EFFECT OF LOSS DISPERSION ON A PULSE SIGNAL SHAPE INSIDE AN UNDERWATER ACOUSTIC CHANNEL

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We consider the shape distortions of narrow–band pulse signals during their propagation in an underwater acoustic channel. It is shown that this distortion is mainly determined by appearance of additional contribution to quadrature components. The contribution amplitude is connected with both a shape of emitted signal and channel parameters, namely, frequency dependence of attenuation. Additional contribution appearing in an underwater acoustic channel due to dispersion is detectable practically for all frequency ranges. The estimations of distortion value are presented for sea water as a function of distance and frequency.

In order to obtain the most complete information about channel through which a signal propagates, one should provide the recording of the signal shape, since this shape carries information about distortions which the signal obtains in acoustic channel. Such a change in a signal shape is usually called dispersive distortions. Inside an underwater acoustic channel, the phase dispersion (frequency dependence of the sonic speed) is practically lacking, whereas the dispersion of attenuation is clearly pronounced.

On examination of narrow–band signals, so–called "quasimonochromatic approximation" is usually used. Such an approximation reduces to the ignoring the dispersion. This approximation is based on the assumption that a signal keeps its shape if channel transfer function slightly depends on frequency within the signal band. In this paper we consider the influence of frequency dependence of transfer function modulus within signal band on its shape.

Before proceeding to the shape variation, we first discuss the concept of signal shape. Signals used in underwater acoustics, waveguide engineering, and so on and considered as a time functions are, as a rule, of special type. Often, such signals are high–frequency oscillations with slowly changing characteristics, such as amplitude and phase (or frequency). These are precisely the slowly changing parameters, which carry information, i. e., can be used for information transfer.

If spectral band of a signal is small in comparison with the carrier frequency, then the rate of variation of information parameters is small in comparison with the rate of variation of a signal as a time function (with carrier frequency). It is common practice to call these signals narrow–band ones.

It is known that to describe such signals of general type, suffice is to specify two "slowly changing" real–valued functions,[1] for example, amplitude and phase. In this paper we will systematically follow the description of signals with using the quadrature components. In the case of narrow–band signals, more specifically, signals with finite spectrum, this description is equivalent to analytical signal,[3] however, the real quadrature components are used here as a starting concepts, instead of envelope and phase.

Thus, in our description a signal is characterized by two "slow" quadrature components modulating the carrier frequency cosine and sine:

$$f(t) = a(t) \cos \omega_0 t + b(t) \sin \omega_0 t. \quad (1)$$

Any linear stationary wave channel can be generally described with its pulse response, that is, response to the δ–pulse. In so doing, the propagation of a signal through such a channel is described with the Duhamel integral

$$u_{\text{p}}(t) = \int_{-\infty}^{t} h(t - \tau)\, u(\tau)\, \mathrm{d}\tau . \quad (2)$$

Here $h(t)$ is the pulse response, $u(t)$ is the input signal, while $u_p(t)$ is the output one. The pulse responce is the Green function of the system, and its Fourier transform (as a response to a monochromatic signal) is the system transfer function (TF).

In the case of stationary channel, the spectrum of echo signal is connected with the spectrum of sounding signal in the following way:

$$U_{\text{p}}(\omega) = H(\omega)\, U(\omega) . \quad (3)$$

Let us represent both sounding and echo signals as a sum of quadrature components. As a sounding signal, we will consider narrow–band amplitude–modulated (AM) pulse (the general case is the sum of two such signals with relative phase shift of $\pi/2$). In the case of amplitude–modulated signal, the amplitudes $a(t)$ and $b(t)$ in Eq. (1) is proportional to each other (it is the property typical of amplitude–modulated signals), and one of the quadratures may be vanished via appropriate choosing the phase. Hense, the sounding pulse we choose in the following form:

$$U(t) = a(t) \cos (\omega_0 t) , \quad (4)$$

omitting the initial phase. Here ω_0 is the carrier frequency, $a(t)$ is the amplitude being the slowly varying time function, whose spectrum $A(\omega)$ is concentrated near zero. In such a case, the spectrum of sounding signal has the following form:

$$U(\omega) = 1/2 \, [\, A(\omega - \omega_0) + A(\omega + \omega_0)] \quad (5)$$

and is concentrated near the frequencies $\pm\, \omega_0$.

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Following the inverse Fourier transformation for echo signal as a time function in Eq. (2) and using Eq. (5), we derive

$$u_p(t) = 1/2 \int_{-\infty}^{\infty} \exp(-i\,\omega\, t)\, H(\omega)[A(\omega-\omega_0) + A(\omega+\omega_0)]\, d\omega =$$

$$= 1/2 \int_{-\infty}^{\infty} \exp(-i\,\omega\, t)\, A(\omega)\,[\exp(-i\,\omega_0\, t)\, H(\omega+\omega_0) +$$

$$+ \exp(i\,\omega_0\, t)\, H(\omega-\omega_0)]\, d\omega\,. \qquad (6)$$

Let us represent TF in exponential form: $H(\omega) = \exp(-\mu(\omega) + i\varphi(\omega))$. Taking into account the conjugation conditions $H(-\omega) = H^*(\omega)$, which follow from real–valued nature of pulse response and properties of the Fourier transformation, we may bring Eq. (6) to the form:

$$u_p(t) = 1/2 \int_{-\infty}^{\infty} \exp(-i\,\chi\, t)\, A(\chi)\, \{\exp[(-i\,\omega_0 t) - i\,\varphi\,(\omega_0+\chi) -$$

$$- \mu(\omega_0+\chi)] + \exp[\,i\,\omega_0\, t + i\,\varphi\,(\omega_0-\chi) - \mu\,(\omega_0-\chi)]\}\, d\chi\,. \quad (7)$$

In this form, integrating over frequency is performed within spectral band of the sounding pulse amplitude and χ is, in fact, the deviation of frequency from the carrier one; in other words, $\chi < \omega_0$ for narrow–band signal. Write the echo signal as a sum of quadrature components taking into account that echo signal has a delay time. Separating out the delay time, we obtain as a result:

$$u_p(t) = a_1(t)\cos(\omega_0\, t - \varphi_0) + b_1(t)\sin(\omega_0\, t - \varphi_0)\,. \quad (8)$$

Here φ_0 is the phase of TF at a carrier frequency; from Eq. (7) we obtain exact formulas for a_1 and b_1, which express them in terms of spectrum of the sounding pulse amplitude:

$$a_1(t) = 1/2 \int \exp[-i\,\chi\,(t-\tau)]\, A(\chi)\, [\exp[-\mu\,(\omega_0+\chi) -$$

$$- i\,\varphi_+(\chi)] + \exp[-\mu\,(\omega_0-\chi) + i\,\varphi_-(\chi)]]\, d\chi\,,$$

$$b_1(t) = 1/2\, i \int \exp[-i\,\chi\,(t-\tau)]\, A(\chi)\, [\exp[-\mu\,(\omega_0+\chi) -$$

$$- i\,\varphi_+(\chi)] - \exp[-\mu\,(\omega_0-\chi) + i\,\varphi_-(\chi)]]\, d\chi\,.$$

Here $\tau = d\varphi(\omega_0)/d\omega_0$ is the group delay and $\varphi_\pm(\chi) = \Phi(\omega_0 \pm \chi) - \Phi(\omega_0) \pm \chi\tau$ is the remainder of TF phase after subtracting the constant and linear over χ terms. Denote the expressions in square brackets by Z and V and rewrite previous expression in more compact form:

$$a_1(t) = 1/2 \int \exp[-i\,\chi\,(t-\tau)]\, A(\chi)\, Z(\chi)\, d\chi\,, \qquad (9)$$

$$b_1(t) = 1/2 \int \exp[-i\,\chi\,(t-\tau)]\, A(\chi)\, V(\chi)\, d\chi\,. \qquad (9a)$$

In the dispersionless case, when there are no distortions, but only time and phase shift, $V = 0$ and $Z = 1$. The distortions appear at $Z \neq \text{const}$ and $V \neq 0$. Let us analyze explicit expressions for Z and V and reveal the essential factors and their behavior depending on the signal bandwidth.

We can draw on the smallness of distortions of a narrow–band signal and write the expansion of Z and V in terms of χ discarding the high–order powers of χ. Derived expressions have the form:

$$Z(\chi) = \exp(-\mu_0)\,(1 - i\gamma\,\varphi''\,\chi^3/2^2)\,, \qquad (10)$$

$$V(\chi) = \exp(-\mu_0)\,(i\gamma\,\chi - \varphi''\,\chi^2/2^2)\,, \qquad (10a)$$

There

$$\gamma = \left.\frac{d\mu(\omega)}{d\omega}\right|_{\omega=\omega_0}, \quad \varphi'' = \left.\frac{d\tau(\omega)}{d\omega}\right|_{\omega=\omega_0}, \quad \mu_0 = \mu(\omega_0).$$

The quantity γ has a dimension of time. Being the derivative of imaginary part of TF phase, this quantity is thereby the imaginary part of the group delay and determines the tilt of amplitude–frequency characteristic of channel (AFCh). The quantity φ'' is the first term in $\varphi(\chi)$ expansion and, in fact, the second derivative of TF phase at the point ω_0. If γ and φ'' are equal to zero, then $b_1 = 0$ and $a_1 = e^{-\mu_0}a(t-\tau)$, that is, output signal is the exact copy of emitted one, delayed for the time τ and attenuated by factor of $e^{-\mu_0}$.

If these quantities are not equal to zero, but small in value, then inphase component in Eq. (9) gains addition proportional to the cube of bandwidth. Since the integration interval in Eqs. (9) and (9*a*) is determined by $A(\omega)$, that is, the bandwidth of sounding signal, and the additional term in Z follows the χ^3 dependence, we can assume that this addition decreases as cubed bandwidth with decreasing bandwidth.

Ignoring the second and higher powers of χ, we obtain explicit expressions for quadrature components of a signal passed through the channel

$$a_1(t) = e^{-\mu_0}\, a(t-\tau)\,, \qquad (11)$$

$$b_1(t) = e^{-\mu_0}\,\gamma\,\frac{da(t-\tau)}{dt}\,. \qquad (11a)$$

Thus, in the case of small distortions, the principal distortion of a narrow–band signal shape is connected with appearance of additional quadrature component. The appearance of the second quadrature component results in appearance of additional phase modulation in a signal. To make visible the phase modulation, we may use the phase diagram.

Let us define the phase diagram as follows. The value $a_1(t)$ may be plotted on one of the axes of a rectangular coordinate system, and the value $b_1(t)$ may be plottes on another one. As it takes place, time plays the role of parameter corresponding to parametric representation of obtained curve. Amplitude–modulated pulse has a form of portion of a curve passing through the origin of coordinates in phase diagram. Occurrence of additional quadrature component, which is not proportional to the first one, results in forming the signal phase diagram as a closed loop. The loop shape depends on predominance of addends in Eq. (10a) and shape of emitted signal. So, if the signal distortion is connected with the variability of absorption coefficient, then the additional quadrature component has a form of time derivative of inphase component, and phase diagram is shaped like ellipse for sounding pulse close to rectangular. As it takes place, the ratio of lobe width to lobe length is proportional to $2\gamma\Delta\omega$. In the case of predominance of the second addend in Eq. (10a), the additional quadrature component is proportional to the second derivative of initial pulse amplitude and the phase diagram has quite a different shape, it allows the visual distinction of such cases.

The phase diagram is especially convenient, because it allows a pictorial representation of phase structure of a signal. In its turn, it enables us to record such small variations, which are not seen in the envelope. So, if in the phase diagram the loop width comprises 10% of loop length, then the second quadrature component totals, in respect of amplitude, about 5% of the first one, whereas the variation in the envelope comprises less than 1% in such a case.

Inside an underwater acoustic channel, the absorption coefficient has a clearly pronounced frequency dependence: the coefficient grows as a squared frequency within wide frequency range. Here, we take into account only the attenuation caused by the absorption, while the scattering and so–called waveguide dispersion can lead to extra effects. In approximation under consideration, the additions to the attenuation factor, which are determined by these effects, are additive. Consider the values of distortions caused by absorption. To estimate these values, we will use the dependence presented in Ref. 3. Figure 1 shows the threshold distance, where the contribution due to absorption dispersion to the quadrature components is equal to 2 (curve *1*) and 50% (*2*), as a function of frequency, for the ratio of signal bandwidth to carrier frequency $df/f = 0.1$. For comparison, curve *3* determines the distance, where attenuation due to absorption comprises 80 dB at given frequency. It seen from the figure that distance, where the signal distortions are measurable, occurs for all frequencies practically. We choose, by convention, the threshold of 2% as a threshold of observable distortions (starting from this value, the phase diagram is visually different from a portion of a straight line), and the threshold of 50% as a threshold of strong distortions, when used approximation becomes unsuitable.

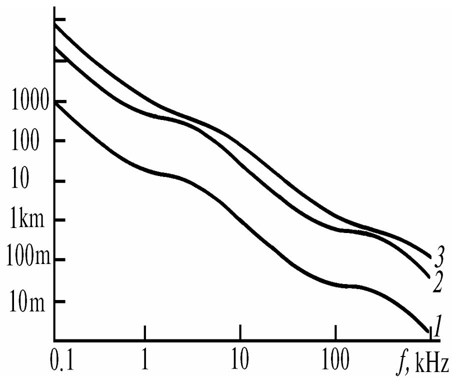

FIG. 1.

We can summarize the above results as follows: AM pulse distortions during its propagation through linear channel mainly reduce to appearance of additional quadrature component reproducing the derivative of inphase component of initial pulse, whose amplitude is proportional to the tilt of channel amplitude–frequency characteristic. The phase diagram corresponding to such a case is shaped as a lobe, whose width is proportional to the steepness of pulse front and the tilt of amplitude–frequency characteristic of channel. In sea water, the distortions caused by the frequency dependence of absorption are measurable practically in all frequency ranges starting with some distance.

On examination of attenuation dispersion, we use only the general properties of transfer function of any linear stationary channels, therefore, such distortions occur in all these channels: in radio engineering, radio physics, fiber optics, acoustics, coherent optics, and so on.

Importance and measurability of discussed distortions in each of these physical systems need to be analyzed separately. It is essential as well to have a feasibility of coherent operation with a signal (with quadrature components).

In the case of short–wave radio sounding of the ionosphere, the effect of attenuation dispersion was experimentally observed.[4] This effect was found to be larger than theoretical estimations and calculations, which took into account only absorption. The distortions of modulus of transmission coefficient due to dispersion were observed in radio engineering lumped circuits as well, where calculated values agreed with measured ones.

In the case of short–wave radio sounding, not the absorption, but inhomogeneous structure of reflection area contributes mainly to the frequency dependence of TF and, consequently, to the signal distortion. Occurrence of medium inhomogeneities leads to observed distortion values, which are larger than theoretically calculated distortions due to absorption.

Analogous situation can take place in an underwater acoustic channel, hence the experimental check on distortions values is necessary for acoustic channel. Such distortions must be measurable even without regard for inhomogeneities and interference effects.

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