow rate $V = \mu E_0 \theta f \tau_c$ (Refs. 5 and 6) for the conditions of operation of repetitively pulsed cataphoretic He–Sr and He–Cd lasers (studied by us in Refs. 5–7), as well as calculated by Eq. (25) time needed for appearance of the metal vapor homogeneous distribution over the tube length. One can see that the values of β meet the condition of homogeneity (23) for the both tubes. At the same time, the actual time t_h for establishment of the homogeneous vapor distribution proves to be rather short, about one second. For comparison, the Table also gives the calculated parameters for the typical operation conditions of the cw He–Cd cataphoretic laser.

Table. Values of β, reduced (τ_h), and actual (t_h) times for establishment of the homogeneous longitudinal distribution of metal vapor, and the vapor flow rate V for repetitively pulsed He–Sr laser (p_{He} = 0.8 atm, T = 900 K, E_0 = 1000 V/cm, f = 30 kHz, τ_c = 0.04 μs, θ = 0.5, D = 3.5 cm²/s), He–Cd laser (p_{He} = 3 Torr, T = 600 K, E_0 = 100 V/cm, f = 10 kHz, τ_c = 0.2 μs, θ = 0.1, D = 185 cm²/s), and cw He–Cd laser (p_{He} = 3 Torr, T = 600 K, E = 20 V/cm, θ = 0.01, D = 185 cm²/s)

Laser	λ, nm	L, cm	d, mm	β	τ_h	t_h, s	V, cm/s
He–Sr	430.5	26	3	201	$4.4\cdot10^{-3}$	0.85	27
He–Cd	537.8 533.7	50	3	19.3	$5.2\cdot10^{-2}$	0.7	71.4
He–Cd	441.6	50	3	193	$4.6\cdot10^{-3}$	0.06	714

It is seen from the calculated results that the longitudinal vapor distribution in the repetitively pulsed He–Sr laser and the cw He–Cd laser must have a high degree of homogeneity (what is observed experimentally), because β takes rather high values. However, in the repetitively pulsed lasers the time needed for appearance of the homogeneous distribution turns to be by an order of magnitude larger than in the cw laser. Nevertheless, the time obtained for the pulsed mode is acceptable, because it is far smaller than the characteristic time for establishment of the temperature of the evaporator and the temperature of the active medium, which is about several minutes. This means that in the process of experimental optimization of cataphoretic active elements of the repetitively pulsed MVL, the axial vapor distribution follows quasi-stationarily variations of parameters of the active medium.

Figure 3 depicts the process of establishment of the longitudinal distribution of Sr and Cd vapor in the repetitively pulsed He–Sr and He–Cd lasers. It is seen that for Sr there is a sufficiently sharp, as compared with Cd, boundary between the discharge area, where the homogeneous distribution has already been established, and the area, where metal vapor is absent. The speed of movement of the boundary is about 25 cm/s. This difference is connected with the difference of an order of magnitude between the values of β (see the Table) for the He–Sr and He–Cd lasers, which determines the degree of homogeneity of the longitudinal vapor distribution. Although the longitudinal distribution for the He–Sr laser turns to be more homogeneous than that for the He–Cd laser, the buffer gas flow rate in the latter, nevertheless, turns to be higher than in the former and roughly equals 71 cm/s.

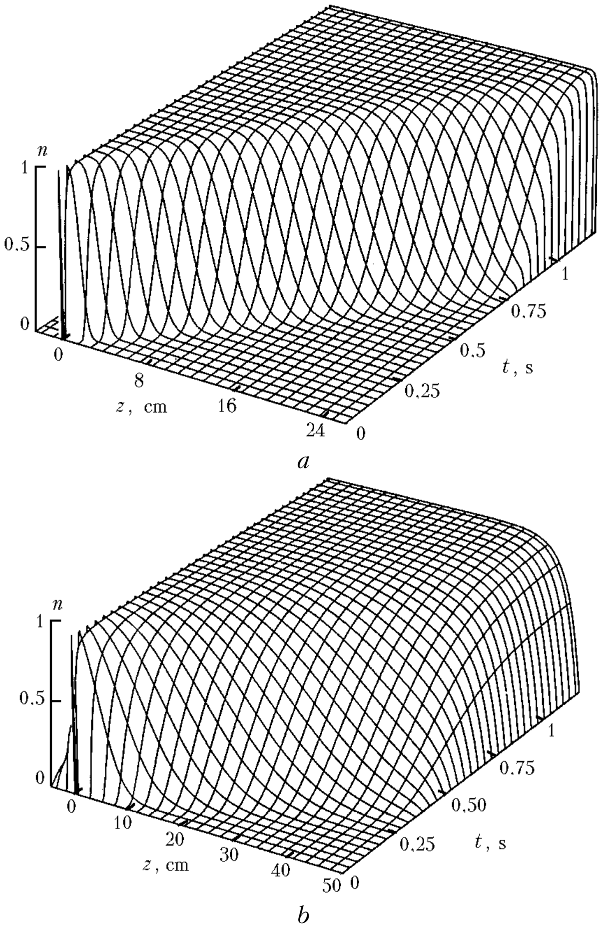

Fig. 3. Process of establishment of the axial distribution of Sr (*a*) and Cd (*b*) vapor in repetitively pulsed cataphoretic He–Sr and He–Cd lasers.

It follows from Eq. (25) that at $\beta^2 \gg |b|$ (it is seen from the Table that this condition is fulfilled for the typical operation conditions of the He–Sr and He–Cd lasers)

$$\tau_h \propto 1/\beta . \qquad (26)$$

Herefrom we obtain the dependence of the actual time for establishment of the homogeneous metal vapor distribution on the active medium parameters:

$$t_h = \frac{L^2}{D}\tau_h \propto \frac{L}{\mu E_0 \theta \tau_c f} \propto \frac{L p_{He}}{\sqrt{T} E_0 \theta \tau_c f}. \qquad (27)$$

Thus, the longer is the active zone and the higher is the buffer gas pressure, the longer time is needed for establishment of the axial distribution of metal vapor. At the same time, the establishing process is faster with increasing gas temperature, electric field strength, length of the current pulse or pulse repetition frequency.

Conclusion

To study the process of establishing in time of the axial distribution of metal vapor in the repetitively pulsed discharge, the mathematical model of longitudinal cataphoresis applicable to the repetitively pulsed MVL has been constructed. The analytical solution of the nonstationary problem of longitudinal cataphoresis is obtained. The axial distributions of the metal vapor at different points of time are calculated. The conditions for the homogeneous vapor distribution are found.

The calculations have shown that for typical operation conditions of the repetitively pulsed He–Sr and He–Cd lasers, the condition of the metal vapor homogeneous distribution over the discharge channel length is fulfilled. The time needed for establishment of the homogeneous distribution is found to be about a second under conditions typical of MVL operation, and its dependence on the active medium parameters is determined.

The obtained results offer much promise for using cataphoresis in the repetitively pulsed metal vapor lasers in formation of homogeneous active media. They allow purposeful selection of excitation conditions providing for a high degree of homogeneity and, consequently, high output characteristics.

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