Comparison of efficiencies of the direct simulation and local estimation methods in inverse scheme for UV radiation fluxes on the Earth's surface

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Two Monte Carlo algorithms for calculation of the intensity of the solar UV flux on the Earth surface, namely, direct simulation and local estimation in the inverse scheme, are analyzed comparatively. The flux intensity is calculated for various optical and geometrical conditions of observation. The efficiency of each algorithm is estimated. The comparison shows that the method of local estimation in the inverse scheme is more expedient for calculation of solar UV flux intensity on the Earth's surface.

Introduction

The study of regularities in propagation of UV radiation through the Earth's atmosphere is the subject of a considerable literature.[1–11] However, most papers consider experimental results. They describe a wide variety of observation conditions: estimation of UV irradiance of the Earth's surface measured from satellites,[4,5] measurements of the intensity of UV radiation with spectral devices,[6,7] integral measurements (A and B regions, erythemal irradiance, etc.),[8–10] measurements in a small solid angle,[11] measurements of the flux from the upper hemisphere, etc. Theoretical results are presented, for example, in Ref. 1.

Only few papers present a comparison of theoretical results with experimental measurements. What's more, measurements are often conducted with different devices, and their reported characteristics not always allow a correct comparison of obtained data.[3] Results are often interpreted using empirical dependences obtained earlier[1,13] or approximate theoretical estimates, whose accuracy is not always known or acceptable for description of particular situations.[12]

This paper is aimed at selecting the method for solving the radiative transfer equation and studying its efficiency for interpretation of experimental results obtained at the Institute of Optical Monitoring SB RAS, when measuring UV radiation fluxes reaching the Earth's surface.

1. Formulation of the problem

In the approximation of ray optics, stationary medium, and continuous radiation sources, the radiation transfer process is described by the integro-differential equation[15]:

$$(\omega, \mathrm{grad} I(\lambda, r, \omega)) = -\sigma(\lambda, r) I(\lambda, r, \omega) + \\ + \frac{\sigma_s(\lambda, r)}{4\pi} \int_\Omega I(\lambda, r, \omega') g(\lambda, r, \omega', \omega) \mathrm{d}\omega' + \Phi_0(\lambda, r, \omega) \quad (1)$$

with the boundary condition $I(\lambda, r, \omega) = 0$, if $r \in \Gamma$ and $(\omega\, n_r) < 0$. Here $\sigma(\lambda, r)$ and $\sigma_s(\lambda, r)$ are, respectively, the volume spectral extinction and scattering coefficients, $x = (r, \omega)$ is the point of the phase space $X = R \times \Omega$ of coordinates $r \in R$ and directions $\omega \in \Omega$, $g(\lambda, r, \omega', \omega)$ is the normalized scattering phase function, $\Phi_0(\lambda, r, \omega)$ is the source density; n_r is the external normal to the surface of the medium Γ at the point r.

Let the Earth's surface be a sphere of radius R_0 (Fig. 1) and the atmosphere be a spherical layer of the radius $R_1 > R_0$. A parallel flux of solar radiation is incident in the direction $\omega^{(S)} = (0, 0, -\theta)$. The observation point is on the Earth's surface at $r^* = (0, 0, R_0)$. The solid observation angle is determined by the given sighting direction $\omega^* = (\theta, \varphi)$ and the aperture angle 2γ, $0 \le \gamma \le \pi/2$; γ is measured from the axis of the aperture angle ω^*.

To specify the parameters in the model of the atmosphere, we divide it into layers $h_{i-1} \le h \le h_i$, $i = 1, \ldots, M$, where h is the height above the surface. The aerosol σ_a and molecular σ_m scattering coefficients and absorption coefficients σ_S are assumed constant for each layer $[h_{i-1}, h_i]$ and equal to $\sigma_{a_i}(\lambda)$, $\sigma_{m_i}(\lambda)$, and $\sigma_{S_i}(\lambda)$, respectively. Each layer is characterized by its own aerosol scattering phase function $g_a(\lambda, \mu)$, and $g_a(\lambda, \mu)$ is assumed linear in the intervals $[\mu_{i-1}, \mu_i]$, $i = 1, \ldots, n$; $\mu_0 = 1$, $\mu_n = -1$. The air scattering phase function for each layer is determined as a weighted-mean of molecular and aerosol scattering phase functions in this layer:

$$g(\lambda, \mu) = \frac{g_a(\lambda, \mu)\sigma_a(\lambda) + g_m(\lambda, \mu)\sigma_m(\lambda)}{\sigma_a(\lambda) + \sigma_m(\lambda)},$$

and $\int_{-1}^{1} g(\lambda, \mu)d\mu = 1$ for each layer. Reflection of radiation from the surface is determined by the law of reflection $p(\mu, \varphi)$ and the surface albedo A.

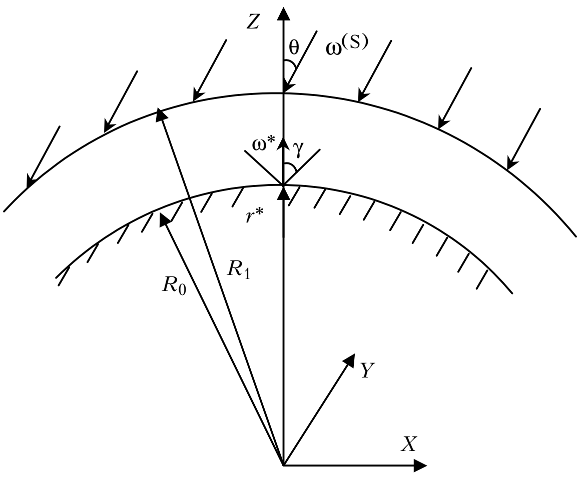

Fig. 1. Observation geometry in the spherical atmosphere.

2. Monte Carlo method for direct and conjugate trajectories

The approximate methods for solution of the radiative transfer equation, based on some simplifying assumptions on optical characteristics of the medium under study as a whole or elementary processes accompanying radiation propagation through the medium, often allow one to describe explicitly and compactly the regularities of the light flux transfer through the atmosphere. However, every approximate method has a limited domain of applicability, which is not consistently strictly defined.[14]

A particular place in the theory of radiative transfer is occupied by the statistical test method or the Monte Carlo method. This method imposes practically no restrictions on the geometry of a medium, composition and spatial distribution of its scattering and absorbing properties, conditions of illumination, and measurement of light fluxes. The main disadvantage of the method, as well as any other numerical experiment, is a particular character of solutions. This disadvantage is compensated for by the above advantages and the possibility of obtaining solutions at node points for drawing analytical approximations.

Simulating optical radiation propagation through disperse media by the Monte Carlo method is described in sufficient detail in Ref. 15. It should be noted that any algorithm of the Monte Carlo method imitating the process of radiative transfer in the atmosphere is based on the scheme of simulation of photon random walk trajectories. The variety of modifications of simulation schemes and special estimates allow the efficiency of the Monte Carlo method to be improved considerably. Selection of a proper algorithm is determined by the geometry of the problem.

In this paper, we consider two such algorithms: the algorithm of direct simulation and the algorithm of local estimation in the inverse scheme. They are chosen based on the experience in solution of direct problems of atmospheric optics[17] and recommendations from Ref. 16, advising to select the proper modification of the method for estimation of characteristics of light fluxes when illuminating a medium by an extended radiation source. The algorithm of direct simulation was taken as a reference one to test the results obtained from local calculations for conjugate trajectories.

2.1. Simulating algorithm for direct trajectories

The algorithm for estimation of light fluxes reaching the Earth's surface in a given solid angle at illumination of the atmosphere by the solar radiation is the simplest for the plane-parallel model of the atmosphere (Fig. 2). In this case, for horizontally homogeneous scattering and absorbing media, the extended source can be replaced by a monodirectional one situated at the point r_0 at the atmospheric top and oriented parallel to the direction of the solar light flux. Then the sought estimate is

$$P(r^*, \Omega) = P(\Omega) = \sum_i p(r_i, \Omega), \quad (2)$$

where r^* is the point, at which the solution is sought; $p(r_i, \Omega)$ is the radiation flux generated at the point r_i of the Earth's surface by a monodirectional source situated at the point r_0. Thus, the algorithm for estimation of fluxes on direct trajectories is reduced to simulating photon random walk trajectories in a medium and counting the trajectories crossing the Earth's surface within a solid angle Ω.

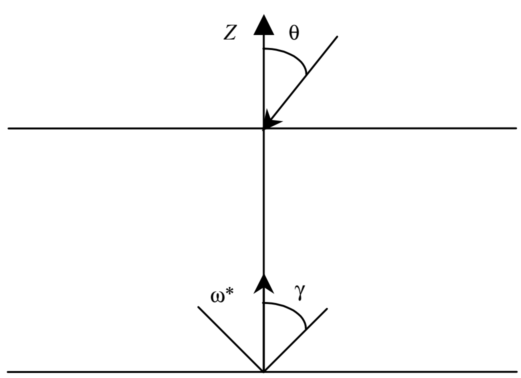

Fig. 2. Observation geometry in the case of the plane-parallel atmosphere.

This algorithm can be used for simultaneous estimation of fluxes within a set N of solid angles Ω_j ($j = 1, ..., N$). The accuracy of estimates in this case apparently increases with the increasing a solid angle Ω_j. It should be noted that simultaneous estimation of fluxes for other directions ω_0 of incidence of solar rays on the atmospheric top is impossible.

Now consider a spherical model of the atmosphere. In this case, we obviously cannot use the condition of a horizontally homogeneous medium to replace the extended source with a monodirectional one, because even at homogeneous optical properties of the medium the sphericity of the atmosphere and the Earth's surface leads to inhomogeneous illumination of the latter (i.e., the equality $p(r_i, \Omega) = p(r_j, \Omega)$ is strictly fulfilled only at $i = j$). Thus, application of Eq. (2) for estimation of $P(r^*, \Omega)$ becomes impossible. Therefore, application of the algorithm based on the idea of direct simulation with replacement of an extended source with a monodirectional one may result in the biased estimate of the fluxes $P(r^*, \Omega)$.

2.2. Algorithm of local estimation for conjugate trajectories

The idea of the algorithms of the Monte Carlo method based on conjugate random walks follows from the optical reciprocity theory[15] and is as follows. In the direct formulation of the problem, the initial point of each photon random walk trajectory is selected randomly from the area of receiver location. Then, according to the same rules as in the scheme of direct simulation, the photon free path is calculated, the probability of its absorption by elements of the medium is selected (or is taken into account through the weighting coefficient), directional cosines of the new direction of motion are re-calculated, etc. To estimate the sought flux $P(r^*, \Omega)$, the probability that a photon leaves the medium through a scattering point in the direction opposite to $\omega^{(S)}$ is calculated:

$$\psi_n = \frac{e^{-\tau(r_n)} g(-\omega^{(S)} \omega_n) q(r_n)}{2\pi} |\Omega| , \qquad (3)$$

where n is the current number of "collision"; $\tau(r_n)$ is the optical length from the point r_n to the atmospheric boundary in the direction $-\omega^{(S)}$ (direction to the Sun); ω_n is the direction of motion of a particle before the "collision" at the point r_n; $q(r) = \dfrac{\sigma_s(r)}{\sigma(r)}$, $\sigma(r)$ and $\sigma_s(r)$ are the volume extinction and scattering coefficients in the atmosphere; $g(\mu)$ is the normalized atmospheric scattering phase function; $|\Omega|$ is the solid angle of observation. Then, the integral intensity of the multiply scattered solar radiation can be estimated:

$$I = M\xi, \quad \xi = \sum_{n=1}^{N} \psi_n ,$$

where N is the number of collisions on the trajectory.

The algorithm of local estimation in the scheme of conjugate random walks possesses some advantages: it can be readily computerized and the estimate (3) has a finite variance. In contrast to the algorithm considered above, it can be used to obtain simultaneously the solutions for a set of $\omega^{(S)}$ values at the same aperture angle and sighting direction. It is easy to show that the accuracy of estimation of the fluxes $P(r^*, \Omega)$ by this method increases with decreasing $|\Omega|$, all other conditions being the same.

3. Comparison of the efficiency of algorithms of the direct simulation and local estimation in the inverse scheme

In spite of the listed advantages and disadvantages of each method, deciding between them is not obvious. The analysis of the local estimation algorithm in the inverse scheme in comparison with other algorithms of the Monte Carlo method[16] has proved its efficiency in solution of various problems. Nevertheless, the efficiency of some or other method as applied to solution of a particular problem should be necessarily estimated in test numerical experiments. The criterion of the efficiency of an algorithm is usually its labor-consuming $S = Dt$, where t is the consumed time and D is the average value of the relative root-mean-square error of the obtained results, in %.

Keeping in mind the above remarks concerning the possibility of application of the discussed algorithms in numerical experiments for the spherical system "atmosphere–Earth's surface," we compared the calculated results (the results of the direct simulation were taken as the reference ones) and the efficiency of the algorithms for the geometry of a simplified problem (see Fig. 2). In the case of spherical geometry, this scheme was imitated by the Earth's curvature length tending to infinity. A total of 160 numerical experiments were conducted at $R_0 = 10^9 R_E$. The results given by the both algorithms converged asymptotically with the increasing number of modeled trajectories.

The flux intensity was calculated depending on the receiver's field of view γ (5, 10, 45, 90°) and for a set of the angles of solar radiation incidence on the atmospheric top (0, 20, 40, 60, 80°). Computations were made for the simplified model of the scattering medium with the optical characteristics close to the radiative fog. The medium was assumed homogeneous, and the computations were carried out in relative units (the value of extraterrestrial solar constant was equal to unity) for the optical depths of 0.5, 1, 2, and 4. The accuracy of the obtained solution and the time-consumption were under control. The efficiency of algorithms was determined from calculation of the intensity for one angle of reception and one angle of direction to the Sun. In the simulation, we controlled the fulfillment of the law of energy conservation: the number of photons entering the atmosphere must be equal to the number of photons leaving it and absorbed in it. Simulation was performed by packets, the *i*th random solution of the problem was the solution estimated as a mean value for the *i*th packet. For all the conditions of simulation, the number of trajectories in a packet was equal to 2000, and the number of packets was 50. Computations were performed with the help of 530-MHz Pentium III computer.

The results of computations demonstrate a close agreement between the methods of direct simulation and conjugate random walks (Fig. 3). The analysis of the obtained results (Table) shows that the algorithm of conjugate random walks is more efficient than the algorithm of direct simulation for small solid angles of observation (5, 10°). As the solid angle increases, the efficiency of the direct simulation algorithm increases, while the efficiency of the algorithm of conjugate random walks decreases because of the increasing time-consumption and variance of the results. The increase of the solar zenith angle increases the time-consumption and insignificantly increases the variance of the results for the direct simulation method, while the scatter of results for the method of conjugate random walks at the large solar zenith angle becomes noticeable at the almost unchanged computational time. This fact should be taken into account when using the algorithm of conjugate random walks for calculation of the intensity at several solar zenith angles. In this case, the accuracy of the results can be controlled only for the maximum solar zenith angle (for all other Sun elevations the variance of the results is smaller). The increase of the scattering optical depth in the atmosphere weakly affects the variance, but leads to the increase of the computational time in the both algorithms because of the increasing number of scattering events for every modeled trajectory.

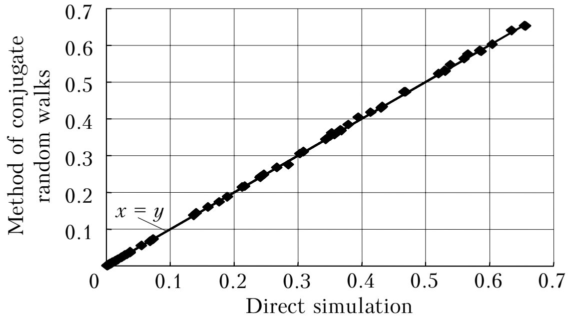

Fig. 3. Comparison of calculated results.

When interpreting the measured intensities of UV radiation fluxes, it is often needed to estimate a flux for different values of Sun elevation and the same solid angle of observation or for different sighting directions at a small angular aperture of the detector. Since the method of conjugate random walks is efficient for small solid angles of observation and it allows the flux to be estimated simultaneously for several zenith angles, the algorithm of conjugate random walks is more efficient than the algorithm of direct simulation in this case.

Thus, taking into account both the results of comparison of the considered algorithms and the fact that the method of direct simulation leads to a biased estimate in the case of spherical geometry, we come to conclusion that the method of conjugate random walks is preferable in estimating fluxes of UV radiation on the Earth's surface.

Table. Comparison of the efficiency of the methods of direct simulation and conjugate random walks (τ = 2)

$\gamma/2$	θ	Direct simulation				Conjugate random walks			
		Mean	D, %	t, s	S	Mean	D, %	t, s	S
5	0	3.25E-02	12.24	1.32	16.16	3.22E-02	2.58	2.2	5.67
5	20	7.07E-03	29.32	1.43	41.93	7.20E-03	4.55	2.25	10.23
5	40	4.39E-03	28.01	1.54	43.13	4.55E-03	4.96	2.25	11.16
5	60	3.63E-03	39.15	1.76	68.90	3.52E-03	8.19	2.31	18.91
5	80	2.61E-03	46.69	1.7	79.38	2.59E-03	18.44	2.31	42.59
10	0	6.85E-02	7.98	1.32	10.54	6.74E-02	2.46	2.25	5.52
10	20	2.81E-02	11.53	1.43	16.49	2.87E-02	4.37	2.26	9.87
10	40	1.78E-02	17.46	1.54	26.89	1.83E-02	5.39	2.25	12.13
10	60	1.40E-02	19.68	1.76	34.64	1.39E-02	7.01	2.3	16.11
10	80	1.01E-02	25.25	1.71	43.18	1.03E-02	16.43	2.31	37.95
45	0	4.30E-01	2.39	1.37	3.28	4.30E-01	3.34	2.36	7.88
45	20	4.14E-01	2.70	1.43	3.86	4.18E-01	3.74	2.42	9.05
45	40	3.52E-01	3.42	1.54	5.27	3.61E-01	5.99	2.42	14.49
45	60	2.67E-01	3.47	1.76	6.11	2.68E-01	6.48	2.42	15.69
45	80	1.90E-01	4.21	1.76	7.42	1.88E-01	14.64	2.47	36.16
90	0	6.56E-01	1.51	1.37	2.06	6.52E-01	4.49	2.63	11.80
90	20	6.55E-01	1.72	1.43	2.46	6.53E-01	4.50	2.69	12.09
90	40	6.34E-01	1.90	1.54	2.92	6.40E-01	4.96	2.69	13.33
90	60	5.60E-01	2.05	1.76	3.60	5.64E-01	7.67	2.69	20.63
90	80	3.79E-01	2.52	1.7	4.28	3.85E-01	13.27	2.74	36.37

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