RECONSTRUCTION OF THE SURFACE REFLECTANCE FROM MEASUREMENT DATA

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Solution of the inverse problem of reconstructing reflectance of the underlying surface from measured radiance data in the visible spectrum range is proposed. The statement of a problem is based on a model of radiative transfer above the surface with non–Lambertian inhomogeneous reflection. The measurements can be carried out both by ground and remote methods. This problem is solved by inverting optical transfer operator. Solution of the problem for the case of non–Lambertian homogeneous reflection is considered as a particular case.

INTRODUCTION

The reflectance of Earth's landscapes is normally characterized by the reflection coefficient, i.e., the optical characteristic that depends only on physical properties of the surface itself. Since natural and artificial objects on the Earth's surface have quite various reflectivities their reflection coefficients, generally speaking, depend on the horizontal coordinates on the surface as well as on the angular coordinates of an observer and an external source of illumination. Under conditions of natural illumination the reflection coefficient cannot be directly determined because the total reflected radiation flux is formed by photons scattered in the atmosphere and reflected from the surface. Measured values normally are brightness of upward radiation and the brightness coefficient of the atmosphere–underlying surface system, which are determined by the reflecting properties of the surface and optical conditions of the atmosphere as well. The study of the relationships between the reflecting and radiation characteristics of the surface is necessary to develop algorithms for atmospheric correction of data obtained from ground–based remote from measurements.

A model of radiative transfer in the atmosphere above the surface with non–Lambertian inhomogeneous reflection is the basis for statement and solution of the inverse problem on reconstruction of the reflection coefficient from measurement data. Mathematical aspects of a radiation transfer model were considered in a number of papers.[1–6] Different statements of the inverse problem were considered in Refs. 3, 5–7, and 9. In the general case, it can be stated that solution of the inverse problem is reduced to inversion of the atmospheric optical transfer operator transforming the reflection coefficient into the brightness field of upward radiation. The method of determination of the surface reflection coefficient from the measured brightness of upward radiation or from a known coefficient of brightness of the atmosphere–underlying surface system is the subject of the present investigation.

MATHEMATICAL MODEL OF RADIATIVE TRANSFER

Spectral brightness of natural radiation in the visible range, $I \equiv I(z, r, \mathbf{s}, \mathbf{s}_0)$, in the atmosphere–underlying system obeys the boundary–value problem for integral differential equation of radiative transfer[4]

$$LI = SI \; ; \; I\big|_{\Gamma_0} = \pi S_\lambda \delta(\mathbf{s} - \mathbf{s}_0) \; ; \qquad I\big|_{\Gamma_h} = R_\rho I \; . \tag{1}$$

Here $L = (\nabla, \mathbf{s}) + \alpha(z)$ is the transfer operator; S: $SI = \dfrac{\sigma(z)}{4\pi} \times \int\limits_{\Omega} f(z, \mathbf{s}, \mathbf{s}')\, I(z, \mathbf{r}, \mathbf{s}, \mathbf{s}_0)\, d\mathbf{s}'$ is the scattering operator; R_ρ: $R_\rho I = \dfrac{1}{\pi} \int\limits_{\Omega_+} \rho(\mathbf{r}, \mathbf{s}', \mathbf{s}_0)\, I(h, \mathbf{r}, \mathbf{s}, \mathbf{s}_0)\, d\mathbf{s}'$ is the reflection operator; $\alpha(z)$ and $\sigma(z)$ are the extinction and volume scattering coefficients; $f(z, \mathbf{s}, \mathbf{s}')$ is the scattering phase function; $\rho(\mathbf{r}, \mathbf{s}, \mathbf{s}_0) \equiv \rho$ is the reflection coefficient, $0 < \rho < 1$; πS_λ is the extraterrestrial solar constant, λ is the wavelength; $\Gamma_0 = \{z = 0, \mathbf{s} \in \Omega_+\}$; $\Gamma_h = \{z = h, \mathbf{s} \in \Omega_-\}$, Ω is the unit sphere; Ω_+ and Ω_- are the lower and upper hemispheres; z is a vertical coordinate; $\mathbf{r} = \{x, y\}$ is the vector of horizontal coordinates; $z = 0$, $z = h$ are the upper and lower boundaries of the atmosphere; $\mathbf{s} = \{\mu, \mathbf{s}_\perp\}$ is the unit vector of light propagation, $\mathbf{s}_\perp = \sqrt{1 - \mu^2}\, \{\cos\varphi, \sin\varphi\}$; $\mu = \cos\theta$; $\mathbf{s}_0 = \{\zeta, \sqrt{1 - \zeta^2}, 0\}$ is the direction of incidence of the solar radiation flux; $\zeta = \cos \tau_0$; θ and φ are the zenith and azimuthal angles; and, θ_0 is the zenith angle of the sun.

Let the reflection coefficient be

$$\rho(\mathbf{r}, \mathbf{s}, \mathbf{s}_0) = q(\mathbf{r}, \mathbf{s}_0)\, P(\mathbf{r}, \mathbf{s}, \mathbf{s}_0) \; , \tag{2}$$

where $q(\mathbf{r}, \mathbf{s}_0) = \dfrac{1}{\pi} \int\limits_{\Omega_-} \rho(\mathbf{r}, \mathbf{s}, \mathbf{s}_0)\, \mu\, d\mathbf{s}$ is the surface albedo, $P(\mathbf{r}, \mathbf{s}, \mathbf{s}_0)$ is the reflection phase function satisfying the condition $\dfrac{1}{\pi} \int\limits_{\Omega_-} P(\mathbf{r}, \mathbf{s}, \mathbf{s}_0)\, \mu\, d\mathbf{s} = 1$. Any surface can be considered as superposition of N types of the basic physical surfaces with the known reflection phase functions $P_n(s, s_0)$ whose presence at each point of the surface is controlled by the albedo functions. Therefore, without loss of generality one can assume

$$\rho(\mathbf{r}, \mathbf{s}, \mathbf{s}_0) = \sum_{n=1}^{N} q_n(\mathbf{r})\, P_n(\mathbf{s}, \mathbf{s}_0) \; . \tag{3}$$

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The coefficient of reflection by a homogeneous surface can, by analogy, be represented as

$$\overline{r}(\mathbf{s},\ \mathbf{s}_0) = \sum_{n=1}^{N} \overline{q}_n\, P_n(\mathbf{s},\ \mathbf{s}_0)\ . \tag{4}$$

Here the values $\overline{q}_n$ are the weights of the corresponding modes. The determination of the summing limit N is an independent problem. At $n = 1$, from Eq. (3) we have the most rough representation

$$\rho(\mathbf{r},\ \mathbf{s},\ \mathbf{s}_0) = q(\mathrm{r})\, P(\mathbf{s},\ \mathbf{s}_0) \tag{5}$$

and for a homogeneous surface

$$\overline{r}(\mathbf{s},\ \mathbf{s}_0) = \overline{q}\ P(\mathbf{s},\ \mathbf{s}_0)\ , \tag{6}$$

where $\overline{q}$ is the average surface albedo. Representation (5) contains the internal contradiction. On the one hand, the surface is characterized only by the average reflection phase function $P(\mathbf{s},\ \mathbf{s}_0)$. On the other hand, according to Eq. (5), the surface is virtually inhomogeneous because the albedo $q(r)$ depends on r. Therefore, if one has only the average reflection phase function $P(\mathbf{s},\ \mathbf{s}_0)$ ($n = 1$) Eq. (6) is actually more grounded and practical.

The measured coefficient of the spectral brightness of the surface is defined as follows:

$$\rho_{\mathrm{m}}(\mathbf{r},\ \mathbf{s},\ \mathbf{s}_0) = \pi I(h,\ \mathbf{r},\ \mathbf{s},\ \mathbf{s}_0)/E(\mathbf{r},\ \mathbf{s}_0)\ ,\ \mathbf{s} \in \Omega_-\ , \tag{7}$$

where $E(\mathbf{r},\ \mathbf{s}_0) = \int\limits_{\Omega_+} I(h,\ \mathbf{r},\ \mathbf{s},\ \mathbf{s}_0))\,\mu\, \mathrm{d}\mathbf{s}$ is the surface irradiance.

For the directions $s \in \Omega_+$ the equality

$$I = I' + I_{\mathrm{d}} \tag{8}$$

is valid, where $I' \equiv I'(z,\ \mathbf{r},\ \mathbf{s},\ \mathbf{s}_0)$ and $I_{\mathrm{d}} = \pi S_\lambda \delta(\mathbf{s} - \mathbf{s}_0)\mathrm{e}^{-\tau/\xi}$ are the brightnesses of the scattered and direct components of radiation in the atmosphere, $\tau = \int\limits_0^z \alpha(z')\mathrm{d}z'$ is the optical coordinate, and $\zeta = \cos\theta_0$. After substitution of Eq. (8) into Eq. (1) for the brightness of the scattered component which involves the directly reflected radiation for $s \in \Omega$, we obtain

$$LI' = SI' + SI_{\mathrm{d}}\ ; \quad I'|_{\Gamma_0} = 0\ ; \quad I'|_{\Gamma_h} = R_\rho(I' + I_{\mathrm{d}})\ . \tag{9}$$

By substituting the bottom boundary value condition from Eq. (9) into Eq. (7) and taking into account that the equality $I = I'$ is valid for $s \in \Omega_-$ we can find the relationship $\rho_{\mathrm{m}}(\mathbf{r},\ \mathbf{s},\ \mathbf{s}_0) \equiv \rho_{\mathrm{m}}$ with $\rho(\mathbf{r},\ \mathbf{s},\ \mathbf{s}_0)$

$$\rho_{\mathrm{m}}(\mathbf{r},\ \mathbf{s},\ \mathbf{s}_0) = \pi\, R_\rho(I' + I_{\mathrm{d}})/\mathrm{E}\ , \tag{10}$$

or in the extended form

$$\rho_{\mathrm{m}}(\mathbf{r},\ \mathbf{s},\ \mathbf{s}_0) = \frac{\pi}{E}\Bigg[\rho(\mathbf{r},\ \mathbf{s},\ \mathbf{s}_0)\ T' + \frac{1}{\pi}\int\limits_{\Omega_+} \rho(\mathbf{r},\ \mathbf{s},\ \mathbf{s}')\ I(h,\ \mathbf{r},\ \mathbf{s}',\ \mathbf{s}_0)\,\mu'\, \mathrm{d}\mathbf{s}'\Bigg]\ , \tag{11}$$

where $T' = \zeta S_\lambda T(\zeta)$, $T_0(\zeta) = \mathrm{e}^{-\tau_0/\xi}$, and $\tau_0 = \int\limits_0^h \alpha(z')\mathrm{d}z'$.

The earth's surface irradiance by virtue of Eq. (8) is

$$E(\mathbf{r},\ \mathbf{s}_0) = \pi T' + \int\limits_{\Omega_+} I'(h,\ \mathbf{r},\ \mathbf{s}',\ \mathbf{s}_0))\,\mu'\mathrm{d}\mathbf{s}'\ .$$

In order to find the relationship of $\rho(r,\ \mathbf{s},\ \mathbf{s}_0)$ and the brightness of upward radiation at any height $h - z$ above the surface we have to solve the boundary value problem (Eqs. (1) or (9)). The solution of the boundary problem (Eq. (1)) is reduced to the solution of the simplest boundary value problems[5,10–12] using methods of multiple rereflections and space–frequency characteristics. Thus, using the method of multiple rereflections the boundary value problem (Eq. (1)) for $s \in \Omega_-$ can be represented in the form[2,6]

$$I = D + Z(\mathbf{r} - \tilde{\mathbf{r}},\ \mathbf{s},\ \mathbf{s}_0)\, T(\mu) + \int\limits_{\Omega_-}\int\limits_{-\infty}^{\infty} \tilde{O}_\delta(z,\ \mathbf{r} - \tilde{\mathbf{r}} - \mathbf{r}',\ \mathbf{s},\ \mathbf{s}')\, Z(\mathbf{r},\ \mathbf{s}',\ \mathbf{s}_0)\, \mathrm{d}\mathbf{r}'\, \mathrm{d}\mathbf{s}'\ , \tag{12}$$

where D is the brightness of the atmospheric haze, $Z(r,\ \mathbf{s},\ \mathbf{s}_0) = I(h,\ r,\ \mathbf{s},\ \mathbf{s}_0)$ is the surface brightness,

$$Z(\mathbf{r},\ \mathbf{s},\ \mathbf{s}_0) = \sum_{n=0}^{\infty} (\mathrm{Q}_h R_\rho)^n R_\rho(D + I_{\mathrm{d}}) = (\hat{E} - R\Theta_h)^{-1} R_\rho(D + I_{\mathrm{d}}), \tag{13}$$

$\hat{E}$ is the unit operator, Θ_h is the integral operator acting according to the rule Θ_h:

$$\Theta_h Z = \int\limits_{\Omega_-}\int\limits_{-\infty}^{\infty} O_h(\mathbf{r} - \mathbf{r}',\ \mathbf{s},\ \mathbf{s}')\, Z(\mathbf{r}',\ \mathbf{s}',\ \mathbf{s}_0)\, \mathrm{d}\mathbf{r}'\, \mathrm{d}\mathbf{s}'\ ,$$

$$T(\mu) = \mathrm{e}^{-(\tau_0 - \tau)/\eta}\ ; \quad \eta = |\mu|\ ; \quad O_h(\mathbf{r},\ \mathbf{s},\ \mathbf{s}') = O_\delta(h,\ \mathbf{r},\ \mathbf{s},\ \mathbf{s}')\ ,$$

$O_\delta(z,\ \mathbf{r},\ \mathbf{s},\ \mathbf{s}')$ is the pulse–transient function of a system, where the radiation is transferred and has a unidirectional point source of light at its low boundary

$$O_\delta(\mathrm{z},\ \mathbf{r},\ \mathbf{s},\ \mathbf{s}') = T(\mu')\, \delta(\mathbf{r} - \tilde{\mathbf{r}})\, \delta(\mathbf{s} - \mathbf{s}')) + \tilde{O}_\delta(z,\ \mathbf{r} - \tilde{\mathbf{r}} - \mathbf{r}',\ \mathbf{s},\ \mathbf{s}'). \tag{14}$$

$\tilde{O}_\delta(z,\ \mathbf{r},\ \mathbf{s},\ \mathbf{s}')$ is the diffusion component of the pulse transient function; $\tilde{\mathbf{r}} = \mathbf{s}_\perp(h - z)/\eta$ is the displacement vector. Functions D and $\tilde{O}_\delta$ obey the equations

$$LD = SD + SI_{\mathrm{d}}\ ;\ L\tilde{O}_\delta = S\tilde{O}_\delta + \frac{\sigma(z)}{4\pi}\, T(\mu')\, \delta(\mathbf{r} - \tilde{\mathbf{r}})\, f(z,\ \mathbf{s},\ \mathbf{s}')$$

with the zero boundary conditions.

The main idea of representation (12) is supplemented by the following analytic representations:

$$R_\rho(D + I_{\mathrm{d}}) \equiv E_\rho(\ \mathbf{r},\ \mathbf{s},\ \mathbf{s}_0) = = 2\int\limits_0^1 \rho_0(\mathbf{r},\ \mu,\ \mu')\, D^0(h,\ \mu',\ \mathrm{z})\,\mu'\, \mathrm{d}\mu' + \zeta\rho(\mathbf{r},\ \mathbf{s},\ \mathbf{s}_0)\, S_\lambda \mathrm{e}^{-\tau_0/\xi}; \tag{15}$$

$$Z(\mathbf{r},\ \mathbf{s},\ \mathbf{s}_0) \equiv E_\rho(\mathbf{r},\ \mathbf{s},\ \mathbf{s}_0) +$$

$$+\sum_{n=1}^{\infty} \underbrace{\int_{\Omega_-}\int_{-\infty}^{\infty} \dots \int_{\Omega_-}\int_{-\infty}^{\infty}}_{n} Q(r,\ r - r_1,\ s,\ s_1)\ Q(r_1,\ r_1 - r_2,\ s_1,\ s_2)\ \dots$$

$$\dots Q_n(r_{n-1},\ r_{n-1} - r_n,\ s_{n-1},\ s_n)\ E_\rho(r_n,\ s_n,\ s_0)\,dr_n\,ds_n \dots dr_1\,ds_1,$$

where

$$Q(\mathbf{r},\ \mathbf{r} - \mathbf{r}_1,\ \mathbf{s},\ \mathbf{s}_1) = \frac{1}{\pi}\int_{\Omega_+} \rho(\mathbf{r},\ \mathbf{s}_n,\ \mathbf{s}_0)\ O_h(\mathbf{r} - \mathbf{r}_1,\ \mathbf{s}',\ \mathbf{s}_1)\ \mu'\ d\mathbf{s}'\ ,$$

$$\rho^0(\mathbf{r},\ \mu,\ \zeta) = \frac{1}{2\pi}\int_0^{2\pi} \rho(\mathbf{r},\ \mathbf{s},\ \mathbf{s}_0)\ d\varphi\ ,$$

$$D^0(h,\ \mu,\ \zeta) = \frac{1}{2\pi}\int_0^{2\pi} D(h,\ \mathbf{s},\ \mathbf{s}_0)\ d\varphi\ .$$

In the case of a homogeneous Lambertian surface representation (12) is reduced to the form

$$\overline{I} = D + \overline{Z}(\mathbf{s},\ \mathbf{s}_0)\ T(\mu) + \int_{\Omega_-} A_{\delta,0}(z,\ \mathbf{s},\ \mathbf{s}')\ \overline{Z}(\mathbf{s}',\ \mathbf{s}_0)\ d\mathbf{s}'\ , \quad (16)$$

where $\overline{I} = I\big|_{\rho=\overline{\rho}_0}$ is the radiance,

$$\overline{Z}(\mathbf{s},\ \mathbf{s}_0) = E_{\overline{\rho}}(\mathbf{s},\ \mathbf{s}_0) +$$

$$+\sum_{n=1}^{\infty} \underbrace{\int_{\Omega_-}\dots\int_{\Omega_-}}_{n} \overline{Q}(s,\ s_1)\ \overline{Q}(s_1,\ s_2)\dots\overline{Q}(s_{n-1},\ s_n)\ E_{\overline{\rho}}(s_n,\ s_0)ds_n \dots ds_1, \quad (17)$$

$$E_{\overline{\rho}}(\mathbf{s}_n,\ \mathbf{s}_0) = 2\int_0^1 \overline{\rho}(\mu,\ \mu')\,D^0(h,\ \mu',\ \zeta)\,\mu'd\mu' + \zeta\,S_\lambda \overline{\rho}(\mathbf{s},\ \mathbf{s}_0)e^{-\tau_0/\xi},$$

$$\overline{Q}(\mathbf{s},\ \mathbf{s}_1) = \frac{1}{\pi}\int_{\Omega_+} \overline{\rho}(\mathbf{s},\ \mathbf{s}')\ \Psi_{\delta,0}(h,\ \mathbf{s}',\ \mathbf{s}_1)\mu' d\mathbf{s}'\ , \quad (18)$$

$$A_{\delta,0}(z,\ \mathbf{s},\ \mathbf{s}') = \Psi_{\delta,0}(z,\ \mathbf{s},\ \mathbf{s}') - \delta(\mathbf{s} - \mathbf{s}')\ T(\mu')\ ,$$

$$\overline{\rho}^{\,0}(\mu,\ \mu') = \frac{1}{2\pi}\int_0^{2\pi} \overline{\rho}(\mathbf{s},\ \mathbf{s}')\ d\varphi\ ,$$

$$\Psi_{\delta,0}(z,\ \mathbf{s},\ \mathbf{s}') = \int_{-\infty}^{\infty} O_\delta(z,\ \mathbf{r}',\ \mathbf{s},\ \mathbf{s}')\ d\mathbf{r}'\ .$$

In a limiting case of an isotropic reflection representations (12) and (16) and accompanying formulas give the well–known results.[10,11]

Formulas (12) and (16) are compact, however, when applied to calculations they imply computations of multidimensional sums and integrals. To simplify the computations some approximate models of reflection are used. The approximate models take into account the anisotropy of the reflection coefficient only for unscattered radiation,[13] or use the single–reflection approach.[1,5,6] These assumptions enable one to simplify the operator of direct problem solution.

Let us make use of a non–Lambertian single reflection approach, according to which the radiation singly reflected from the surface is taken into account by the reflection coefficient $\rho(r,\ \mathbf{s},\ \mathbf{s}_0)$ and the multiply rereflected from the surface radiation is accounted for by the albedo $q(r,\ \mathbf{s}_0)$. As is shown in Ref. 1, this approach provides the calculation error of brightness I less than 1%. For solving boundary value problem (1) we use the method of space–frequency characteristics. To use this method it is necessary to factor the dependences on horizontal and angular variables. Such a factorization is done by expansion (3). The use of this method within the framework of the non–Lambertian single–reflection approach we have

$$I = D + \sum_{n=1}^{N} \overline{q}_n \left[\overline{\Psi}_{\delta,0,\ n}(z,\ \mathbf{s},\ \mathbf{s}_0) + \frac{\overline{q}\Psi_0(z,\ \mu)}{1 - \overline{q}c_0}\,\overline{C}_{\delta,0,\ n}(\zeta) \right] +$$

$$+ \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty} \sum_{n=1}^{N} \hat{q}_n(\mathbf{p}) \left\{ \overline{\Psi}_{\delta,\ n}(z,\ \mathbf{p},\ \mathbf{s},\ \mathbf{s}_0) + \frac{\Psi(z,\ \mathbf{p},\ \mathbf{s})}{1 - \overline{q}C(\mathbf{p})} \right. \times$$

$$\left. \times \left[\overline{q}\ \overline{C}_{\delta,n}(\mathbf{p},\ \mathbf{s}_0) + \frac{\sum_{n'=1}^{N} \overline{q}_{n'}\ \overline{C}_{\delta,0,\ n'}(\zeta)}{1 - \overline{q}c_0} \right] \right\} e^{-i(\mathbf{p},\mathbf{r})}\ d\mathbf{p}, \quad (19)$$

where

$$\overline{\Psi}_{\delta,0,\ n}(z,\ \mathbf{s},\ \mathbf{s}_0) = \int_{\Omega_-} \Psi_{\delta,0}(z,\ \mathbf{s},\ \mathbf{s}')\ E_{P_n}(\mathbf{s}',\ s_0)\ d\mathbf{s}'\ ,$$

$$\Psi_0(z,\ \mu) = 2\pi\int_0^1 \Psi^0_{\delta,0}(z,\ \mu,\ \mu')\ d\mu')\ ,\ c_0 = 2\int_0^1 \Psi_0(h,\ \mu)\ \mu\ d\mu\ ,$$

$$\overline{C}_{\delta,0,n}(\zeta) = 2\int_0^1 \overline{\Psi}^0_{\delta,0,\ n}(h,\ \mu',\ \zeta)\mu' d\mu'\ ,$$

$$E_{P_n}(\mathbf{s},\ s_0) = 2\int_0^1 P^0_n(\mu,\ \mu')\ D^0(h,\ \mu',\ \zeta)\mu' d\mu' + \zeta P_n(\mathbf{s},\ s_0)\ S_\rho e^{-\tau_0/\xi},$$

$$\overline{\Psi}^0_{\delta,0,\ n}(z,\ \mu,\ \zeta) = \frac{1}{2\pi}\int_0^{2\pi} \overline{\Psi}_{\delta,0,\ n}(z,\ \mathbf{s},\ \mathbf{s}_0)\ d\varphi\ ,$$

$$P^0_n(\mu,\ \zeta) = \frac{1}{2\pi}\int_0^{2\pi} P_n(\mathbf{s},\ s_0)\ d\varphi\ ,$$

$$\overline{q}_n = \langle q_n(\mathbf{r})\rangle\ ,\ \tilde{q}_n(\mathbf{r}) = q_n(\mathbf{r}) - \overline{q}_n\ ,$$

$$\hat{q}_n(\mathbf{p}) = \int_{-\infty}^{\infty} \tilde{q}_n(\mathbf{r})\ e^{i(\mathbf{p},\mathbf{r})}\ d\mathbf{r}\ ,$$

$$\overline{q} = \sum_{n=1}^{N} \overline{q}_n\ ,\ \Psi^0_{\delta,0}(z,\ \mu,\ \mu') = \frac{1}{2\pi}\int_0^{2\pi} \Psi_{\delta,0}(z,\ \mathbf{s},\ \mathbf{s}')\ d\varphi'\ ,$$

$$\overline{\Psi}_{\delta,\,n}(z,\,\mathbf{p},\,\mathbf{s},\,\mathbf{s}_0) = \int\limits_{\Omega_-} \Psi_\delta(z,\,\mathbf{p},\,\mathbf{s},\,\mathbf{s}')\,E_{p_n}(\mathbf{s}',\,s_0)\,d\mathbf{s}' \,,$$

$$\overline{C}_{\delta,\,n}(\,\mathbf{p},\,\mathbf{s}_0) = \frac{1}{\pi}\int\limits_{\Omega_+} \overline{\Psi}_{\delta,\,n}(h,\,\mathbf{p},\,\mathbf{s}',\,\mathbf{s}_0)\,\mu' d\mathbf{s}' \,,$$

$$C(\mathbf{p}) = \frac{1}{\pi}\int\limits_{\Omega_+} \Psi(h,\,\mathbf{p},\,\mathbf{s})\,\mu\,d\mathbf{s} \,,$$

$$\Psi(z,\,\mathbf{p},\,\mathbf{s}) = \int\limits_{\Omega_+} \Psi_\delta(z,\,\mathbf{p},\,\mathbf{s},\,\mathbf{s}')\,d\mathbf{s}' \,,$$

$\Psi_\delta(z,\,\mathbf{p},\,\mathbf{s},\,\mathbf{s}') = \int\limits_{-\infty}^{\infty} O_\delta(z,\,\mathbf{r},\,\mathbf{s},\,\mathbf{s}')\,e^{i(\mathbf{p},\,\mathbf{r})}\,d\mathbf{r}$ is the main space–frequency characteristics, $\mathbf{p} = \{p_x,\,p_y\}$ is the vector of a two–dimensional spatial frequency.

In the case of a Lambertian surface it directly follows from Eq. (19) that

$$\overline{I} = D + \sum_{n=1}^{N} \overline{q}_n \left[\overline{\Psi}_{\delta,0,\,n}(z,\,\mathbf{s},\,\mathbf{s}_0) + \frac{\overline{q}\Psi_0(z,\,\mu)}{1-\overline{q}c_0}\,\overline{C}_{\delta,0,\,n}(\zeta) \right]. \quad (20)$$

It we omit the subscripts n and n' in Eqs. (19) and (20) and in the auxiliary relations as well as the summing over them we obtain the solution corresponding to representations (5) and (6).

According to the above–presented mathematical models the numerical calculations of the radiation brightness are reduced to calculations of the basic radiation characteristics D, Ψ_0, Ψ, $\Psi_{\delta,0}$, Ψ_δ (or O_δ) entering into solutions of Eqs. (12), (16), (19), and (20). These functions are independent of ρ and determine the action of atmospheric optical transmission operator. Algorithms for calculation of these characteristics and the corresponding software have been developed in Refs. 5, 9, 11, 12, and 14.

The whole set of the above relations forms a mathematical model of the radiative transfer which is used below for the statement of the inverse problem.

RECONSTRUCTION OF THE REFLECTION COEFFICIENT FROM GROUND–BASED MEASUREMENT DATA

To reconstruct the reflection coefficient ρ from measurements of the brightness coefficient ρ_m one should solve the integral equation

$$\rho(\mathbf{r},\,\mathbf{s},\,\mathbf{s}_0) = -\frac{1}{\pi T'}\int\limits_{\Omega_+} r(\mathbf{r},\,\mathbf{s},\,\mathbf{s}')\,I(h,\,\mathbf{r},\,\mathbf{s}',\,\mathbf{s}_0)\,\mu'\,d\mathbf{s}' + {}$$

$$+ \frac{E(\mathbf{r},\,\mathbf{s}_0)}{\pi T'}\,\rho_m(\mathbf{r},\,\mathbf{s},\,\mathbf{s}_0) \quad (21)$$

with respect to $\rho(\mathbf{r},\,\mathbf{s},\,\mathbf{s}_0)$. Equation (21) follows from Eq. (11). Under Lambertian reflection $\rho(\mathbf{r},\,\mathbf{s},\,\mathbf{s}_0) \equiv \rho(\mathbf{r},\,\mathbf{s})$ Eq. (21) degenerates to the identity $q(\mathbf{r}) \equiv \rho_m(\mathbf{r},\,\mathbf{s},\,\mathbf{s}_0)$ and the statement of the problem on atmospheric correction of the ground measurements ρ_m becomes senseless. The same result takes place under $\rho(\mathbf{r},\,\mathbf{s},\,\mathbf{s}_0) \equiv \rho(\mathbf{r},\,\mathbf{s})$. Therefore, the difference between ρ and ρ_m is determined by the dependence ρ and ρ_m on $\mathbf{s}_0$ alone. Difference between ρ and ρ_m has an important consequence: the experimental ground support data of satellite measurements obtained from a non–Lambertian surface ρ_m are not identical to the results of reconstructing ρ from remote measurements.

Integral equation (21) is nonlinear because the functions $I(h,\,\mathbf{r},\,\mathbf{s},\,\mathbf{s}_0)$ and $E(\mathbf{r},\,s_0)$ implicitly depend on $\rho(\mathbf{r},\,\mathbf{s},\,\mathbf{s}_0)$. The solution of this equation can be found by the iteration method. First of all one has to eliminate the unknown function $I(h,\,\mathbf{r},\,\mathbf{s}',\,\mathbf{s}_0)$ from the equation which is determined by the solution of boundary value problem (9). The brightness can be represented by the sum $I = D + \sum\limits_{n=1}^{\infty} I^{(n)}$, where $I^{(n)}$ are the brightness components composed by photons n times rereflected from the surface. Numerical calculations show that for real values of the optical thickness of the atmosphere and surface albedo in downward radiation ($\mathbf{s} \in \Omega_+$) it is sufficient to take only $I^{(1)}$ without a noticeable error. In view of this circumstance let us use the sum

$$I = D + I^{(1)} \,, \quad (22)$$

where the component $I^{(1)}$ is found from the solution of the boundary value problem[8]

$$LI^{(1)} = SI^{(1)} \,; \quad I^{(1)}|_{\Gamma_0} = 0 \,; \quad I^{(1)}|_{\Gamma_h} = R_\rho(D + I_\delta) \,. \quad (23)$$

By substituting Eq. (21) into Eq. (20) we obtain

$$\rho(\mathbf{r},\,\mathbf{s},\,\mathbf{s}_0) = -\frac{1}{\pi T'}\int\limits_{\Omega_+} r(\mathbf{r},\,\mathbf{s},\,\mathbf{s}')\,[D(\mathrm{h},\,\mathbf{s}',\,\mathbf{s}_0) + {}$$

$$+ I^{(1)}(h,\,\mathbf{r},\,\mathbf{s}',\,\mathbf{s}_0)]\,\mu'\,d\mathbf{s}' + \frac{E^{(1)}(\mathbf{r},\,\mathbf{s}_0)}{\pi T'}\,\rho_m(\mathbf{r},\,\mathbf{s},\,\mathbf{s}_0) \,, \quad (24)$$

where

$$E^{(1)}(\mathbf{r},\,\mathbf{s}_0) = \pi T' + \int\limits_{\Omega_+} [D(h,\,\mathbf{s}',\,\mathbf{s}_0) + I^{(1)}(h,\,\mathbf{r},\,\mathbf{s}',\,\mathbf{s}_0)]\,\mu'\,d\mathbf{s}' \,.$$

Let us first write down the solution of the linearized equation neglecting the value $I^{(1)}(h,\,\mathbf{r},\,\mathbf{s}',\,\mathbf{s}_0)$

$$\rho^{(0)}(\mathbf{r},\,\mathbf{s},\,\mathbf{s}_0) = \frac{1}{\pi T'}\Bigg[E^{(0)}(\mathbf{s}_0)\,\rho_m(\mathbf{r},\,\mathbf{s},\,\mathbf{s}_0) + {}$$

$$+ \int\limits_{\Omega_+} E^{(0)}(\mathbf{s}')\,\rho_m(\mathbf{r},\,\mathbf{s},\,\mathbf{s}')\sum_{k=1}^{\infty} D_k(h,\,\mathbf{s}',\,\mathbf{s}_0)\,\mu'\,d\mathbf{s}' \Bigg], \quad (25)$$

where

$$E^{(0)}(\mathbf{s}_0) = \pi T' + \int\limits_{\Omega_+} D(\mathrm{h},\,\mathbf{s}',\,\mathbf{s}_0)\,\mu'\,d\mathbf{s}' \,;$$

$$D_1(\mathbf{s}',\,\mathbf{s}_0) = -\frac{1}{\pi T'}\,D(h,\,\mathbf{s}',\,\mathbf{s}_0) \,, \quad \mathbf{s}' \in \Omega_+ \,;$$

$$D_k(\mathbf{s}',\,\mathbf{s}_0) = \int\limits_{\Omega_+} D_{k-1}(\mathbf{s},\,\mathbf{s}'')\,D(h,\,\mathbf{s}'',\,\mathbf{s}_0)\mathrm{m}''\,d\mathbf{s}'' \,, \; k > 1 \,.$$

Substituting now $\rho^{(0)}(\mathbf{r}, \mathbf{s}', \mathbf{s}_0)$ into Eq. (23) we can approximately find the function $I^{(1)}$ and then $E^{(1)}(\mathbf{r}, \mathbf{s}_0)$. As a result the solution of integral equation (24) takes the form

$$\rho(\mathbf{r}, \mathbf{s}, \mathbf{s}_0) \approx \rho^{(1)}(\mathbf{r}, \mathbf{s}, \mathbf{s}_0) = \frac{1}{\pi T'}\left[E^{(1)}(\mathbf{r}, \mathbf{s}_0)\, \rho_m(\mathbf{r}, \mathbf{s}, \mathbf{s}_0) + \right.$$

$$\left. + \int\limits_{\Omega_+} E^{(1)}(\mathbf{r}, \mathbf{s}')\, \rho_m(\mathbf{r}, \mathbf{s}, \mathbf{s}') \sum_{k=1}^{\infty} I_k(h, \mathbf{r}, \mathbf{s}', \mathbf{s}_0)\, \mu'\, d\mathbf{s}'\right], \qquad (26)$$

where

$$I_1(\mathbf{r}, \mathbf{s}', \mathbf{s}_0) = -\frac{1}{\pi T'}\left[D(h, \mathbf{s}', \mathbf{s}_0) + I^{(1)}(h, \mathbf{r}, \mathbf{s}', \mathbf{s}_0)\big|_{\rho=\rho^{(0)}}\right],$$

$$I_k(\mathbf{r}, \mathbf{s}', \mathbf{s}_0) = \int\limits_{\Omega_+} I_{k-1}(\mathbf{r}, \mathbf{s}', \mathbf{s}'')\, I_1(h, \mathbf{r}, \mathbf{s}'', \mathbf{s}_0)\, \mu''\, d\mathbf{s}'', \; k > 1 .$$

Ground–based measurements are carried out at a fixed point $\mathbf{r}$. The influence of the adjacency effect[15] caused by the surface albedo inhomogeneities is quite insignificant since the brightness of downward radiation weakly depends on the horizontal coordinates. Therefore, it is reasonable to have solutions for the reflection coefficients and brightness coefficients averaged over the horizontal coordinates $\overline{\rho}(\mathbf{s}, \mathbf{s}_0)$ and $\overline{\rho}_m(\mathbf{s}, \mathbf{s}_0)$. For a homogeneous non–Lambertian surface we have

$$\overline{\rho}(\mathbf{s}, \mathbf{s}_0) = \frac{1}{\pi T'}\left[\overline{E}^{(1)}(\mathbf{s}_0)\, \overline{\rho}_m(\mathbf{s}, \mathbf{s}_0) + \right.$$

$$\left. + \int\limits_{\Omega_+} \overline{E}^{(1)}(\mathbf{s}')\, \overline{\rho}(\mathbf{s}, \mathbf{s}') \sum_{k=1}^{\infty} \overline{I}_k(h, \mathbf{s}', \mathbf{s}_0)\, \mu'\, d\mathbf{s}'\right], \qquad (27)$$

where

$$\overline{\rho}^{(0)}(\mathbf{s}, \mathbf{s}_0) = \frac{1}{\pi T'}\left[E^{(0)}(\mathbf{s}_0)\, \overline{\rho}_m(\mathbf{s}, \mathbf{s}_0) + \right.$$

$$\left. + \int\limits_{\Omega_+} E^{(0)}(\mathbf{s}')\, \overline{\rho}_m(\mathbf{s}, \mathbf{s}') \sum_{k=1}^{\infty} D_k(h, \mathbf{s}', \mathbf{s}_0)\, \mu'\, d\mathbf{s}'\right],$$

$$\overline{E}^{(1)}(\mathbf{s}_0) = \pi T' + \int\limits_{\Omega_+} \left[D(h, \mathbf{s}', \mathbf{s}_0) + \overline{I}^{(1)}(h, \mathbf{s}', \mathbf{s}_0)\right] \mu'\, d\mathbf{s}',$$

$$\overline{I}_1(\mathbf{s}', \mathbf{s}_0) = -\frac{1}{\pi T'}\left[D(h, \mathbf{s}', \mathbf{s}_0) + \overline{I}^{(1)}(h, \mathbf{s}', \mathbf{s}_0)\big|_{\overline{\rho}=\overline{\rho}^{(0)}}\right],$$

$$\overline{I}_k(\mathbf{s}', \mathbf{s}_0) = \int\limits_{\Omega_+} \overline{I}_{k-1}(\mathbf{s}', \mathbf{s}'')\, \overline{I}_1(h, \mathbf{s}'', \mathbf{s}_0)\, \mu''\, d\mathbf{s}'', \; k > 1$$

and the function $I^{(1)}$ is determined from the solution of the boundary value problem

$$\left\{\overline{L}\, \overline{I}^{(1)} = S\overline{I}^{(1)};\; \overline{I}^{(1)}\big|_{\Gamma_0} = 0;\; \overline{I}^{(1)}\big|_{\Gamma_h} = \overline{R}_\rho(D + I_d)\right\},$$

$$\overline{L} = \mu \frac{d}{dz} + \alpha(z), \quad \overline{R}_\rho \overline{I} = \frac{1}{\pi}\int\limits_{\Omega_+} \overline{\rho}(\mathbf{s}, \mathbf{s}')\, \overline{I}(h, \mathbf{s}', \mathbf{s}_0)\, \mu'\, d\mathbf{s}' .$$

Formulas (25) are the basic calculational relations for algorithms of atmospheric correction of ground–based measurements provided that the atmospheric optical parameters are known.

RECONSTRUCTION OF THE REFLECTION COEFFICIENT FROM REMOTE MEASUREMENTS

Let the brightness $I(z, \mathbf{r}, \mathbf{s}, \mathbf{s}_0)$ be known. The reflection coefficient $\rho(\mathbf{r}, \mathbf{s}, \mathbf{s}_0)$ is required to be determined. The atmospheric optical parameters are assumed to be known.

Let us make use of model (12)–(15). From Eq. (13) we have

$$(\hat{E} - R_\rho \Theta_h)\, Z(\mathbf{r}, \mathbf{s}, \mathbf{s}_0) = R_\rho(D + I_d) =$$

$$= Z(\mathbf{r}, \mathbf{s}, \mathbf{s}_0) - R_\rho \Theta_h Z(\mathbf{r}, \mathbf{s}, \mathbf{s}_0) .$$

Since $R_\rho\Theta_h Z(\mathbf{r}, \mathbf{s}, \mathbf{s}_0) = \frac{1}{\pi}\int\limits_{\Omega_+} \rho(\mathbf{r}, \mathbf{s}, \mathbf{s}') \int\limits_{\Omega_-}\int\limits_{-\infty}^{\infty} O_\delta(h, \mathbf{r}-\mathbf{r}', \mathbf{s}', \mathbf{s}'') \times$ $\times\, d\mathbf{r}' d\mathbf{s}'' \mu' d\mathbf{s}'$ based on Eq. (15), one obtains

$$2\int_0^1 \rho^0(\mathbf{r}, \mu, \mu')\, D^0(h, \mu', \zeta)\, \mu' d\mu' + \zeta\rho(\mathbf{r}, \mathbf{s}, \mathbf{s}')\, S_\lambda\, e^{-\tau_0/\zeta} =$$

$$= -\frac{1}{\pi}\int\limits_{\Omega_+} \rho(\mathbf{r}, \mathbf{s}, \mathbf{s}')\, J(h, \mathbf{r}, \mathbf{s}', \mathbf{s}_0)\, \mu' d\mathbf{s}' + Z(\mathbf{r}, \mathbf{s}, \mathbf{s}_0), \qquad (28)$$

where

$$J(h, \mathbf{r}, \mathbf{s}, \mathbf{s}_0) = \int\limits_{\Omega_-}\int\limits_{-\infty}^{\infty} O_\delta(h, \mathbf{r}-\mathbf{r}', \mathbf{s}, \mathbf{s}')\, Z(\mathbf{r}', \mathbf{s}', \mathbf{s}_0)\, d\mathbf{r}' d\mathbf{s}' .$$

Let us rewrite equality (12) as the equation with respect to the function $Z(\mathbf{r}, \mathbf{s}, \mathbf{s}_0)$

$$Z(\mathbf{r} - \tilde{\mathbf{r}}, \mathbf{s}, \mathbf{s}_0)\, T(\mu) +$$

$$+ \int\limits_{\Omega_-}\int\limits_{-\infty}^{\infty} \tilde{O}_\delta(z, \mathbf{r} - \tilde{\mathbf{r}} - \mathbf{r}', \mathbf{s}, \mathbf{s}')\, Z(\mathbf{r}', \mathbf{s}', \mathbf{s}_0)\, d\mathbf{r}' d\mathbf{s}' =$$

$$= I - D, \; \mathbf{s} \in \Omega_- . \qquad (29)$$

The function $\rho(\mathbf{r}, \mathbf{s}, \mathbf{s}_0)$ is found by solving successively Eqs. (29) and (28).

In the case of homogeneous reflection $\rho(\mathbf{r}, \mathbf{s}, \mathbf{s}_0) \equiv q(\mathbf{r})$ from formulas (28) and (29) it follows[10]

$$q(\mathbf{r}) = \frac{U(\mathbf{r})}{E_0 + \int\limits_{-\infty}^{\infty} \overline{O}_h(\mathbf{r} - \mathbf{r}')\, U(\mathbf{r}')\, d\mathbf{r}'}, \qquad (30)$$

where

$$U(r) = Z(\mathbf{r}, \mathbf{s}, \mathbf{s}_0)\big|_{\rho \equiv q(r)},$$

$$E_0 = 2\int_0^1 D^0(h, \mu, \zeta)\, \mu d\mu + \zeta S_\lambda\, e^{-\tau_0/\zeta},$$

$$\overline{O}_h(\mathbf{r}) = \frac{1}{\pi} \int_{\Omega_+} \overline{O}(h, \mathbf{r}, \mathbf{s})\, \mu \mathrm{d}\mathbf{s} \ ,$$

$$\overline{O}(z, \mathbf{r}, \mathbf{s}) = \int_{\Omega_-} O_\delta(z, \mathbf{r}, \mathbf{s}, \mathbf{s}')\, \mathrm{d}\mathbf{s}' \ .$$

Equations (28) and (29) are already new mathematical objects. Numerical procedures for their solving have not yet been developed. The following conclusion can obviously be drawn from Eqs. (28) and (29): in order to reconstruct the function $\rho(\mathbf{r}, \mathbf{s}, \mathbf{s}_0)$ the brightness measurements have to be available for all $s \in \Omega_-$. The present instrumentation provides the measurements to be done only for a number of discrete directions $\{s_i\}$, where $s_i \in \Omega_-$. This must be taken into account when developing the algorithm for calculating the functions $Z(\mathbf{r}, \mathbf{s}, \mathbf{s}_0)$ and $\rho(\mathbf{r}, \mathbf{s}, \mathbf{s}_0)$. For $\rho(\mathbf{r}, \mathbf{s}, \mathbf{s}_0) \equiv \overline{\rho}(\mathbf{s}, \mathbf{s}_0)$ the equations for $\overline{Z}(\mathbf{s}, \mathbf{s}_0)$ and $\overline{\rho}(\mathbf{s}, \mathbf{s}_0)$ can be directly obtained from Eqs. (18) and (19) or from Eqs. (16)–(18). These equations have the form

$$2\int_0^1 \overline{\rho}^{\,0}(\mu, \mu')\, D^0(h, \mu', \zeta)\, \mu'\mathrm{d}\mu' + \zeta S_\lambda \overline{\rho}(\mathbf{s}, \mathbf{s}_0)\, \mathrm{e}^{-\tau_0/\zeta} =$$

$$= -\frac{1}{\pi}\int_{\Omega_+} \overline{\rho}(\mathbf{s}, \mathbf{s}')\, \overline{J}(h, \mathbf{s}', \mathbf{s}_0)\, \mu'\mathrm{d}\mathbf{s}' + \overline{Z}(\mathbf{s}, \mathbf{s}_0) \ , \qquad (31)$$

where

$$\overline{J}(h, \mathbf{s}, \mathbf{s}_0) = \int_{\Omega_-} \Psi_{\delta,0}(h, \mathbf{s}', \mathbf{s}_0)\, \overline{Z}(\mathbf{s}', \mathbf{s}_0)\, \mathrm{d}\mathbf{s}'$$

and

$$\overline{Z}(\mathbf{s}, \mathbf{s}_0)\, T(\mu) + \int_{\Omega_-} A_{\delta,0}(z, \mathbf{s}, \mathbf{s}')\, \overline{Z}(\mathbf{s}', \mathbf{s}_0)\, \mathrm{d}\mathbf{s}' = \overline{I} - D \ , \ \mathbf{s} \in \Omega_- . \qquad (32)$$

From the point of view of calculations a more simple approach to solving the inverse problem is based on models (18) and (19) constructed using the method of space–frequency characteristics. Let us demonstrate the abilities of the method of reconstructing the reflection coefficient using an example of a homogeneous surface. For this consider Eq. (20). In order to determine N unknown values $\overline{q}_n$ it is necessary to have N independent angular measurements $\overline{I}_i$. All the values obtained for $s = s_i$ are denoted below by the subscript i. Based on Eq. (20) we have the system of equations

$$\sum_{n=1}^{N} \overline{q}_n \left[\overline{\Psi}_{\delta,0,\,n}(z, \mathbf{s}_i, \mathbf{s}_0) + \frac{\overline{q}\Psi_0(z, \mu_i)}{1 - qc_0}\, \overline{C}_{\delta,0,\,n}(\zeta) \right] = \overline{I}_i - D_i . \qquad (33)$$

For realistic values of τ_0 and $\overline{q}$ the inequalities $\overline{q}\overline{C}_{\delta,0,\,n}(\zeta) \ll 1$ and $\overline{q}c_0 \ll 1$, whence

$$\frac{\overline{q}\Psi_0(z, \mu_i)}{1 - \overline{q}c_0}\, \overline{C}_{\delta,0,\,n}(\zeta)) \ll \overline{\Psi}_{\delta,0,\,n}(z, \mathbf{s}_i, \mathbf{s}_0)$$

are valid.

Taking into account the latter inequality we solve system (33) by the iteration method, and neglecting at the first iteration the term $\dfrac{\overline{q}\Psi_0(z, \mu_i)}{1 - \overline{q}c_0}\overline{C}_{\delta,0,\,n}(\zeta)$ compared to $\overline{\Psi}_{\delta,0,\,n}(z, \mathbf{s}_i, \mathbf{s}_0)$. Let us replace system (33) by the following:

$$A^{(j)}\, \overline{\mathbf{q}}^{(j)} = \mathbf{b} \ , \qquad (34)$$

where $\overline{q}^{\,(j)} = \{\overline{q}_1, \ldots, \overline{q}_n\}$ is the sought vector; $A^{(j)} = \{a^{(j)}_{i,\,n}\}$ is the N by N matrix with elements

$$a^{(j)}_{i,\,n} = \overline{\Psi}_{\delta,0,\,n}(z, \mathbf{s}_i, \mathbf{s}_0) + (1 - \delta_{j-1,0}) \frac{\overline{q}_n^{(j-1)}\Psi_0(z, \mu_i)}{1 - \overline{q}_n^{(j-1)}\, c_0}\overline{C}_{\delta,0,\,n}(\zeta)) \ ,$$

$\delta_{l,\,0} = \begin{cases} 1, & l = 0 \\ 0, & l \ge 1 \end{cases}$ is the Kronecker symbol; $\mathbf{b} = \{b_i\}$ is the vector with the components $b_i = \overline{I}_i - D_i$, $1 \le i \le N$, $j (j \ge 1)$ is the iteration number, $\overline{q}_n^{\,(0)}$ are arbitrary limited numbers, for example, $\overline{\mathbf{q}}_n^{\,(0)} = 1$ for all $n \ge 1$.

Solution of system (34) has the form

$$\overline{q}_n^{\,(j)} = \sum_{i=1}^{N} b_i A^{(j)}_{i,\,n} / \det A^{(j)} \ , \ 1 \le n \le N \ , \qquad (35)$$

where $A^{(j)}_{i,\,n}$ is the algebraic cofactor of the element $a^{(j)}_{i,\,n}$. By passing to a limit we obtain the solution of system (33): $\overline{q}_n = \lim_{j\to\infty} \overline{q}_n^{\,(j)}$, where $1 \le n \le N$. Because of a weak nonlinearity of system (33) it is quite sufficient to make the second iteration, i.e., $\overline{q}_n \approx \overline{q}_n^{\,(2)}$, where $1 \le n \le N$. After determining all the values the reflection coefficient is calculated using formula (4). Radiative characteristics $\overline{\Psi}_{\delta,0,\,n}(z, s_i, \mathbf{s}_0)$, $\Psi_0(z, \mu)$, c_0, $C_{\delta,0,\,n}(\zeta)$ and D_i entering into the solution are computed by accessible means of the numerical analysis.[7,9,11,12,14]

Similar approach is used for solving an inverse problem in the general case of an inhomogeneous surface based on model (19).

CONCLUSION

The mathematical model of radiative transfer in the plane–parallel atmosphere above the surface with inhomogeneous non–Lambertian reflection is presented. The model is used for statement and solution of the inverse problem on reconstructing the reflection coefficient of the underlying surface from photometric measurements. For reconstructing the angular structure of the reflection coefficient one should use a set of angular measurements of the brightness of upward radiation or the brightness coefficient of the atmosphere–underlying surface system. The reconstruction algorithms are realized by means of numerical analysis of the boundary–value problems of the radiative transfer theory. These algorithms can be used for atmospheric correction of the ground–based and spaceborne measurements.

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