Scintillation index of incoherent general type beams in turbulence

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Scintillations are found for incoherent general type beams in weakly turbulent horizontal atmospheric links. Incoherent cosh-Gaussian (IChG) and incoherent cos-Gaussian (ICG) beams exhibit lower scintillations for larger absolute displacement parameters. IChG beam yields lower fluctuations than the ICG beam for the same absolute displacement parameter. Narrower ring incoherent annular (IA) beam scintillates less than the wider ring IA. Increase in the source size lowers the scintillations for all types of the incoherent beams. As the wavelength increases, the scintillations of IChG, ICG and IA beams first increase, then start to decrease and eventually the scintillation indices merge towards a certain value. Raising the structure constant first increases the intensity fluctuations for all the mentioned beams where further rises in the structure constant result in the same level of scintillation index.

1. Introduction

Intensity fluctuations is an important parameter to be taken into account in analyzing the performance of an atmospheric optical communications link. It is known that the intensity fluctuations differ as the source field profile or the source coherence property changes. The scintillations of coherent sinusoidal-Gaussian[1,2] and annular beams[2,3] in turbulent links are studied. Partial coherence is introduced into such beam types to examine the effect of the degree of coherence on the propagation behaviour in free space[4–6] and in random media.[7,8] It is of interest to understand the scintillation noise behaviour when incoherent sources of different intensity profiles are employed in atmospheric optics links. In this respect, we have recently reported the scintillation index variations of incoherent flat-topped Gaussian sources in atmospheric turbulence.[9] In this paper we investigate the intensity fluctuations in weak turbulence under incoherent sinusoidal-Gaussian and incoherent annular excitations.

2. Formulation

The deterministic source field expression yielding sinusoidal-Gaussian and annular beam profiles is obtained as a special case of the general-type beam field formula[10]:

$$u_{sd}(\mathbf{s}) = u_{sd}(s_x, s_y) = \sum_{\ell=1}^{N} A_\ell \exp\left[-\left(\frac{s_x^2}{2\alpha_{sx\ell}^2} + jV_{x\ell}s_x\right)\right]\exp\left[-\left(\frac{s_y^2}{2\alpha_{sy\ell}^2} + jV_{y\ell}s_y\right)\right], \quad (1)$$

where $\mathbf{s} = (s_x, s_y)$ is the transverse source coordinate, A_ℓ is the amplitude of the ℓth component of the source field, $V_{x\ell}$ and $V_{y\ell}$ are the displacement parameters, $\alpha_{sx\ell}$ and $\alpha_{sy\ell}$ are the Gaussian source sizes, $j = (-1)^{0.5}$ and N denotes the number of beams. Representing the random part of the source field by $u_{sr}(\mathbf{s})$, the incident field can be expressed in terms of a product of the deterministic and the random source fields as

$$u_s(s) = u_{sd}(s)u_{sr}(s). \quad (2)$$

Using the incident field given in Eq. (2) and applying the extended Huygens–Fresnel principle, instantaneous received intensity as measured by a detector whose response time is much longer than the source coherence time is found to be

$$I(\mathbf{p}, L) = \frac{1}{(\lambda L)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d^2\mathbf{s}_1 d^2\mathbf{s}_2 \Gamma_2^s(\mathbf{s}_1,\mathbf{s}_2)\times \exp\left\{\frac{jk}{2L}\left[|\mathbf{p}-\mathbf{s}_1|^2 - |\mathbf{p}-\mathbf{s}_2|^2\right]\right\}\exp\left[\psi(\mathbf{s}_1,\mathbf{p}) + \psi^*(\mathbf{s}_2,\mathbf{p})\right], \quad (3)$$

where * represents the complex conjugate, L is the link length, λ is the wavelength, $k = 2\pi/\lambda$ is the wave number, $\mathbf{p} = (p_x, p_y)$ is the transverse receiver coordinate, and $\psi(\mathbf{s}, \mathbf{p})$ is the random complex phase of a spherical wave propagating from the source point to the receiver point found by Rytov method. The second order source mutual coherence function is $\Gamma_2^s(\mathbf{s}_1,\mathbf{s}_2) = \langle u_{sd}(\mathbf{s})u_{sd}^*(\mathbf{s})\rangle_s$ which can be expressed for an incoherent source by[11]

$$\Gamma_2^s(\mathbf{s}_1,\mathbf{s}_2) = \lambda^2 I[(\mathbf{s}_1+\mathbf{s}_2)/2]\delta(\mathbf{s}_1-\mathbf{s}_2).$$

Here δ is the delta function. $<\ >_s$ is the ensemble average over the source statistics, $I[(\mathbf{s}_1+\mathbf{s}_2)/2]$ is the intensity at the source coordinate $(\mathbf{s}_1+\mathbf{s}_2)/2$.

Inserting into Eq. (3), δ function representation of $\Gamma_2^s(\mathbf{s}_1,\mathbf{s}_2)$ together with the deterministic source field expression for the general-type beam field formula given by Eq. (1), assuming that the source and the medium statistics are independent, using

$$<\exp\left[\psi(\mathbf{s}_1,\mathbf{p})+\psi^*(\mathbf{s}_2,\mathbf{p})\right]>_m=$$
$$=\exp\left[-0.5D_\psi(\mathbf{s}_1-\mathbf{s}_2)\right] \simeq \exp\left[-\rho_0^{-2}(\mathbf{s}_1-\mathbf{s}_2)^2\right]$$

as given in Ref. 12 (where $<\ >_m$, D_ψ, $\rho_0 = (0.545C_n^2k^2L)^{-3/5}$ and C_n^2 are the ensemble average over the medium statistics, wave structure function, the coherence length of a spherical wave propagating in the turbulent medium and the structure constant, respectively), performing the integrations over $\mathbf{s}_1$ and $\mathbf{s}_2$, the average intensity at the receiver plane is found as

$$<I(\mathbf{p},L)>=$$
$$=\frac{2\pi}{L^2}\sum_{\ell=1}^{N}\sum_{\ell'=1}^{N}\frac{A_\ell A_{\ell'}^*}{(B_{x\ell\ell'}B_{y\ell\ell'})^{1/2}}\exp\left[-0.5\left(\frac{W_{x\ell\ell'}^2}{B_{x\ell\ell'}}+\frac{W_{y\ell\ell'}^2}{B_{y\ell\ell'}}\right)\right], \quad (4)$$

where

$$W_{x\ell\ell'} = V_{x\ell} - V_{x\ell'}^*,\ W_{y\ell\ell'} = V_{y\ell} - V_{y\ell'}^*,$$
$$B_{x\ell\ell'} = \alpha_{sx\ell}^{-2} + \alpha_{sx\ell'}^{-2},\ B_{y\ell\ell'} = \alpha_{sy\ell}^{-2} + \alpha_{sy\ell'}^{-2}. \quad (5)$$

In finding $<I^2(\mathbf{p}, L)>$, the instantaneous intensity given by Eq. (3) is used as the starting point to obtain

$$<I^2(\mathbf{p},L)>=\frac{1}{(\lambda L)^4}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}d^2\mathbf{s}_1 d^2\mathbf{s}_2\Gamma_2^s(\mathbf{s}_1,\mathbf{s}_2)\times$$
$$\times\exp\left\{\frac{jk}{2L}\left[|\mathbf{p}-\mathbf{s}_1|^2-|\mathbf{p}-\mathbf{s}_2|^2\right]\right\}\times$$
$$\times\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}d^2\mathbf{s}_3 d^2\mathbf{s}_4\Gamma_2^s(\mathbf{s}_3,\mathbf{s}_4)\exp\left\{\frac{jk}{2L}\left[|\mathbf{p}-\mathbf{s}_3|^2-|\mathbf{p}-\mathbf{s}_4|^2\right]\right\}\times$$
$$\times<\exp\left[\psi(\mathbf{s}_1,\mathbf{p})+\psi^*(\mathbf{s}_2,\mathbf{p})+\psi(\mathbf{s}_3,\mathbf{p})+\psi^*(\mathbf{s}_4,\mathbf{p})\right]>_m. \quad (6)$$

Using $\Gamma_2^s(\mathbf{s}_1,\mathbf{s}_2) = \lambda^2 I\left[(\mathbf{s}_1+\mathbf{s}_2)/2\right]\delta(\mathbf{s}_1-\mathbf{s}_2)$ and performing the integrations over $\mathbf{s}_1$, $\mathbf{s}_2$, $\mathbf{s}_3$ and $\mathbf{s}_4$, we obtain

$$<I^2(\mathbf{p},L)>\approx<I(\mathbf{p},L)>^2 +$$
$$+\frac{16\pi^2\sigma_\chi^2}{L^4}\sum_{\ell=1}^{N}\sum_{\ell'=1}^{N}\sum_{\ell_1=1}^{N}\sum_{\ell_1'=1}^{N}A_\ell A_{\ell'}^* A_{\ell_1}A_{\ell_1'}^* H_x(\rho_0)H_y(\rho_0), \quad (7)$$

where $\sigma_\chi^2 = 0.124C_n^2k^{7/6}L^{11/6}$ is the variance of the log amplitude fluctuations for a spherical wave,

$$H_x(\rho_0) = \frac{1}{\sqrt{B_{x\ell_1\ell_1'}E_{x\ell\ell'}+\dfrac{2B_{x\ell\ell'}}{\rho_0^2}}}\times$$
$$\times\exp\left\{-0.5\left[\frac{W_{x\ell\ell'}^2}{E_{x\ell\ell'}}+\frac{\left(W_{x\ell_1\ell_1'}-\dfrac{2W_{x\ell\ell'}}{\rho_0^2E_{x\ell\ell'}}\right)^2}{\left(B_{x\ell_1\ell_1'}+\dfrac{2B_{x\ell\ell'}}{\rho_0^2E_{x\ell\ell'}}\right)}\right]\right\}, \quad (8)$$

$$W_{x\ell_1\ell_1'} = V_{x\ell_1} - V_{x\ell_1'}^*,\ B_{x\ell_1\ell_1'} = \alpha_{sx\ell_1}^{-2} + \alpha_{sx\ell_1'}^{-2},$$
$$E_{x\ell\ell'} = B_{x\ell\ell'} + \frac{2}{\rho_0^2}, \quad (9)$$

and $H_y(\rho_0)$, $W_{y\ell_1\ell_1'}$, $B_{y\ell_1\ell_1'}$, $E_{x\ell\ell'}$ are obtained when x is replaced by y in Eqs. (8) and (9). In reaching Eq. (7), we have utilized $\psi(\mathbf{s},\mathbf{p}) = \chi(\mathbf{s},\mathbf{p}) + jS(\mathbf{s},\mathbf{p})$ where ψ, χ and S are the wave, log-amplitude and phase fluctuations, respectively. Inserting ψ in Eq. (6), for χ Gaussian distributed in weak turbulence, $<\ >_m$ in Eq. (6) can be approximated by

$$<\exp[2\chi(\mathbf{s}_1,\mathbf{p})]\exp[2\chi(\mathbf{s}_2,\mathbf{p})]>_m \approx 1 + 4B_\chi(\mathbf{s}_1,\mathbf{s}_2), \quad (10)$$

where $B_\chi(\mathbf{s}_1,\mathbf{s}_2)$ is the two source spherical wave covariance function of the log-amplitude fluctuations given by[13]:

$$B_\chi(\mathbf{s}_1,\mathbf{s}_2) = <\left[\chi(\mathbf{s}_1,\mathbf{0}) - <\chi(\mathbf{s}_1,\mathbf{0})>_m\right]\times$$
$$\times\left[\chi(\mathbf{s}_2,\mathbf{0}) - <\chi(\mathbf{s}_2,\mathbf{0})>_m\right]>_m = \sigma_\chi^2\exp\left(-\frac{|\mathbf{s}_1-\mathbf{s}_2|^2}{\rho_0^2}\right). \quad (11)$$

Using Eqs. (4) and (7), the scintillation index is found to be

$$m^2 = \frac{<I^2(\mathbf{p},L)> - <I(\mathbf{p},L)>^2}{<I(\mathbf{p},L)>^2} = \frac{4\sigma_\chi^2G(\rho_0)}{G(\rho_0=\infty)}, \quad (12)$$

where

$$G(\rho_0) = \sum_{\ell=1}^{N}\sum_{\ell'=1}^{N}\sum_{\ell_1=1}^{N}\sum_{\ell_1'=1}^{N}A_\ell A_{\ell'}^* A_{\ell_1}A_{\ell_1'}^* H_x(\rho_0)H_y(\rho_0), \quad (13)$$

and $G(\rho_0 = \infty)$ denotes $G(\rho_0)$ evaluated at $\rho_0 = \infty$. Note that Eq. (12) is independent of transverse plane coordinate $\mathbf{p}$, thus our numerical results of the next section are equally applicable to on-axis as well as off-axis positions.

3. Results

The scintillation index provided in Eq. (12) which is valid for horizontal and weakly turbulent links is calculated for incoherent cosh-Gaussian (IChG), cos-Gaussian (ICG) and annular (IA) beams and the results are presented in this section. As known, turbulence is an integrated effect formed by the wavelength, link length and the structure constant, and it is named weak, when the spherical wave scintillation index is much less than unity. Parameter sets to form these beam combinations are obtained from the general beam formulation as explained in Ref. 14.

Figs. 1–3 provide of the scintillation index variations of IChG, ICG and IA beams versus the link length. Being valid for all link lengths, IChG and ICG beams of fixed sizes yield smaller scintillations when the absolute displacement parameter increases. Again for all the link lengths, narrower ring IA beam has lower scintillations than the wider ring IA beam under the condition that the primary beam sizes are the same. Comparison of the intensity fluctuations versus the link length behaviour of IChG, ICG and IA beams are given in Fig. 4. For the same absolute displacement parameter and at a fixed link length, the scintillations of IChG beam is lower than the scintillations of ICG beam.

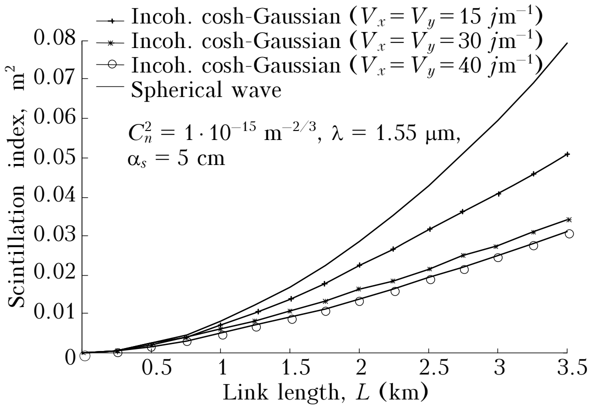

Fig. 1. Scintillation index of incoherent cosh-Gaussian beams versus the link length at various displacement parameters.

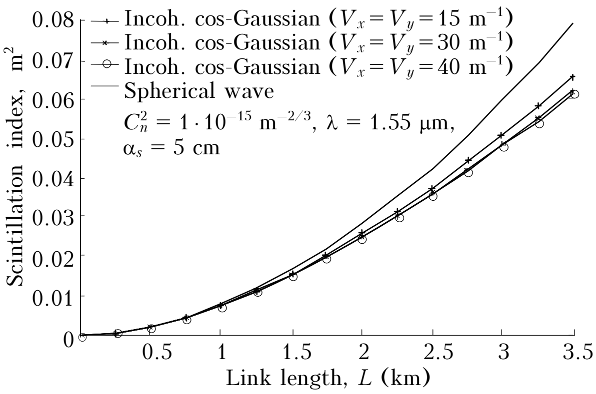

Fig. 2. Scintillation index of incoherent cos-Gaussian beams versus the link length at various displacement parameters.

Figs. 5 and 6 show the intensity fluctuations of IChG, ICG and IA beams versus the source size. For all of the IChG, ICG and IA beams, the scintillation index becomes smaller for larger source sizes. This supports the transmitter aperture averaging effect.[15] IChG beams have lower fluctuations than the ICG beams when the source size and the absolute displacement parameters are the same. At a fixed source size and absolute displacement parameter, IChG beams scintillate less than the ICG beams. As seen from Fig. 6, for all ring sizes, the intensity fluctuations become smaller when the primary beam size becomes larger. This reduction is pronounced when the ring becomes narrower.

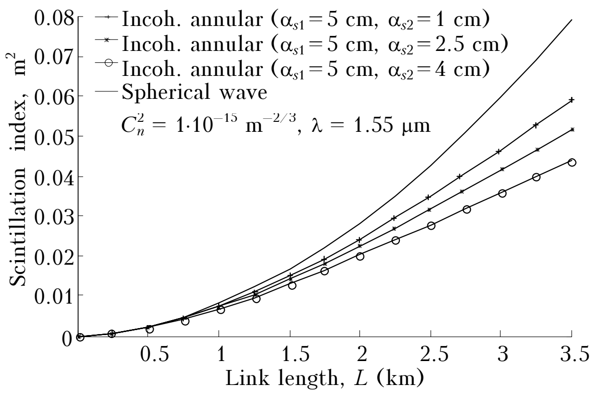

Fig. 3. Scintillation index of incoherent annular beams versus the link length at various source size ratios.

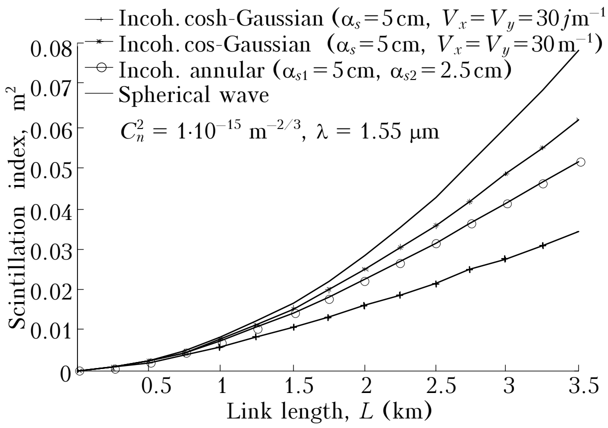

Fig. 4. Comparison of the scintillation indices of incoherent cosh-, cos-Gaussian and annular beams versus the link length.

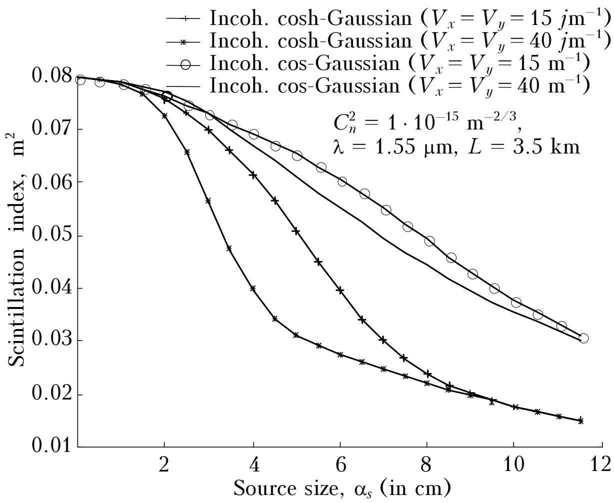

Fig. 5. Scintillation index of incoherent cosh-, cos-Gaussian beams versus the source size.

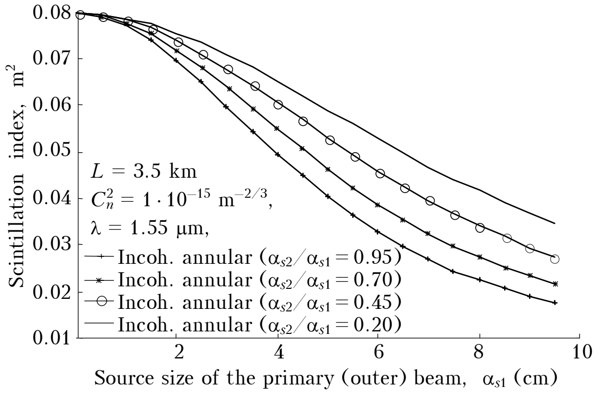

Fig. 6. Scintillation index of incoherent annular beams versus the source size.

In Figs. 7 and 8, the wavelength dependence of the scintillations of IChG, ICG and IA beams are provided. Increase in the wavelength first causes the scintillations of IChG and ICG beams to increase, however when the wavelength is further increased, the scintillations decrease, and eventually merging towards a certain value.

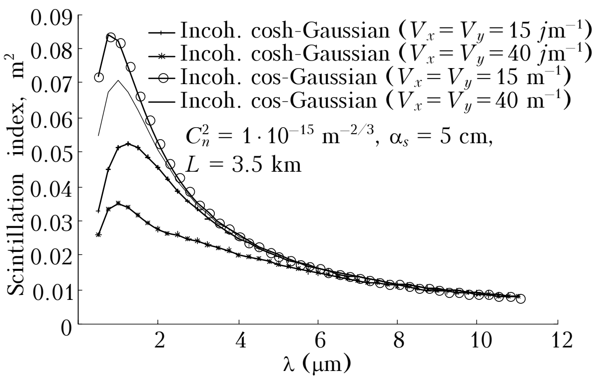

Fig. 7. Scintillation index of incoherent cosh-, cos-Gaussian beams versus the wavelength.

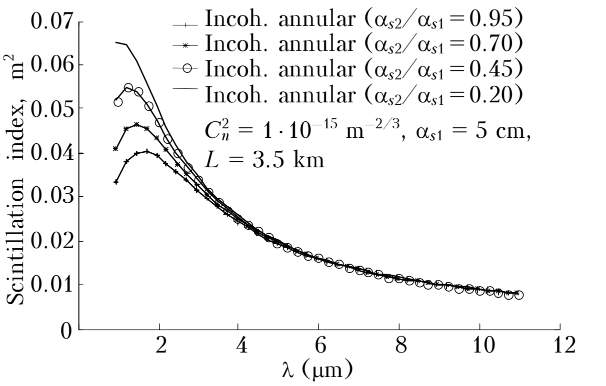

Fig. 8. Scintillation index of incoherent annular beams versus the wavelength.

For IA beams, similar trend in scintillations versus the wavelength is found as noted in the IChG and ICG beam scintillation behaviour versus the wavelength. Finally, in Figs. 9 and 10, the change in the intensity fluctuations of IChG, ICG and IA beams in weak turbulence is investigated when the structure constant changes. Increase in the structure constant first increases the scintillations of IChG, ICG and IA beams, whereas when the structure constant is increased further, the intensity fluctuations for all IChG, ICG and IA beams will stay at the same scintillation level. Comparing the intensity fluctuations of IChG, ICG and IA beams with their coherent counterparts and the coherent Gaussian beam reveals that the intensity fluctuations IChG, ICG and IA beams are lower when the source sizes are large.

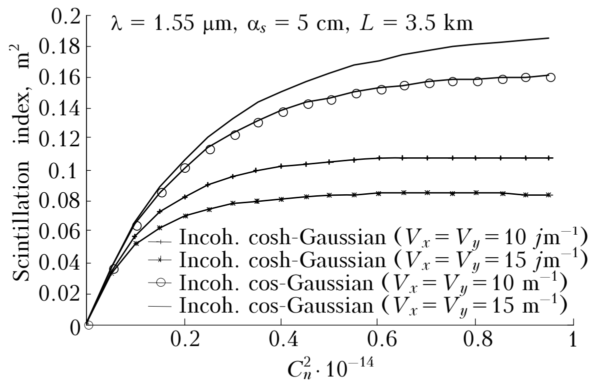

Fig. 9. Scintillation index of incoherent cosh-, cos-Gaussian beams versus the structure constant.

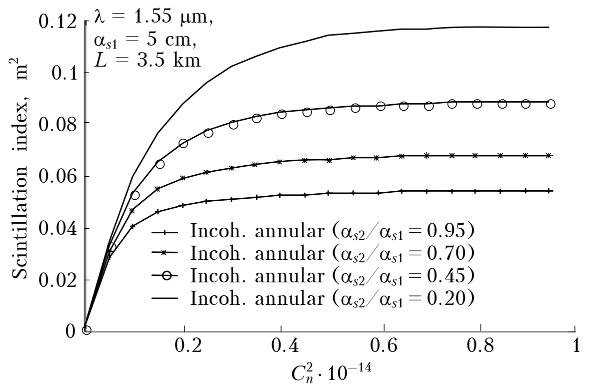

Fig. 10. Scintillation index of incoherent annular beams versus the structure constant.

Conclusion

For incoherent cosh-Gaussian, cos-Gaussian and annular beams, the scintillation index in weakly turbulent horizontal atmospheric optical links is formulated. Our formulation is derived for a detector having a response time much longer than the source coherence time, and under the assumption that the source and the medium statistics are independent. Also, delta correlation is utilized to mimic the spatial partial incoherence of IChG, ICG and IA beams.

Our study covers the investigation of the changes in the scintillations of IChG, ICG and IA beams when the link length, source size, wavelength and the structure constant vary. For both IChG and ICG beams, at a given source size, as the absolute displacement parameter increases, the scintillations decrease, and this holds to be true for all the link lengths. At the same link length and at the same absolute displacement parameter, intensity fluctuations of IChG beam is lower than the fluctuations of the ICG beam. In IA beam fluctuations, at all the link lengths, when the size of the primary beam is kept constant, IA beams with narrower ring structure will fluctuate less than the IA beams possessing wider ring structure.

When the scintillation index is examined versus the source size, it is found that for all types of incoherent beams investigated, large sized incidences result in smaller intensity fluctuations in turbulence. Comparing IChG beams with ICG beams, it is seen that when the source size and the absolute displacement parameter are kept constant, IChG beams fluctuate less than the ICG beams. For both IChG and ICG beams, when the source size is kept constant, the scintillation index decreases when the absolute displacement parameter increases. When the size of the primary beam in IA beams is increased, the intensity fluctuations become smaller, and this trend occurs for all ring sizes, the reduction in the scintillations is more when the ring of IA beam becomes narrower.

Dependence of the intensity fluctuations of IChG, ICG beams on the wavelength shows that the fluctuations become larger as the wavelength increases until a certain wavelength value, after which the trend reverses, i.e., the increase in the wavelength causes the fluctuations to decrease. When the wavelength is further increased, all types of IChG and ICG beams tend to attain the same scintillation index values, mainly due to the domination of the source incoherence in the determination of the intensity fluctuations. For IA beams, the intensity fluctuations follow a similar trend as in the IChG and ICG beam scintillations versus the wavelength.

In weak turbulence, increase in the structure constant in general raises the scintillations of IChG, ICG and IA beams up to a certain structure constant, above which the scintillations remain at the same level. This resembles the saturation effect of scintillation as in the coherent beam case. However, here the saturation level of the scintillation index is determined by the corresponding incoherent beam.

As a final note, the comparison of the scintillations of IChG, ICG and IA beams with their coherent counterparts indicate that incoherent general beams with large source sizes exhibit smaller intensity fluctuations.

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