CORRECTION ERROR IN IMAGING OF EXTENDED OBJECTS THROUGH THE TURBULENT ATMOSPHERE

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The effect of anisoplanatism on the performance of an adaptive optics system intended for imaging of extended objects through the atmospheric turbulence has been studied. The main attention has been paid to estimation of the residual error induced by anisoplanatism, disregarding other factors degrading the system performance. A method for compromise compensation minimizing the residual error of correction has been suggested. By expanding phase distortions in a system of Zernike polynomials, the contribution of correction for particular aberrations has been evaluated to the residual error. An analytical method for evaluating the system performance for an arbitrary turbulence spectrum has been developed. Numerical calculations have been done for a circular extended area on the object using the Gaussian quadrature formulas. The results are presented for the von Karman and Kolmogorov turbulence spectra with different values of the outer scale.

1. INTRODUCTION

Analyzing the efficiency of an adaptive optics system intended for correction of images, it is usually assumed that the angular size of an object is less than that of the isoplanatic zone. In a randomly inhomogeneous medium, phase distortions of optical waves coming from various points of this object can be considered identical. Distribution of optical inhomogeneites along the path of radiation propagation can be obtained by recording phase distortions of a reference wave.[1]

When the size of an object is greater than that of the isoplanatic zone (an extended object), optical paths for waves coming from different points of this object are different. So phase aberrations acquired by these waves differ notably. This effect is referred to as anisoplanarity of optical system.[2] When recording these distortions and compensating for them using a phase corrector, considerable difficulties arise due to this effect. Complete information on distortions of optical waves coming from different points of the object cannot be obtained using a single reference source. The schematic with several reference sources designed for use in astronomy[3] can be used to obtain this information.

Next difficulty is that even if the complete information on the distortions is available, we cannot correct them simultaneously for all points of the object. So there are fundamental limitations on the size of the object whose image can be corrected by a single phase corrector. These limitations are due to anisoplanarity of the adaptive optics system considering the atmospheric inhmogeneities.

The present article is devoted to an analysis of these fundamental limitations on the quality of phase correction disregarding difficulties associated with wavefront detection. We assume that the complete information about distortions is available.

It is possible to propose various algorithms for correcting the image of an extended object. Wavefront corrector can be adjusted using the data of phase distortion measurements for wave coming from a reference source placed near the center of the object.[1] In such a system, the image quality is improved only in the central region, whereas in peripheral regions the image may undergo even greater distortions. In the paper, we consider another correction algorithm by which the corrector is adjusted to maximize the quality criterion allowing for all parts of the object. In this algorithm, the RMS phase error averaged with some weight over the object is taken as a characteristic of distortion.

This approach is based on the expansion of wavefront distortions in a system of Zernike polynomials.[4] The residual error is evaluated by analyzing the behavior of the correlation functions for Zernike coefficients.

2. PROBLEM FORMULATION. CORRECTION ALGORITHM

Let us consider an object with angular size greater than that of the isoplanatic zone. The object is placed at a finite distance from an adaptive optics system and is separated from it by a distorting layer of the atmospheric turbulence. It is necessary to correct the image of an extended region of the object so that to

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maximize the given quality criterion considering all the points of this region. In inhomogeneous media optical waves coming from different points of the object to the aperture acquire different phase distortions. For the distortions of a spherical wave coming from a point $\mathbf{r}'$ and observed at a point $\mathbf{r}$ in the aperture plane, let us introduce a designation $\varphi(\mathbf{r}, \mathbf{r}')$. Let us also assume that we know these distortions. So the mean square residual error of the wavefront of this wave is

$$\varepsilon^2(\mathbf{r}') = \frac{1}{S_A} \int\limits_{S_A} \mathrm{d}^2 r \, \langle(\varphi(\mathbf{r}, \mathbf{r}') - u(\mathbf{r}))^2\rangle \, , \qquad (1)$$

where integration is performed over the aperture and $u(\mathbf{r})$ is the phase profile introduced by a correcting mirror.

Using the expansion of the mirror profile and phase distortions $\varphi(\mathbf{r}, \mathbf{r}')$ in a system of Zernike polynomials,[4] we obtain for the residual error

$$\varepsilon^2(\mathbf{r}') = \sum_{q=0}^{\infty} \langle(a_q(\mathbf{r}') - u_q)^2\rangle \, , \qquad (2)$$

where u_q are the control parameters of the corrector, $a_q(\mathbf{r}')$ are the coefficients of expansion of phase distortions $\varphi(\mathbf{r}, \mathbf{r}')$ in the system of Zernike polynomials

$$\varphi(\mathbf{r}, \mathbf{r}') = \sum_{q=0}^{\infty} a_q(\mathbf{r}') \, Z_q(\mathbf{r}) \, . \qquad (3)$$

Let us assume that S' is the extended area on the object surface. (The whole object can be meant by this area). As a measure of distortions, let us take the mean square phase error

$$\sigma^2 = \int\limits_{S'} \omega(\mathbf{r}') \, \varepsilon^2(\mathbf{r}') \, \mathrm{d}^2 r' \qquad (4)$$

averaged with the weight $\omega(\mathbf{r}')$ over this area. The weight $\omega(\mathbf{r}')$ that meets the requirement

$$\int\limits_{S'} \omega(\mathbf{r}') \, \mathrm{d}^2 r' = 1 \qquad (5)$$

defines which points are more important in image forming and which are less important. Substituting Eq. (2) into Eq. (4), we can observe that different aberrations defined by the subscript q yield additive contributions to the total mean square error

$$\sigma^2 = \sum_q \sigma_q^2 \, , \qquad (6)$$

where the error of correction for the qth aberration is

$$\sigma_q^2 = \int\limits_{S'} \omega(\mathbf{r}') \, \langle(a_q(\mathbf{r}') - u_q)^2\rangle \, \mathrm{d}^2 r'. \qquad (7)$$

Let us pay attention to the fact that Zernike coefficients $a_q(\mathbf{r}')$ are functions of coordinates of the point of the object, whereas the control parameters u_q that define the surface shape of the flexible mirror are independent of these coordinates. The problem arises to choose the control parameters from the known (as assumed) coefficients $a_q(\mathbf{r}')$. Let as take them in such a way that the value of the mean square error (Eq. (7)) be minimum. We obtain that the parameter u_q minimizing Eq. (7) can be found using the following formula:

$$u_q = \int\limits_{S'} \omega(\mathbf{r}') \, a_q(\mathbf{r}') \, \mathrm{d}^2 r'. \qquad (8)$$

Then the equation describing the error in correction of the qth aberration has the form

$$\sigma_q^2 = \langle a_q^2\rangle - \int\limits_{S'} \int\limits_{S'} \omega(\mathbf{r}') \, \omega(\mathbf{r}'') \, \langle a_q(\mathbf{r}') \, a_q(\mathbf{r}'')\rangle \, \mathrm{d}^2 r' \, \mathrm{d}^2 r''. \qquad (9)$$

So the algorithm for "compromise" correction involves the choice of the control parameters using the phase perturbations averaged with the weight $\omega(\mathbf{r}')$. Let us consider the case in which all the points of the region S' are equally important for observations. To do so, the function $\omega(\mathbf{r}')$ should be chosen in the following way:

$$\omega(\mathbf{r}') = 1/S' \, . \qquad (10)$$

Equation (4) describing the error and Eq. (8) defining the control parameter assume the forms

$$\sigma^2 = \frac{1}{S'} \int\limits_{S'} \varepsilon^2(\mathbf{r}') \, \mathrm{d}^2 r', \qquad (11)$$

$$u_q = \frac{1}{S'} \int\limits_{S'} a_q(\mathbf{r}') \, \mathrm{d}^2 r'. \qquad (12)$$

Substituting u_q from Eq. (12) into Eq. (7) which describes the error of the qth aberration, we obtain

$$\sigma_q^2 = \langle a_q^2\rangle \left\{ 1 - \left(\frac{1}{S'}\right)^2 \iint\limits_{S'S'} K_q(\mathbf{r}', \mathbf{r}'') \, \mathrm{d}^2 r' \, \mathrm{d}^2 r'' \right\} , \qquad (13)$$

where

$$K_q(\mathbf{r}', \mathbf{r}'') = \frac{\langle a_q(\mathbf{r}')\, a_q(\mathbf{r}'')\rangle}{\langle a_q^2\rangle} \qquad (14)$$

is the correlation function of Zernike coefficients for different points in the area of observations. Due to uniformity of the turbulence, it depends only on the relative positions of points on the object. Let us change the variables

$$\mathbf{s} = \mathbf{r}' - \mathbf{r}'', \quad \mathbf{s}_1 = (\mathbf{r}' + \mathbf{r}'')/2\ . \qquad (15)$$

With these substitutions, Eq. (13) has the form:

$$\sigma_q^2 = \langle a_q^2\rangle \left\{ 1 - \left(\frac{1}{S^*}\right)^2 \int\limits_{S^*} K_q(\mathbf{s})\ \mathrm{d}^2 s \right\}, \qquad (16)$$

where S^* is the area derived from S' after we have made substitutions (15).

So we have found that the mean square error resulting from compromise correction for an extended area can be represented through the normalized correlation function of Zernike coefficients averaged over the area.

3. CORRELATION FUNCTIONS FOR ZERNIKE POLYNOMIALS

In the above section we have obtained the equation for the residual correction error through the correlation function $K_q(\mathbf{s})$ of Zernike coefficients for two spherical waves coming from different points of the object to the receiving aperture, where $\mathbf{s}$ is the distance between the points. " efore analyzing the residual error for the specific area, let us consider the correlation functions $K_q(\mathbf{s})$ for different Zernike modes.

Correlation functions for Zernike coefficients can be related to the structure function of phase perturbations measured in the aperture plane. In so doing, we should exclude the mean phase, i.e., the phase shift constant over the whole aperture that does not influence the image formation[5]:

$$K_q(\mathbf{s}) = -\frac{1}{2\pi^2 \langle a_q^2\rangle} \times$$

$$\times \iint\limits_{\substack{r_1\le 1\\ r_2\le 1}} Z_q(\mathbf{r}_1)\, Z_q(\mathbf{r}_2)\, D_\varphi(R(\mathbf{r}_1 - \mathbf{r}_2), \mathbf{s})\ \mathrm{d}^2 r_1\, \mathrm{d}^2 r_2, \qquad (17)$$

where R is the radius of the circular aperture, integration is performed over a circle with a unit radius, and

$$D_\varphi(\mathbf{r}, \mathbf{s}) = \langle(\varphi(R\mathbf{r}_1, \mathbf{r}') - \varphi(R\mathbf{r}_2, \mathbf{r}''))^2\rangle \qquad (18)$$

is the structure function for two beams one of them coming from the point $\mathbf{r}'$ of the object to the point $R\mathbf{r}_1$ in the aperture plane and the other coming from the point $\mathbf{r}''$ to $R\mathbf{r}_2$. Due to uniformity of turbulence, the structure function depends only on two arguments

$$\mathbf{r} = R\ (\mathbf{r}_1 - \mathbf{r}_2), \quad \mathbf{s} = \mathbf{r}' - \mathbf{r}''. \qquad (19)$$

In the approximation of geometric optics, the structure function of the phase for a wave traveled a small distance $\mathrm{d}h$ can be expressed through the spectrum of optical inhomogeneities[4]:

$$D_\varphi(\tilde{\mathbf{r}}(h)) = \frac{0.046}{r_0^{5/3}(h)} \iint F_n(k)\,[1 - \exp[i\,2\pi\mathbf{k}\,\tilde{\mathbf{r}}(h)]]\,\mathrm{d}^2 k, \qquad (20)$$

where

$$\tilde{\mathbf{r}}(h) = \frac{h}{H}\,\mathbf{r} + \left(1 - \frac{h}{H}\right)\mathbf{s} \qquad (21)$$

is the current distance between the beams at altitude h, H is the optical path length, and $F_n(k)$ is the spatial spectrum of inhomogeneities. In addition, we introduced the designation

$$\frac{1}{r_0^{5/3}(h)} = \frac{2.92}{6.88}\, k_0^2\, c_n^2(h)\ \mathrm{d}h\ , \qquad (22)$$

where $r_0(h)$ is "local" Fried's radius, k_0 is the wave number, and $c_n^2(h)$ is the structure constant.

" elow we consider two types of the atmospheric turbulence spectra, namely, the Kolmogorov spectrum

$$F_n(k) = k^{-11/3} \qquad (23)$$

and the von Karman spectrum with the outer turbulence scale L_0

$$F_n(k) = k^{-11/3}\left[1 + \left(\frac{1}{k\,L_0}\right)^2\right]^{-11/6}. \qquad (24)$$

The inner scale of turbulence considered in the Tatarskii spectrum influences only insignificantly the correlation functions of low order Zernike polynomials.

The structure function of the phase in the aperture plane is derived by integration of Eq. (20) over the path length

$$D_\varphi(\mathbf{r}, \mathbf{s}, H) = \frac{0.046}{r_0^{5/3}}\frac{1}{C_n^2}\int\limits_0^H\!\!\iint c_n^2(h)\, F_n(k)\ \times$$

$$\times \left[1 - \exp\left[i\ 2\pi\ \mathbf{k}\left(\frac{h}{H}\,\mathbf{r} + \left(1 - \frac{h}{H}\right)\mathbf{s}\right)\right]\right]\mathrm{d}^2 k\ \mathrm{d}h, \qquad (25)$$

where

$$C_n^2 = \int\limits_0^H c_n^2(h)\ \mathrm{d}h, \qquad (26)$$

$$\frac{1}{r_0^{5/3}} = \frac{2.92}{6.88}\, k_0^2\, C_n^2. \qquad (27)$$

Substituting Eq. (25) into Eq. (17) and taking the Fourier transform of Zernike polynomials, we obtain the integral equation for the correlation function

$$K_q(\mathbf{s}) = \frac{1}{\langle a_q^2\rangle}\left(\frac{2R}{r_0}\right)^{5/3} 4\pi^{8/3}\, 0.046\, \frac{1}{C_n^2}\,(n+1)\,(-1)^{n-m} \times$$

$$\times \int\limits_0^H c_n^2(h) \int\limits_0^\infty x\, F_n\left(\frac{x}{2\pi R}\right) \frac{J_{n+1}^2\left(x\dfrac{h}{H}\right)}{\left(x\dfrac{h}{H}\right)^2} \left\{J_0\left(\left(1-\frac{h}{H}\right)x\frac{s}{R}\right) + \right.$$

$$\left. + l(-1)^m J_{2m}\left(\left(1-\frac{h}{H}\right)x\frac{s}{R}\right)\cos(2m\varphi)\right\} \mathrm{d}x\mathrm{d}h, \tag{28}$$

where $q \equiv (n, m, l)$ is the triple subscript defining Zernike mode, $J_n(\xi)$ is the "essel function of the integer order, and $\mathbf{s} = (s, \varphi)$ is the vector specifying the relative positions of points in polar coordinates. Let us note that in this form the correlation function depends on the ratio $\frac{s}{R}$. So the main parameter of the problem is the ratio of the object linear size to the aperture radius.

Let us assume that the structure constant c_n^2 is independent of altitude h. Such assumption is valid for short propagation paths or for paths (the Z axis) parallel to the Earth's surface.

Plots of corresponding correlation functions obtained for the Kolmogorov spectrum are shown in Fig. 1. It should be noted that the higher is the serial number of aberration, the faster is the decrease of the correlation function with increasing distance between the points. So the small-scale aberrations are more sensitive to anisoplanatism. With increasing distance between objects, the correlation functions of tilts decrease more slowly as compared with aberrations of higher orders. Figure 1 illustrates the case in which the X axis of the coordinate system defined by Zernike polynomials is parallel to the vector $\mathbf{s}$.

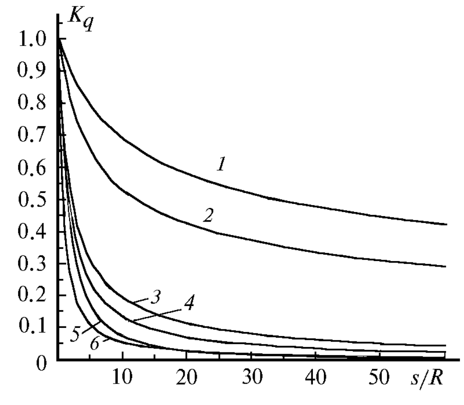

FIG. 1. Normalized correlation functions of Zernike coefficients for different aberrations: X-tilt (1), Y-tilt (2), Y-astigmatism (3), defocusing (4), X-astigmatism (5), and spherical aberration (6).

4. CORRECTION ERROR FOR IMAGE OF A CIRCULAR AREA

In the previous section, we have considered the correlation function of Zernike polynomials for two waves coming form two different points of the object surface. Let us consider in more detail Eq. (16) describing the residual error of correction for image of an extended area. We assume that this area is a circle with radius R'. Equation (16) can be written in the form

$$\sigma_q^2 = \langle a_q^2\rangle\left[1 - f_q\left(\frac{R'}{R}\right)\right], \tag{29}$$

where $\langle a_q^2\rangle$ is the mean square error of Zernike polynomials independent of the object shape,

$$\langle a_q^2\rangle = \beta_q\left(\frac{2R}{r_0}\right)^{5/3}, \tag{30}$$

β_q is the numerical factor defined by the turbulence spectrum. Function

$$f_q\left(\frac{R'}{R}\right) = \frac{1}{4S'}\int\limits_{4S'} K_q(\mathbf{s})\,\mathrm{d}^2\mathbf{s}, \quad S' = \pi R'^2, \tag{31}$$

is the correlation function of Zernike polynomials averaged over the circle. Let us note that

$$0 < f_q\left(\frac{R'}{R}\right) < 1, \tag{32}$$

so

$$0 < \sigma_q^2 < \langle a_q^2\rangle . \tag{33}$$

After averaging, we obtain the following equation for f_q:

$$f_q\left(\frac{R'}{R}, \frac{L_0}{R}\right) = \left(\frac{2R}{r_0}\right)^{5/3} \frac{1}{\langle a_q^2\rangle}\, 0.046\, 4\pi^{8/3}\,(n+1) \times$$

$$\times \int\limits_0^\infty x^{-14/3} J_{n+1}^2(x) \int\limits_0^1 y^{5/3}\left[1 + \left(\frac{2\pi y}{x}\left(\frac{L_0}{R}\right)^{-1}\right)^2\right]^{-11/6} \times$$

$$\times \left[J_0\left(2\frac{1-y}{y}x\frac{R'}{R}\right) + J_2\left(2\frac{1-y}{y}x\frac{R'}{R}\right)\right]\mathrm{d}y\mathrm{d}x. \tag{34}$$

It should be noted that after averaging over the area symmetric about the center, the dependence on the subscript specifying the azimuth component m and on the parity subscript l vanished ($q = (n, m, l)$ are subscripts specifying Zernike mode). This means that correlation functions averaged over the circular aperture are identical for modes with identical radial numbers n. The Kolmogorov spectrum corresponds to the case $(L_0/R)^{-1} = 0$. Functions $f_q\,(R'/R\,,\,L_0/R)$ are shown in Fig. 2 for the Kolmogorov spectrum and $n = 1$, 2, 3, and 4. The higher is the serial number of aberration, the faster is the decrease of the corresponding correlation function with the increase of the radius of the area whose image is formed. So the size of the area for which it makes sense to correct the small-scale aberrations is less than that for low-order aberrations.

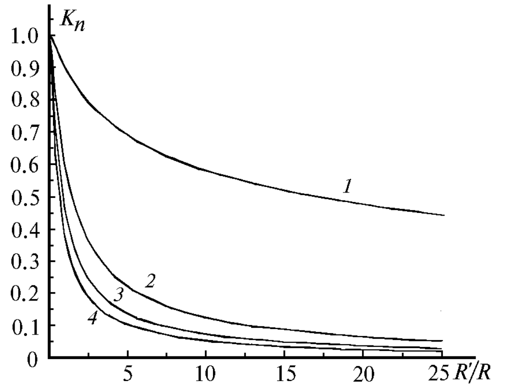

FIG. 2. Averaged correlation functions for different aberrations in the case of the Kolmogorov spectrum of turbulence for $n = 1$ *(1)*, 2*(2)*, 3*(3)*, *and* 4*(4)*.

For large areas $(R'/R > 10)$ in the case of the Kolmogorov spectrum for the correlation functions averaged over the circle we obtained approximate formulas. With the approximation by a hyperbola of the form $f_q\,(R'/R\,,\,\infty) \approx \alpha_n\,(R'/R)^{-\beta_n}$ in this interval, the results are the following:

$$f_1(x) \approx 1.33/\sqrt[3]{x}\ ,\quad f_2(x) \approx 1.32/x\ ,\quad f_3(x) \approx 0.76/x\ ,$$
$$f_4(x) \approx 0.54/x\ ,\quad x = R'/R\ . \tag{35}$$

These formulas confirm the conclusions drawn above. As n increases, the coefficients α_n decrease. Asymptotic curves for correlation functions of tilts $(n = 1)$ differ from those for other aberrations.

Correlation functions of tilts in the case of the von Karman spectrum and different outer scales of turbulence are shown in Fig. 3. In the interval $L_0/R \approx 10^2 - 10^3$, aberrations of higher orders are not influenced by the outer scale. Having the largest scale, tilts are most sensitive to variations of the outer scale of turbulence.

When the lowest N aberrations are corrected by the above-described algorithm, the total residual error of correction for the circular area can be represented as

$$\sigma_N^2\left(\frac{R'}{R}\right) = \sum_{q=1}^{\infty}\langle a_q^2\rangle - \sum_{q=1}^{N}\langle a_q^2\rangle + \sum_{q=1}^{N}\langle a_q^2\rangle\left[1 - f_q\left(\frac{R'}{R}\right)\right]. \tag{36}$$

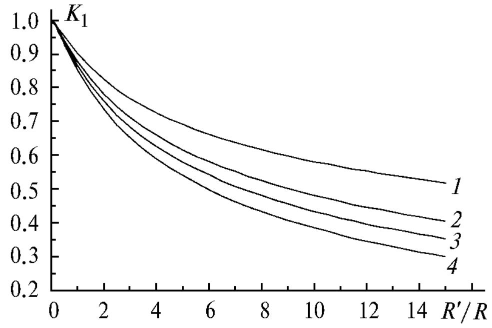

FIG. 3. Averaged correlation functions of tilts for different values of the outer scale in the case of the von Karman spectrum of turbulence. $L/R = \infty$ *(1)*, 500 *(2)*, 200 *(3)*, *and* 100 *(4)*.

The residual error σ_N^2 as a function of the number of corrected modes is shown in Fig. 4 for areas with different size. The error is shown in units of $Q = (2\,R/r_0)^{5/3}$, where r_0 is Fried's radius of a plane wave. Allowing for the factor $\frac{3}{8}$, at $R' = 0$ all the points of the plot coincide with the well-known residual errors of correction for aberrations of a plane wave.[6] This factor appears due to the finite distance between the system and the object, so the waves propagating from the object are spherical. At $R' = 0$ and $N = 21$, the error is equal to $0.01Q$. This value can be taken as maximum accuracy for correction. For small objects $(R' \le 0.25R)$, the value of the residual error is almost the same as for $R' = 0$. The difference is no greater then $0.01Q$. So in this case the objects can be considered as point-like. As the size of the object increases, correction for higher order aberrations becomes inefficient because it does not practically change the value of the total residual error. For example, for $R' \ge 2R$ the correction for aberrations with radial number $n = 3$ decreases the error approximately by $0.001Q$, that is, ten times lower than the maximum accuracy of correction. For $R' \ge 5R$, the correction for aberrations higher than tilt makes no sense because in this region the correction for astigmatism and defocusing contributes to the total error less than $0.001Q$.

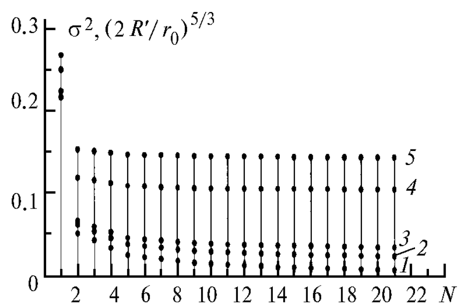

FIG. 4. Residual error of correction as a function of the number of corrected modes N computed for different size of the area R': $R' = 0$ *(1)*, $0.25R$ *(2)*, $0.5R$ *(3)*, $2.5R$ *(4)*, *and* $5R$ *(5)*.

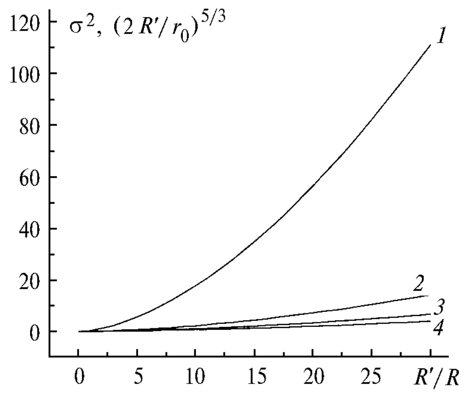

FIG. 5. Total residual error as a function of the aperture radius normalized by the radius of the area, R/R', computed for different numbers of corrected modes: without correction (1) and with correction for tilt (2); for tilts, defocusing, and two types of astigmatism (3); for all aberrations with radial number n = 1, 2, and 3 (4).

To follow the dependence of the total mean square error (Eq. (36)) on the aperture radius, let us write the error in the following form:

$$\sigma_N^2 = \left(\frac{2\,R'}{r_0}\right)^{5/3} \times$$

$$\times\left\{\left(\frac{R}{R'}\right)^{5/3} \sum_{q=1}^{N} \beta_q \left[1 - f_q\left(\frac{R'}{R}\right)\right] + \left(\frac{R}{R'}\right)^{5/3} \sum_{q=N+1}^{\infty} \beta_q\right\}, \quad (37)$$

where the mean square error is represented in the form with explicitly expressed dependence on the aperture radius

$$\langle a_q^2\rangle = (2\,R'/r_0)^{5/3}\,(R/R')^{5/3}\,\beta_q. \quad (38)$$

The value of error as the function of the parameter (R/R') is shown in Fig. 5 in units of $(2\,R'/r_0)^{5/3}$ for different numbers of corrected modes. Curve $N = 0$ is for a beam without correction, $N = 2$ is for the correction for tilts, $N = 5$ is for the correction for tilts and all aberrations with radial number $n = 2$ (defocusing and astigmatism), $N = 9$ is for the correction for all modes with n = 1, 2, and 3.

5. CONCLUSION

To use the potentialities of the adaptive optics system, the modified algorithm for control with the phase corrector has been proposed to correct the image of an extended object. The control parameters are chosen allowing for distortions of waves reflected from all points of the object. For a particular aberration, the residual error is expressed through the correlation function of Zernike coefficients for two points of the object averaged over the area whose image is formed. The error increases monotonically as the size of the area increases.

Applying the theory of wave propagation in a distorting medium, the integral equation has been obtained which relates the averaged correlation function for the coefficients of aberrations with the spectrum of the optical turbulence. The averaged correlation functions of small-scale aberrations decrease with increasing area size faster than those of large-scale aberrations. So the modal correction for higher-order aberrations makes sense only when the size of the area is small. If the area size is five times (or more) larger than the aperture radius, the correction for tilts only makes sense. Computation of the correlation function has been performed for the von Karman spectrum of the turbulence with different outer scales. The outer scale influences notably the correlation functions of tilts. The effect of the outer scale on aberrations of higher orders is negligible.

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