QUASI–PERIODIC FREQUENCY VARIATIONS IN REMOTE SENSING OF THE TURBULENCE

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This paper considers two mechanisms of quasi–periodicity in remote sensing of the turbulence: modulation of radiation by inhomogeneity waves in a medium and interference of elementary waves in the process of the radiation phase front splitting by large–scale inhomogeneities. The dominating role of the second mechanism is illustrated by the example of sensing of the solar atmosphere.

INTRODUCTION

One effective technique for remote sensing of turbulent media is that by which coherent optical or microwave radiation has passed once through the examined inhomogeneities. In case of well–developed turbulence, the energy parameters of transmitted radiation (correlation and spectrum) vary monotonically, on the average, in so far as the spectrum of the well–developed turbulence is nearly Kolmogorov one.[1–3] However, a detailed analysis of individual realization, e.g., of phase and frequency records, reveals the components that deviate from this monotonic dependence. For example, quasi–periodic components are found that are most clearly manifested in the turbulent media with wide spectra of inhomogeneities, such as the Earth's atmosphere[4] or interplanetary or solar plasma.[5]

Experimental data indicate that quasi–periodicity in sensing of the turbulent media is quite regular rather than stationary in character. Amplitudes of the components found, e.g., in frequency records, may be rather high, thus affecting the accuracy of optical and radio navigation systems. To explain this phenomenon by the existence of some periodic waves that modulate the phase fronts in the medium is not the only alternative available. Such a manifestation may also accompany the interference of the partial waves generated in the process of the initial phase front splitting by large inhomogeneities. The present paper compares these two approaches explaining quasi–periodicity of wave frequency and phase in sensing of the turbulence of the solar atmosphere.

1. INHOMOGENEITY WAVES IN A TURBULENT MEDIUM

In order to account for the existence of inhomogeneity waves propagating through turbulent media, we may modify the spatial spectrum of the developed turbulence $\Phi(\boldsymbol{\kappa})$, introducing its spatiotemporal spectrum[6]

$$F(\boldsymbol{\kappa};\,\omega) = \Phi(\kappa)\,\frac{\delta(\omega - \mathbf{k}\mathbf{v} - \omega(\mathrm{k})) + \delta(\omega - \mathbf{k}\mathbf{v} + \omega(\mathbf{k}))}{2}\,,$$

where $\mathbf{v}$ is the drift velocity of frozen inhomogeneities, and the function $\omega = \omega(\boldsymbol{\kappa})$ describes the frequency dispersion of waves in the medium. For linear dispersion $\omega(\boldsymbol{\kappa}) = |\kappa_z| v_s$ typical of magnetosonic waves propagating with the velocity v_s, the energy spectrum of temporal phase fluctuations of a sensing wave, calculated by the well–known technique[1–3] in the geometric optics approximation, has the form

$$W_\varphi(\omega) = 2\pi L\kappa^2 \int\!\!\int_{-\infty}^{+\infty} F(\kappa_x = 0,\,\kappa_y,\,\kappa_z;\,\omega)\,d\kappa_y\,d\kappa_z = \frac{|\gamma_+|\;W^{(0)}_\varphi(\gamma_+\,\omega) + |\gamma_-|\;W^{(0)}_\varphi(\gamma_-\,\omega)}{2}\,.$$

Here $W^{(0)}_\varphi(\omega)$ is the spectrum in the absence of inhomogeneity waves, L is the depth of the turbulent layer, and $\kappa = 2\pi/\lambda$ is the wave number. The spectrum of frequency fluctuations $\Omega = d\varphi/dt$ is obtained by simple multiplication

$$W_\Omega(\omega) = W_\varphi(\omega)\,\omega^2\,.$$

It is seen that neither $W_\varphi(\omega)$ nor $W_\Omega(\omega)$ exhibits singularities at any frequency. In other words, there is no quasi–periodicity in the realization. Such a conclusion follows from the fact that the corresponding correlation functions have the form

$$B_\varphi(\tau) = \{\mathrm{sgn}\,(\gamma_+) B^{(0)}_\varphi(\tau/\gamma_+) + \mathrm{sgn}\,(\gamma_-) B^{(0)}_\varphi\,(\tau/\gamma_-)\}\;/\;2\,,$$

$$B_\Omega(\tau) = \{\gamma_+\,|\gamma_+|\;B^{(0)}_\Omega(\tau/\gamma_+) + \gamma_-\,|\gamma_-|\,B^{(0)}_\Omega(\tau/\gamma_-)\}\;/\;2\,,$$

where $\gamma_\pm = v/(v \pm v_s)$, while $B^{(0)}_\varphi(\tau)$ and $B^{(0)}_\Omega(\tau)$ are the corresponding correlation functions in the absence of waves. Neither the spectrum nor the correlation points to quasi–periodicity.

We have $\omega(\kappa) = \Omega_0 = \mathrm{const}$ for the dispersion in the absence of waves, and

$$W_\varphi(\omega) = \{W^{(0)}_\varphi\,(\omega - \Omega_0) + W^{(0)}_\varphi\,(\omega + \Omega_0)\}\;/\;2$$

for the spectral power density of the phase fluctuations provided the turbulence occurs at eigenfrequencies.

For example, the phase correlation acquires the form

$$B_\varphi(\tau) = B^{(0)}_\varphi(\tau)\,\cos\,\Omega_0\,\tau\,.$$

Oscillations should then be found at the frequency Ω_0 in the realization. We now proceed to a verification of this conclusion from the available observational data.[5]

Quasi–periodic components were identified for microwave sensing of the turbulence of the solar atmosphere, with their relative contribution reaching 20–60% and their characteristic period increasing from 12 to 250 s away from the Sun and following nearly–power law

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$$T = 2\pi/\Omega_0 = a_T(\rho/R_0)^{\alpha_T} , \qquad (1)$$

where $a_T = 0.15 \pm 0.05$ s, $\alpha_T = 2.1 \pm 0.2$, ρ is the impact parameter, and R_0 is the radius of the solar photosphere.[5] Figure 1 shows the measured values that yielded the above dependence (1). Measurements were conducted at $\lambda = 32$ cm from aboard "Venera–15" and "Venera–16" spacecrafts. The characteristic period was retrieved from the records of frequency fluctuations using the correlation (empty circles) and spectral (filled circles) techniques.

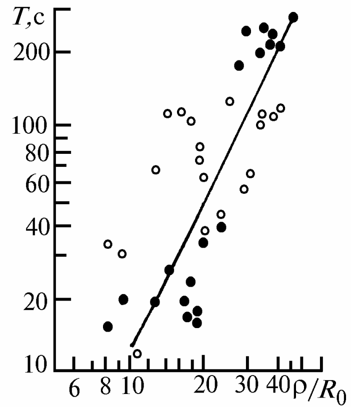

FIG. 1. Period of microwave frequency fluctuations vs. the impact parameter.

Assuming the frequency Ω_0 to be equal to that of the ionic cyclotron oscillations

$$\Omega_0 = \bar{B}e/(mc) ,$$

we see that in addition to the electronic charge e and the proton mass m, it depends on the magnetic field strength $\bar{B}$. It is usually assumed[7] that the magnetic field strength decreases with distance from the Sun, following a quadratic law $\bar{B} \sim \rho^{-2}$. This entails a quadratic increase of the characteristic period of frequency variations, $T = 2\pi/\Omega_0 \sim \rho^2$, which agrees qualitatively with dependence (1). To check the quantitative agreement, we take the magnetic field strength at the distance $\rho = 10R_0$ from the Sun. According to Ref. 7, we have $\bar{B} = 10^{-2}$ G. A close value follows from Ref. 8. Assuming plasma ions to be hydrogen nuclei, we find $F_0 = \Omega_0/2\pi = 15$ Hz, so that $T = 0.066$ s. This is two orders of magnitude less than the periods shown in Fig. 1. Thus, the observed quasi–periodicity cannot be attributed to the above–considered turbulence; its nature and origin should be sought in larger–scale perturbations of the solar atmosphere. Their spectrum is rather wide.

2. INTERFERENCE

It is known from the theory of wave propagation that quasi–periodic nonstationary perturbations often result from wave interference.[9] In case of multibeam field, the total radiation phase φ is represented as

$$\varphi = \omega_0 t - \kappa L - \delta\varphi + \psi ,$$

where ω_0 is the carrier frequency, kL is the spatial phase run–on, and $\delta\varphi$ are phase variations due to the small–scale turbulence. The component $\psi = \arg U$ is related to interference of the partial waves with amplitudes A_j and frequency shifts Δf_j, so that

$$U = \sum_j A_j \exp(i2\pi\, \Delta f_j\, t) .$$

The temporal correlation function for this model is

$$B(\tau, t) = B_\varphi(\tau) + B_\psi(\tau, t) ,$$

where $B_\varphi(\tau)$ is the autocorrelation of the turbulent phase fluctuations and

$$B_\psi(\tau, t) = \frac{1}{T}\int_t^{t+T} \delta\psi(t' + \tau)\, \delta\psi(t')\, dt'$$

describes the phase variations caused by interference. Analogous result is obtained for the frequency $f = d\varphi/d(2\pi t)$. When two waves having different amplitudes A_1 and A_2 and frequency shifts Δf_1 and Δf_2 interfere, for the temporal frequency autocorrelation we have

$$B_f(\tau, t = 0) = \sigma^2(1 - \beta) / (1 + 4\sigma^2\beta) ,$$

where

$$\sigma^2 = \Delta f^2 \nu^2 / 2(1 - \nu^2), \quad \beta = 2\sin^2(2\pi\, \Delta f\, \tau) / (1 - \nu^2) ,$$

and the values $\nu = A_1/A_2$ and $\Delta f = \Delta f_1 - \Delta f_2$ depend on the ratio of the amplitudes and the difference between the frequencies of the two partial waves. There appears quasi–periodicity, and its characteristic period $T = 1/\Delta f$ is determined by the difference between the two frequency shifts. Provided the initial amplitudes are close to each other ($\nu \sim 1$), the frequency dispersion of quasi–periodicity sharply increases.

Curves *2* and *3* in Fig. 2 show the calculated frequency variations for two and three interfering waves with $\Delta f_1 = -0.0033$ Hz, $\Delta f_2 = 0.0033$ Hz, and $A_2/A_1 = 0.7$; $\Delta f_1 = -0.0033$ Hz, $\Delta f_2 = 0.0033$ Hz, $\Delta f_3 = 0.030$ Hz, $A_2/A_1 = 0.5$, and $A_3/A_2 = 0.15$. For comparison, curve *1* shows the frequency recorded in sensing of the solar atmosphere with the impact parameter $\rho = 17.4\, R_0$ and subsequently averaged over a 13s period. A noticeable similarity is found between the two records, and the possibility becomes apparent of several periods simultaneously. A single period is only observed in the case of two waves.

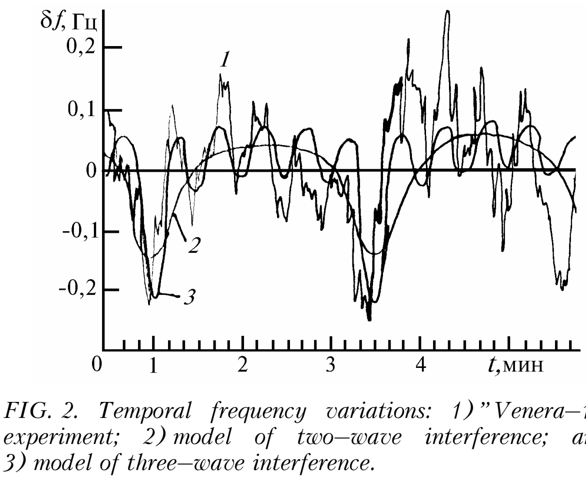

FIG. 2. Temporal frequency variations: 1) "Venera–15" experiment; 2) model of two–wave interference; and, 3) model of three–wave interference.

We now proceed to the estimate of the period of oscillations using the model of random refraction by large–scale inhomogeneities with a lens–like effect. The existence of such a mechanism was noted in Ref. 10, where it was used to explain the saturated fluctuations of the wave intensity in microwave sensing of the solar atmosphere. In Ref. 11 it was demonstrated that the following approximation may be used to estimate small angles of refraction:

$$\xi = 1.4 \cdot 10^{-15} \lambda^2 N_e ,$$

where $N_e = N_0(R_0/\rho)^2$ is the radial dependence of the electronic plasma number density in the solar atmosphere. Here ξ is measured in radians, λ – in meters, and N_e – in m^{-3}. The angle of beam deviation ξ due to each individual inhomogeneity is of random sign. The relative deviation of the two adjacent beams may be estimated as 2ξ. When such inhomogeneities move across the sensing beam at the velocity v_L, the difference between the frequency shifts is estimated as[3]

$$\Delta f = 2 \xi v_L / \lambda .$$

Since $\Delta f = 1/T$, we have an estimate for T

$$T = 3.6 \cdot 10^{14} (\rho / R_0)^2 / (\lambda v_L N_0) .$$

Accounting for the normalization condition[12] $N_e(\rho = 10R_0) = 1.2 \cdot 10^{16}$ m^{-3}, for $v_L = 7.8$ km/s and $\lambda = 0.32$ m we have

$$T = 0.12 (\rho / R_0)^2 .$$

The corresponding dependence is shown in Fig. 1 by the slant straight line.

Good qualitative and quantitative agreement between the experimental (1) and theoretical (2) dependences counts in favor of quasi–periodicity of interference origin in sensing of the turbulence of the solar atmosphere.

CONCLUSION

The study in the example of the solar atmosphere demonstrates that in addition to the wave processes in the medium itself, quasi–periodic frequency variations in sensing of turbulent media may result from interference of waves generated in the process of the initial phase front splitting by large–scale inhomogeneities. The appearance of such quasi–periodicity is indicative of the phase splitting and may be used to study large–scale portion of the spectrum of turbulence from frequency and phase data.

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