UNIVERSAL ALGORITHM FOR CALCULATING OPTICAL CHARACTERISTICS OF TWO-LAYER SPHERICAL PARTICLES WITH HOMOGENEOUS CORE AND COVER.

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We propose here a universal algorithm indented for calculation of the optical characteristics of two-layer spherical particles with homogeneous core and cover. The algorithm allows calculating without any limitation on the radius and values of the complex refractive indices of the core and the cover, while being sufficiently accurate for simulations of the optical properties of atmospheric aerosol. The algorithm is based on the conversion of formulas for the coefficients of Mie series which allows to avoid calculations of the Riccati-Bessel functions of the complex argument, that provides its stability when computing. The asymptotic formulas have been derived for the case of big particles with strongly absorbing cover. The algorithm has been tested by comparing with independent calculations as well as using special tests for the cases of very small and very big particles.

The algorithms for calculating optical characteristics of two-layer spherical particles of the "sphere in a shell" type have a great significance for simulating optical properties of the atmospheric aerosol along with the algorithms for calculating the characteristics of homogeneous spheres based on the classical Mie theory. Such particles can appear, for example, as a result of the processes of moistening aerosol particles in humid air and, as calculations show, have optical characteristics essentially different from those of dry particles.[1] Theoretical solution of the problem on scattering of electromagnetic waves on two-layer spheres was obtained long ago (for example, see formulas in Ref. 2).

However, big difficulties arise when trying to reduce these solutions to computer programs. Below we analyze the causes of these difficulties. It finally results in the fact that all the algorithms[2–5] available have a limited applicability, one can not use them for big particles and for particles with strongly absorbing cover. Such limitations restrict our abilities to simulate optical characteristics of the atmospheric aerosol. In this paper we propose a new algorithm free of any limitations on the parameters calculated in the range of their possible values in the problems of the numerical simulation of the atmospheric aerosol optical characteristics.

The model of a two-layer particle is two homogeneous concentric spheres. The inner sphere is the core, and the external one is the cover. the particle is characterized by the following parameters: r is the outer radius (radius of the cover); g is the ratio of the inner to outer radii of the sphere ($0 < g < 1$); M_1 is the complex refractive index of the core; M_2 is the complex refractive index of the cover and M is the refractive index of the surrounding medium (let it be real). Following Ref. 2, let us set the complex refractive indices in the form of two different parameters: real and imaginary parts, and formally unite them by the plus sign: $M_1 = n_1 + i\kappa_1$, and $M_2 = n_2 + i\kappa_2$. Such a nonstandard form of the expression for the complex refractive index has some advantages for practical programming.

The difference between the cases with two-layer spheres and the homogeneous ones is only in the formulas used for calculation of the coefficients a_n and b_n of the Mie series. The structure of the scattering phase matrices, formulas for calculation of the scattering and extinction cross-sections, elements of the scattering phase matrix and the coefficients of the expansion of the scattering phase functions into the series over the Legendre polynomials are unchanged.[2,6]

Absorption and scattering of light by the sphere with cover are characterized by six dimensionless parameters:

$$y = 2\pi M\, r/\lambda \quad x = gy \quad m_1 = M_1/M \quad m_2 = M_2/M, \qquad (1)$$

where λ is the wavelength (the values x and y are real, the complex numbers are the pair of parameters).

Let us write the initial formulas presented in Ref. 2 on the page 609 for calculation of a_n and b_n in the form convenient for practical calculations, by slightly modifying the designations

$$A_n = \psi_n(m_2 x)\,\frac{mD_n(m_1 x) - D_n(m_2 x)}{mD_n(m_1 x)\,\chi_n(m_2 x) - \chi_n'(m_2 x)}\,,$$

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$$B_n = \psi_n(m_2\, x)\, \frac{mD_n(m_2\, x) - D_n(m_1\, x)}{m\, \chi_n'(m_2\, x) - D_n(m_1\, x)\, \chi_n(m_2\, x)}\,, \quad (2)$$

$$c_n = \frac{D_n(m_2\, y) - A_n\, \chi_n'(m_2\, y)/\psi_n(m_2\, y)}{1 - A_n\, \chi_n(m_2\, y)/\psi_n(m_2\, y)}\,,$$

$$d_n = \frac{D_n(m_2\, y) - B_n\, \chi_n'(m_2\, y)/\psi_n(m_2\, y)}{1 - B_n\, \chi_n(m_2\, y)/\psi_n(m_2\, y)}\,, \quad (3)$$

$$a_n = \frac{(c_n/m_2 + n/y)\, \psi_n(y) - \psi_{n-1}(y)}{(c_n/m_2 + n/y)\, \xi_n(y) - \xi_{n-1}(y)}\,,$$

$$b_n = \frac{(m_2\, d_n + n/y)\, \psi_n(y) - \psi_{n-1}(y)}{(m_2\, d_n + n/y)\, \xi_n(y) - \xi_{n-1}(y)}\,, \quad (4)$$

where $m = m_2/m_1$; $\psi_n(z)$, $\psi_n'(z)$, $\chi_n(z)$, $\chi_n'(z)$, $\xi_n(z)$ are the Riccati-Bessel functions of the complex argument and their derivatives; $D_n(z) = \psi_n'(z)/\psi_n(z)$. The below recursion formulas[2] are valid for the Riccati-Bessel functions:

$$\psi_{n+1}(z) = \frac{2n + 1}{z}\, \psi_n(z) - \psi_{n-1}(z), \;\; \psi_{-1}(z) = \cos z,$$

$$\psi_0(z) = \sin z,$$

$$\chi_{n+1}(z) = \frac{2n + 1}{z}\, \chi_n(z) - \chi_{n-1}(z), \;\; \chi_{-1}(z) = -\sin z,$$

$$\chi_0(z) = \cos z,$$

$$\xi_n(z) = \psi_n(z) - i\, \chi_n(z), \quad (5)$$

$$\psi_n'(z) = \psi_{n-1}(z) - \frac{n\psi_n(z)}{z}\,,$$

$$\chi_n'(z) = \chi_{n-1}(z) - \frac{n\chi_n(z)}{z}\,. \quad (6)$$

The main difficulty in calculations is the necessity of calculating the Riccati-Bessel function of the complex argument, because the overflow or fast accumulation of the calculational error appears at big imaginary part, z. One can overcome these problems in the case of homogeneous spheres by introducing the logarithmic derivative, i.e. the function $D_n(z)$. the following relationship can be derived for it from Eqs. (5) and (6):

$$D_n(z) = -\frac{n}{z} + \frac{1}{n/z - D_{n-1}(z)} \qquad D_0(z) = \cot z. \quad (7)$$

This relationship makes it possible to calculate $D_n(z)$ using the inverse recurrence formula[2,7]:

$$D_{n-1}(z) = \frac{n}{z} - \frac{1}{n/z + D_n(z)}\,, \quad (8)$$

that is sufficiently stable to the accumulation of the error in calculations.

By introducing the additional function $\alpha_n(z)$, according to Ref. 7

$$\alpha_n(z) = z\, \frac{\psi_{n-1}(z)}{\psi_n(z)}\,, \quad D_n(z) = \frac{\alpha_n(z) - n}{z}\,, \quad (9)$$

we obtain from Eqs. (5) and (6):

$$\alpha_n(z) = 2n + 1 - \frac{z^2}{\alpha_{n+1}(z)}\,, \quad (10)$$

from which the expansion of $\alpha_n(z)$ into the continuous fraction[7] follows

$$\alpha_n(z) = $$
$$= 2n+1 - \frac{z^2}{2n+3-z^2/\{2n+5-[z^2/(2n+7-\ldots)]\}}\,. \quad (11)$$

In order to practically calculate the continuous fraction (11), the authors of Ref. 7 suggested an alternative form for it, which leads to the following scheme taking into account the known recursion algorithm for calculation of the continuous fractions:

$$\alpha_n(z) = \lim_{k\to\infty} (P_k/Q_k);$$

$$P_k = t_k\, P_{k-1} + P_{k-2}\,, \;\; Q_k = t_k\, Q_{k-1} + Q_{k-2}\,,$$

$$P_{-1} = 1, \;\; Q_{-1} = 0, \;\; P_0 = 2n + 1, \;\; Q_0 = 1,$$

$$t_k = 2n + 2k + 1 \text{ for even } k,$$

$$t_k = -(2n + 2k + 1)/z^2 \quad \text{for odd } k. \quad (12)$$

Iterations are being done, when calculating by Eq. (12), until the difference between the ratio of the absolute values $\alpha_n(z)$ obtained for the values $k - 1$ and k and is less then a preset small value. We took this value 10^{-9}, that provided practical coincidence of the results of calculation of $D_n(z)$ by the formula of inverse recursion (8) and by the continuous fraction (12).

Using the functions $D_n(z)$ and $\alpha_n(z)$ makes it possible to calculate for homogeneous spheres at any possible values of radius and complex refractive index. Our experience of such calculations shows that it is optimal to use the combination of the inverse recursion (8) and the direct calculation of $D_n(z)$ by Eqs. (9) and (12) both for the initial start of the inverse recursion and for repeated (reference) starts after some thousand iterations of the inverse recursion.

Let us apply similar approach to the case with two-layer spheres. Let us introduce the function $C_n(z) = \chi_n'(z)/\chi_n(z)$ in addition to $D_n(z)$. The recursion formulas analogous to Eqs. (7) and (8) follows for the function $C_n(z)$ from Eqs. (5) and (6). But there are additional calculation difficulties here, because one can not start the inverse recursion for $C_n(z)$ from zero, otherwise we obtain $C_n(z) = D_n(z)$ for any n, what is not valid.

Let us note that the formula of the inverse recursion (8) makes it possible to formally consider both positive and negative values of the index n. Then from the relation between the initial values

$$C_0(z) = -\tan z = -1/\cot z = -1/D_0(z) = D_{-1}(z)$$

for formula (8) and the recursion formula analogous to Eq. (7) it follows, for the function $C_n(z)$, that

$$C_n(z) = D_{-n-1}(z). \tag{13}$$

This makes it possible to consider the function $D_n(z)$ only with any integer values of the index.

To calculate $D_n(z)$ at negative values n, the formula of inverse recursion (8) was used. the stability of calculations by Eq. (8) was examined by the comparison of the results with that obtained by means of the continuous fraction (12). The coincidence of practically all significant digits was obtained at any actual values of real and imaginary parts of z.

To pass from positive to negative n values when calculating $D_n(z)$, it is better not to use the values $D_0(z)$ and $D_{-1}(z)$ (cot$z$ and $-$tan$z$, respectively), since when calculating them for some values z the overflowing or the loss of accuracy can occur. The explicit formula is recommended for passing from $D_1(z)$ to $D_{-2}(z)$:

$$D_{-2}(z) = \frac{z^2 - z^4 - zD_1(z) - 1}{z + z^2\,(1 + z^2\,D_1(z))}\,.$$

Then, by introducing an additional function $F_n(z_1, z_2) = \dfrac{\psi_n(z_1)\chi_n(z_2)}{\psi_n(z_2)\chi_n(z_1)}$, dividing the nominator and denominator in Eq. (2) by $\chi_n(m_2x)$ and taking into account that $\chi_n'(z)/\psi_n(z) = C_n(z)\chi_n(z)/\psi_n(z)$, we can write Eqs. (2) and (3) in the form:

$$A_n = \frac{mD_n(m_1\,x) - D_n(m_2\,x)}{mD_n(m_1\,x) - D_{-n-1}(m_2\,x)}\,;$$

$$B_n = \frac{D_n(m_1\,x)/m - D_n(m_2\,x)}{D_n(m_1\,x)/m - D_{-n-1}(m_2\,x)}\,;$$

$$c_n = \frac{D_n(m_2\,y) - D_{-n-1}(m_2\,y)\,A_n\,F_n(m_2\,x,\,m_2\,y)}{1 - A_n\,F_n(m_2\,x,\,m_2\,y)}\,;$$

$$d_n = \frac{D_n(m_2\,y) - D_{-n-1}(m_2\,y)\,B_n\,F_n(m_2\,x,\,m_2\,y)}{1 - B_n\,F_n(m_2\,x,\,m_2\,y)}. \tag{14}$$

Thus, using Eqs. (4) and (14) is avoiding the calculations of the Riccati-Bessel functions of the complex argument.

The recursion relationship for $F_n(z_1, z_2)$ follows from Eqs. (5) and (6)

$$F_{n+1}(z_1, z_2) = \\ = F_n(z_1, z_2)\,\frac{(D_{-n-1}(z_1) + n/z_1)\,(D_n(z_2) + n/z_2)}{(D_n(z_1) + n/z_1)\,(D_{-n-1}(z_2) + n/z_2)}\,. \tag{15}$$

Then we obtain for calculating the initial value $F_1(z_1, z_2)$ from $D_0(z) = \cot z$, Eqs. (5), and (8), taking into account that $z_1 = m_2\,x$ and $z_2 = m_2\,y$,

$$F_1(z_1, z_2) = g^3\,\frac{1 + (z_2 + z_2^3)\,D_1(z_2)}{1 + (z_1 + z_1^3)\,D_1(z_1)}\,.$$

According to Ref. 2, the number of terms to be summed in the Mie series for a two-layer sphere is selected in the same way for a homogeneous sphere of the radius r, by the empirical formula

$$N = 2 + y + 4\,y^{1/3}.$$

The accumulation of the computer error can occur when calculating the values A_n and B_n at small values of the parameter g, i.e. when the core radius is much less then the cover radius. It is the result of the recursion going beyond the limits of the appropriate values of the index. So, using the recommendations from Ref. 2, let us introduce the parameter

$$N_x = p\,(2 + x + 4\,x^{1/3})\,,$$

where p is some "amplifying" coefficient, for which the value $p = 2$ is recommended. We have selected it by tracking the process of approaching of the expressions $A_nF_n(m_2\,x,\,m_2\,y)$ and $B_nF_n(m_2\,x,\,m_2\,y)$ to zero. Then, starting with the numbers $n > N_x$, let us assume that $A_n\,F_n(m_2\,x,\,m_2\,y) = 0$, $B_n\,F_n(m_2\,x,\,m_2\,y) = 0$, and Eqs. (4) for a_n and b_n coincide with the formulas for uniform particles. Therefore, using this asymptotics for small g, the sphere inhomogeneity is taken into account only for the first N_x coefficients of the series, i.e. as a small correction of the case of a uniform sphere, that agrees with the physical meaning of the phenomena under consideration.

Let us also note that from Eq. (14) it follows that if $m_1 = m_2$, then $A_n = 0$ and $B_n = 0$, i.e. the formulas for a_n and b_n are also transformed to the formulas for a homogeneous particle.

The case of big particles with strongly absorbing cover is most difficult for calculations. Let us consider it separately.

As follows from the explicit expressions for real and imaginary parts of tangent and cotangent of a complex argument at big imaginary parts, $m_2\,x$, (and then $m_2\,y$) the initial values of the functions $D_0(m_2x)$ and $D_{-1}(m_2\,x)$ are different from the value $-\,i$ by the term of the order of exp $(-\,2x\,\kappa_2)$ that becomes vanishing and can be much less than the computer error in calculation as x and κ_2 increase. The recursion formula for the difference $D_n(z) - D_{-n-1}(z)$ follows from Eq. (7):

$$D_n(z) - D_{-n-1}(z) = (D_{n-1}(z) - D_{-n}(z)) \times$$

$$\times\,(D_n(z) + n/z)\,(D_{-n-1}(z) + n/z)\,,$$

i.e. the difference $D_n(z) - D_{-n-1}(z)$ remains small in the case considered. So, if the value $2x\kappa_2$ is sufficiently big, for example $x\kappa_2 > R$, where R is some value, the choice of which we will discuss below, then one can suppose $D_{-n-1}(m_2 x) = D_n(m_2 x)$ and $D_{-n-1}(m_2 y) = D_n(m_2 y)$ without any loss of the calculations accuracy.

But, in this case the uncertainty of the type of $0/0$ appears when calculating the values c_n and d_n. To remove it, let us take the difference of $D_{-1}(m_2x)$ from $D_0(m_2x)$ and $D_{-1}(m_2y)$ from $D_0(m_2y)$ in Eqs. (14) and (15) in the explicit form and consider only the terms of the first order of smallness. As a result, we obtain, after the transformations, that

$$c_n = D_n(m_2\, y) + \frac{V(mD_n(m_1x) - D_n(m_2\, x))}{\dfrac{T_n(m_2y)}{T_n(m_2\, x)} + (mD_n(m_1x) - D_n(m_2x))\,(S_n(m_2x) - VS_n(m_2y))\,T_n(m_2y)}\,;$$

$$d_n = D_n(m_2\, y) + \frac{V(D_n(m_1\, x)/m - D_n(m_2\, x))}{\dfrac{T_n(m_2\, y)}{T_n(m_2\, x)} + (D_n(m_1\, x)/m - D_n(m_2\, x))\,(S_n(m_2\, x) - VS_n(m_2\, y))\,T_n(m_2\, y)}\,;$$

$$V = (\cos\,(2y\,(1-g)\,n_2) + i\,\sin\,(2y\,(1-g)\,n_2))\,\exp\,(-\,2y\,(1-g)\,\kappa_2);$$

$$T_n(z) = \frac{T_{n-1}(z)}{(D_n(z) + n/z)^2}\,,\ T_0(z) = 1;$$

$$S_n(z) = S_{n-1}(z) + \frac{1}{T_n(z)\,(D_n(z) + n/z)}\,,\quad S_0(z) = i/2. \tag{16}$$

The formulas (16) are the asymptotic formulas for the big particles with strongly absorbing cover and have a simple physical meaning. The fraction in the expressions for c_n and d_n determines the effect of the particle core. It is proportional to the value $\exp(-2y(1-g)\kappa_2)$. Hence, the thicker is the cover, the less is the effect of the core on the optical properties of the particle. In particular, if the value $y(1-g)\kappa_2$ is sufficiently big, namely $y(1-g)\kappa_2 > R$, one can ignore the effect of the core and consider $c_n = d_n = D_n(m_2y)$. As a result, Eq. (4) reduces to formulas for a uniform particle.

One can finally formulate the following logical order of the algorithm proposed. If $y\kappa_2 > 2R$ and $y(1-g)\kappa_2 > R$, one should make calculations in the same way as in the case with uniform particles without taking into account the core, i.e. assuming $c_n = d_n = D_n(m_2\, y)$ in Eq. (4). On the contrary, if $x\kappa_2 > R$, one should make calculations by Eqs. (16) and (4) and if $x\kappa_2 < R$ Eqs (14) and (4) should be used.

The value of the parameter R was specially selected. It should not be too big, because the false results can appear at $x\kappa_2 < R$, but it also should not be too small, otherwise we leave the frameworks of the applicability limits of asymptotic formulas. When testing the algorithm, we selected the value $R = 6$. The error in passing to the asymptotic formulas estimated from the results of numerical simulation was about 1% for this value, what is quite acceptable for practical calculations of the optical characteristics of ensembles of aerosol particles.

But, in spite of all contrivances, the cases of incorrect operation of the algorithms appear at $y > 30000$, for nonabsorbing cores and $x\kappa_2$ less but close to R. it is connected with the fact that the imaginary part of the functions $D_n(z)$ and $D_{-n-1}(z)$ was close to 1 in these cases, and the imaginary part of the value n/z was too small; the smallest digits of n/z are lost when summing. That finally results in the enhanced values of the imaginary parts of the coefficients a_n and b_n, and, hence, in negative absorption. As it was revealed during tests, this error appears already in the coefficients a_n and b_n, with small numbers, (starting with the first one), i.e. it is the result of the aforementioned peculiarity, and not the accumulation of the error during the recursions.

To avoid the consequence of this peculiarity in the algorithm, we applied a mathematically incorrect but practically justified approach. If the contribution from absorption into the sum of the series is negative, then the imaginary parts of a_n and b_n, are multiplied by the coefficient less than 1 that is selected so that the contribution coming from the term of the series is equal to $1 - \exp(-2(1-g)y\kappa_2)$, i.e., the asymptotic value obtained based on geometric optics. Justification of such a fitting is in the fact that it makes it possible to obtain correct values of the scattering parameters and does not affect the value of the extinction factor. The error in the scattering and extinction factors is just a few percent, as the calculation estimates showed, that is quite acceptable for practical calculations, especially if one takes into account that when calculating the optical characteristics of aerosol particles in the atmosphere, the integration is performed over the ensemble where the region $y > 30000$ either is not included into the integral, or its contribution is very small. Moreover, the correction is not always necessary even for $y > 30000$, but only in some cases of the particles with nonabsorbing core and thin absorbing cover.

The comparison of calculations by the algorithm proposed with the results from Ref. 8 was used as a formal test of the algorithm. Good agreement between the results was obtained (the coincidence was estimated visually for the plots, the difference for tables was no larger than 2%, except for two wavelengths where it was of the order of 10%). In addition, we compared our data with the results of calculations by the FORTRAN program from Ref. 2 was carried out. The comparison was carried out for different values of the parameters, and the results coincided accurate to the 5th digit in the range of stable operation of the program from Ref. 2 (including the cases of disagreement with Ref. 8).

In addition to formal tests of the algorithm, it is important to establish the boundaries of its applicability both in the range of small values of the parameter y and for the big ones also.

The asymptotic approximation derived similarly to the asymptotic formulas for uniform spheres presented in Ref. 2 was used for testing in the range of small particles. As to the uniform spheres, the term of the series a_1 has the order y^3, b_1 and a_2 are of the order of y^5, and b_2 is of the order of y^7. Hence, for testing the algorithm at small y it is sufficient to compare only the term a_1 with its asymptotic value that has the following form, accurate to the terms of the order of y^5:

$$a_1 = \frac{2}{3}\, i y^3 \times$$
$$\times \frac{(1 - m_2^2)\,(2\, m_2^2 + m_1^2) + g^3\,(m_2^2 - m_1^2)\,(1 + 2\, m_2^2)}{(2 + m_2^2)\,(2\, m_2^2 + m_1^2) - 2\, g^3\,(m_2^2 - m_1^2)\,(2\, m_2^2 - 1)}\,. \tag{17}$$

The results obtained by the algorithm proposed start to differ from Eq. (17) at some value of the parameters by more than 5% for $y < 7\cdot 10^{-6}$, that is out of the lowest threshold of possible y values for practical calculations (it is interesting to note that for uniform particles this threshold is approximately one order of magnitude less, its increase occurs at big values g, very close to 1).

The system of tests based on the tests from Ref. 2 was used in the range of big y.

Test 1. the extinction and scattering factors Q_e and Q_s should be positive at any values of the parameters, and the conditions $Q_e > Q_s$, if $\kappa_1 \neq 0$ and $\kappa_2 \neq 0$, and $Q_e = Q_s$ if $\kappa_1 = \kappa_2 = 0$ should be fulfilled.

Test 2. The elements S_{12} and S_{34} of the scattering phase matrix should equal zero at the scattering angles of 0 and 180°.

Test 3. The identity

$$\frac{S_{12}}{S_{11}} + \frac{S_{33}}{S_{11}} + \frac{S_{34}}{S_{11}} = 1$$

should hold for all elements of the scattering phase matrix at all angles.

Test 4. The alternative relationship for the expression of the extinction factor through the real part of the element of the complex scattering phase matrix should hold (see Ref. 2).

Test 5. The extinction factor should approach 2 as y increases, except for the case of the hollow particles ($n_1 = 1$, $\kappa_1 = 0$) with thin cover (see test 9).

Test 6. The asymptotic relationship obtained in Ref. 2 for the backscattering factor should hold at big y for particles with a sufficiently thick absorbing cover:

$$Q_b = \frac{(n_2 - 1)^2 + \kappa_2^2}{(n_2 + 1)^2 + \kappa_2^2}\,.$$

Test 7. All optical characteristics of a two-layer particle should equal the corresponding characteristics of a uniform particle at any values g and y, if $n_1 = n_2$ and $\kappa_1 = \kappa_2$.

Test 8 (taken from Ref. 3). Let a particle have an absorbing core $\kappa_1 \neq 0$ and nonabsorbing cover $\kappa_2 = 0$. Then, as the cover increases at the constant core radius, the absorption cross-section (the product of πy^2 and the extinction factor) asymptotically approaches some constant value only depending on the core radius, n_1, κ_1 and n_2.

Test 9. Let us assume that we have a big hollow particle ($n_1 = 1$, $\kappa_1 = 0$) with a very thin cover. Then, if the cover thickness is much less than the light wavelength, the extinction factor should approach zero. The interference phenomena in the thin cover should be observed at the growth of the cover to the thickness comparable with the light wavelength. The dependence of the extinction factor on the cover thickness should be oscillating. The extinction factor should approach 2 for the cover thickness much greater than the wavelength.

The results of operation of the algorithm proposed were examined at different values of the parameters up to $y = 400000$. The examination showed full agreement with all criteria.

The tests 1, 8 and 9 are most "strong". Test 1 was the test, the results of which were the base for the development of the technique for correction of the values a_n and b_n at $y > 30000$. The absorption factor in the test 8 decreases as the cover increases. So the absorption cross-section is the product of a very small value by a very big one, but the test 8 perfectly holds even at $y = 400000$. Moreover, we obtained practically full coincidence between the asymptotic values of the absorption cross-section with the values presented in Ref. 3 for this test. The test 6 that is considered as most "strong" for the uniform particles, is not illustrative in our case, because in the case of thick strongly absorbing cover the calculations are performed by formulas for uniform particles, for which the validity of this test is proved.

Let us present the results of the model calculations as an illustration of the algorithm operation. The wavelength was taken to be equal to 0.35 μm in all calculations.

Dependence of the extinction, scattering and absorption factors of big water particles with thin soot cover (model of the urban fog) $r = 10$ μm, $n_1 = 1.33$, $\kappa_1 = 0$, $n_2 = 1.74$, $\kappa_2 = 0.73$ on the cover thickness (parameter $1 - g$) is shown in Fig. 1.

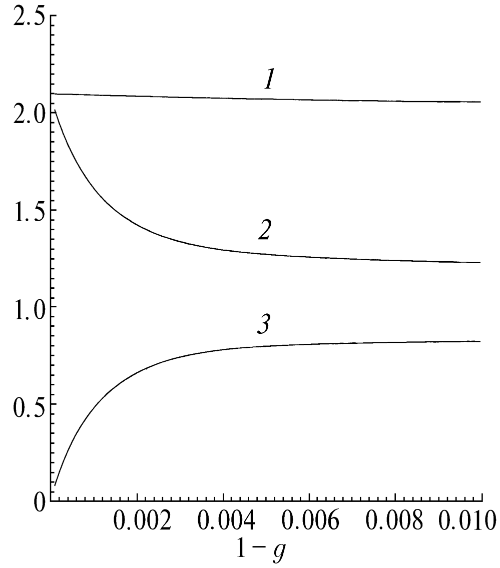

FIG. 1 Dependence of the extinction, scattering, and absorption factors of the particle with $r = 10$ μm, $n_1 = 1.33$, $\kappa_1 = 0$, $n_2 = 1.74$, $\kappa_2 = 0.73$ *on the cover thickness; 1) extinction factor; 2) scattering factor; 3) absorption factor.*

Figure 2 shows the dependence of the scattering and absorption factors of very big ice particles with thin soot cover (model of the comet substance) $n_1 = 1.33$, $\kappa_1 = 0$, $n_2 = 1.74$, $\kappa_2 = 0.73$ on the cover thickness (parameter $1 - g$) for three values of the particle size $r = 25$, 250 and 2500 μm.

An example of the interference in thin films is shown in Fig. 3. It is the dependence of the extinction factor of the hollow particle with the cover of the hardened magma (model of the volcanic aerosol) $n_2 = 1.52$, $\kappa_2 = 0$ for two radii $r = 5$ and 50 μm on the cover thickness.

Thus, the algorithm proposed makes it possible to calculate the optical characteristics of the two-layer spherical aerosol particles with homogeneous core and the cover at any values of the parameters with the accuracy sufficient for the practical calculations. Let us note for care, that big number of the parameters of the problem makes it impossible a detailed consideration of their combinations when testing the algorithm, so no full guarantee can be provided that there is no such values of the parameters for which the algorithm proposed give too big errors or false results.

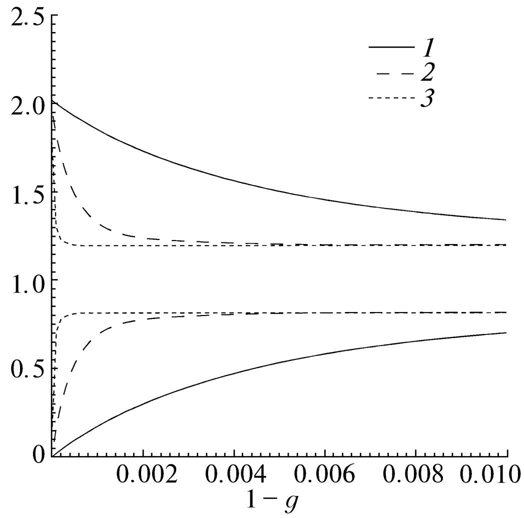

FIG. 2. Dependence of the scattering (curves above unit level) and absorption (curves below unit level) factors of particles with $n_1 = 1.33$, $\kappa_1 = 0$, $n_2 = 1.74$, $\kappa_2 = 0.73$ *on the cover thickness 1)* $r = 25$ μm; *2)* $r = 250$ μm; *3)* $r = 2500$ μm.

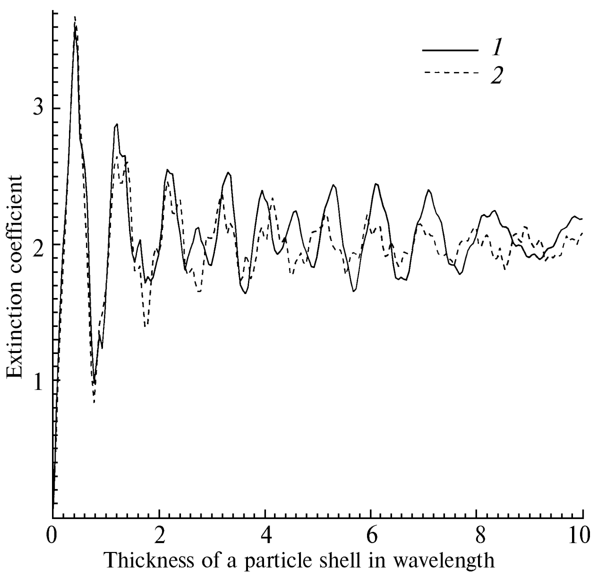

FIG. 3. Dependence of the extinction factor of the hollow particle with $n_1 = 1$, $\kappa_1 = 0$, $n_2 = 1.52$, $\kappa_2 = 0$ *on the cover thickness 1)* $r = 5$ μm; *2)* $r = 50$ μm.

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