Measurement of atmospheric turbulence parameters by vertically-scanning pulsed coherent lidar

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We present a method of estimation of atmospheric turbulence characteristics from the transversal structure function of velocity measured with a scanning (in vertical plane) pulsed coherent lidar. The altitude turbulence profiles are reconstructed from data of measurements of wind velocity by 2-μm lidar with use of the developed method of transversal structure function and the method of turbulence estimation from width of the Doppler spectrum. It is shown that, for conditions of moderate to strong turbulence, the methods give close results.

Introduction

Creation of coherent lidars, whose principle of operation is based on the Doppler effect, opened wide possibilities in the study of the dynamic atmospheric processes. To date, a number of methods have been developed for lidar measurements of the mean wind speed and direction[1] and such parameters of the atmospheric turbulence as kinetic turbulence energy, angular momentum, turbulent energy dissipation rate, and outer turbulence scale.[2–8]

Use of 2-μm pulsed coherent lidar[9] provides the possibility to perform efficient studies of eddies formed in flying aircraft condensation trail (contrail).[10] In this case, the eddies are measured by lidar, scanning in the vertical plane. In view of the fact that the main role in decay of aircraft-caused eddies is played by atmospheric turbulence, the turbulence information, extracted from these same lidar sensing data, is very useful in analysis of measurements of characteristics of eddies, generated by flying aircraft.

Based on the measurements by 2-μm pulsed coherent lidar, scanning in the vertical plane, Smalikho et al.[6] and Banakh et al.[7] reconstructed altitudinal turbulence profiles. Noteworthy, the pertinent characteristics were estimated by two methods: 1) from longitudinal structure function of wind velocity and 2) from the width of the Doppler spectrum.

Frehlich et al.[8] extended the method, proposed by Banakh et al.[3] for continuous lidars, to pulsed lidars and obtained estimates of the profiles of turbulence characteristics with high resolution in altitude from pulsed wind lidar data on transversal structure function of wind velocity. However, the observation geometry, used by Frehlich et al.[8] (scanning with laser beam in azimuth angle) precludes simultaneous lidar measurements of aircraft-produced eddies and turbulence.

In this paper, the turbulent wind characteristics are estimated from data of vertically scanning lidar by two methods: 1) from transversal structure function of wind velocity and 2) from width of the Doppler spectrum. Also, the results of reconstruction of altitudinal turbulence profiles with use of these approaches are compared.

Theory

We will consider the following scheme of pulsed lidar measurement. The ground-based lidar is scanning with the laser beam in the vertical plane, transmitting pulses to the atmosphere at different elevation angles φ. During measurements, we record the coherently detectable signal of backscattering at different ranges R from the lidar.

Using fast Fourier transform, for each shot of the pulse, from the measured signal we estimate its power spectra (Doppler spectra) with a certain step along the optical axis by applying the Gauss window for this.[6] Then, after averaging of the spectra, obtained for 25 neighboring shots of the pulse, first and second spectral moments are estimated. Using the Doppler relation $V = f\lambda/2$, where V is the measured velocity, f is the frequency, and λ is the optical wavelength, from the first spectral moment we can obtain an estimate of the radial speed (projection of the wind velocity on the axis of the sensing beam), which is described by the formula[4–6]:

$$\hat{V}_{\mathrm{D}}(R,\varphi) = V_{\mathrm{D}}(R,\varphi) + e(R,\varphi), \quad (1)$$

where

$$V_{\mathrm{D}}(R,\varphi) = \int_{-\infty}^{+\infty} \mathrm{d}z\, Q_{\mathrm{s}}(z) V_{\mathrm{r}}(R+z,\varphi) \quad (2)$$

is the radial wind speed averaged over the sensed volume; V_{r} is the radial speed at the point ($R + z$, φ) on the plane scanned by the laser beam;

$$Q_s(z) = \Delta z^{-1}\exp(-\pi z^2/\Delta z^2) \quad (3)$$

is the function of low-frequency spatial filter; Δz is the longitudinal size of the sensed volume (generally, $\Delta z \approx 94$ m in our experiments); and e is the random error of velocity estimate.

On account that the velocity V_D and the error e, like the errors $e(R, \varphi_1)$ and $e(R, \varphi_2)$, are statistically independent,[4,5] the ensemble average square of the difference of radial velocity fluctuations, measured at different angles φ_1 and φ_2, on the basis of Eq. (1) can be represented as

$$D_{\hat{V}}(R,\varphi_1,\varphi_2) = <[\hat{V}'_D(R,\varphi_1) - \hat{V}'_D(R,\varphi_2)]^2> = D_V(R,\varphi_1,\varphi_2) + 2\sigma_e^2, \quad (4)$$

where

$$D_V(R,\varphi_1,\varphi_2) = <[V'_D(R,\varphi_1) - V'_D(R,\varphi_2)]^2>;$$

$$\sigma_e^2 = <e^2>; \quad \hat{V}'_D = \hat{V}_D - <\hat{V}_D> \text{ and } V'_D = V_D - <V_D>.$$

Let us now assume that in the considered atmospheric layer at height $h = (R\sin\varphi_1 + R\sin\varphi_2)/2$ the turbulence is homogeneous and isotropic. Then, for small angular spacings $|\varphi_1 - \varphi_2| \ll \pi/2$ and under the condition that $L_V \ll R$, where L_V is the integral turbulence scale,[6] for the structure function D_V we can assume that the change of the scan angle from φ_1 to φ_2 is equivalent to displacement of sensing beam in the direction transversal to the optical axis by the distance $y = R|\varphi_1 - \varphi_2|$; therefore, D_V can be considered as a transversal velocity structure function of argument y, which has a more smooth dependence on the height h in passage from one homogeneous layer to another. Taking into account the assumptions above, from Eqs. (1)–(4) we obtain

$$D_V(h,y) = 2\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} d\kappa_z d\kappa_y S_{zz}(h,\kappa_z,\kappa_y) \times \exp(-2\pi\Delta z^2\kappa_z^2)[1 - \cos(2\pi\kappa_y y)], \quad (5)$$

where $S_{zz}(h,\kappa_z,\kappa_y)$ is the two-dimensional spectrum of wind velocity fluctuations. For the D_V calculations, we will use the spectral Karman model[11]:

$$S_{zz}(h,\kappa_z,\kappa_y) = \frac{1}{6\pi}\frac{\sigma_V^2(8.43L_V)^2}{[1+(8.43L_V)^2(\kappa_z^2+\kappa_y^2)]^{4/3}} \times \left[1 + \frac{8}{3}\frac{(8.43L_V\kappa_y)^2}{1+(8.43L_V)^2(\kappa_z^2+\kappa_y^2)}\right], \quad (6)$$

where the variance of wind velocity σ_V^2 and the integral turbulence scale L_V are functions of the height h. According to this model, the turbulent energy dissipation rate ε is defined by the formula[12]:

$$\varepsilon = 1.972\sigma_V^3/(C_K^{3/2}L_V), \quad (7)$$

where $C_K \approx 2$ is the Kolmogorov constant.

The method of estimation of the variance of wind velocity $\hat{\sigma}_V^2$ from lidar measurements is described in Ref. 6. The integral turbulence scale L_V is estimated from measured transversal structure function, normalized by the variance of wind velocity, i.e., from the formula

$$\tilde{D}_E(y_i) = [\hat{D}_{\hat{V}}(h,y_i) - 2\hat{\sigma}_e^2]/\hat{\sigma}_V^2,$$

by minimizing the functional

$$\rho(L_V) = \sum_{i=1}^{I}[\tilde{D}_E(y_i)/\tilde{D}_T(y_i,L_V) - 1]^2, \quad (8)$$

where $\tilde{D}_T(y_i,L_V) = D_V(h,y_i)/\sigma_V^2$ is calculated from formulas (5)–(6), $y_i = \Delta y i$. After $\hat{\sigma}_V^2$ and $\hat{L}_V$ are obtained, we can calculate the estimate of turbulent energy dissipation rate $\hat{\varepsilon}$ from formula (7).

Below, we compare the results of reconstruction of turbulence parameters, based on the above-mentioned method, with the results of the estimation of turbulence characteristics from Doppler spectrum width. The method of reconstruction of altitude turbulence profiles from measurements of Doppler spectrum width is described in Ref. 6 in detail.

Experiment

Measurements with 2-μm coherent lidar, owned by Institute of Atmospheric Physics DLR, Germany were performed near Istres, France on May 26 and 27, 2005 from 12:52 to 13:12 Local Time (May 26, 2005) and from 15:51 to 16:14 LT (May 27, 2005). Container with lidar was located approximately 700 m away from landing strip of Istres airport. The measurements were made under clear-sky conditions with quite high concentration of atmospheric aerosol, for which the signal-to-noise ratio was fairly acceptable up to the sensing range 2–3 km. The sensing beam scanned in the vertical plane with angular speed $|d\varphi/dt| = 2°/s$ alternatively up and down. Noteworthy, the minimum elevation angle φ was 0° and the maximum elevation angle was 30°; thus, a single scan lasted for 15 s. So, 75 full scans were taken in the first measurement day and 88 scans in the second day.

From the measured signals, the Doppler spectra were estimated for ranges R_k between 500 m and 2 km with step 30 m ($R_k = R_0 + \Delta R k$, where $R_0 = 500$ m, $\Delta R = 30$ m, $k = 1, 2, 3, \ldots, 50$). In the used lidar, the pulse repetition rate was 500 Hz. On account of the fact that the spectra were estimated using 25 pulses, for rate of scanning by the sensing beam 2°/s, the resolution of the estimated spectra in elevation angle was $\Delta\varphi = 0.1°$. Thus, for a given elevation angle, the measured spectra contain information on wind velocity in overlapping ($\Delta R < \Delta z$) volumes with longitudinal size $\Delta z = 94$ m and transversal sizes (in the scanning plane) $R\Delta\varphi$ from 0.87 (for $R = 500$ m) to 3.49 m (for $R = 2000$ m).

From the measured Doppler spectra, we obtained arrays of the estimates of the radial wind velocity $\hat{V}_D(R_k,\varphi_l,n)$ and square of the Doppler spectrum width $\hat{\sigma}_f^2(R_k,\varphi_l,n)$, defined as the second central spectral moment,[6] where $l = 1, 2, 3, \ldots, 300$, $n = 1, 2, 3, \ldots, N$, and N is the scan number (in the first and second measurement days, respectively, $N = 75$ and 88). Then, we determined the average values $<\hat{V}_D(R_k,\varphi_l)>$ and fluctuations

$$\hat{V}'_D(R_k,\varphi_l,n) = \hat{V}_D(R_k,\varphi_l,n) - <\hat{V}_D(R_k,\varphi_l)>$$

of radial velocity, measured by lidar.[6]

The altitude profiles of the statistical characteristics of wind velocity were calculated by averaging the velocity estimates at the points (R_k, φ_l), falling within the horizontal layer 40 m in thickness, with step 20 m in altitude. From $\hat{V}'_D(R_k,\varphi_l,n)$ fluctuations we estimated the variance $\hat{\sigma}_{\hat{V}}^2(h)$ and transversal structure function $\hat{D}_V(h,y_i)$ of the measured velocity, where $y_i = i\Delta y$, $\Delta y = 10$ m, and $i = 1, 2, \ldots, I$. Taking into account the thickness of the averaging layer, the maximum spacing of the points y_I did not exceed 50 m. Method of determination of the error $\hat{\sigma}_e^2(h)$ is explained in Refs. 5 and 6.

The difference $\hat{\sigma}_{\hat{V}}^2(h) - \hat{\sigma}_e^2(h)$ represents an estimate of the variance of wind velocity, averaged over the sensed volume with longitudinal size $\Delta z = 94$ m; hence, it accounts only for the turbulent wind variations with scales exceeding Δz. The fluctuations with the scales less than Δz make their contribution to the broadening of the Doppler spectrum. From the array $\hat{\sigma}_f^2(R_k,\varphi_l,n)$ it is possible to estimate this contribution by calculating the quantity

$$\hat{\sigma}_t^2(h) = (\lambda/2)^2 <[\hat{\sigma}_f^2 - \sigma_0^2 - \sigma_s^2]^2>_E,$$

where $<\ldots>_E$ means averaging of the data, measured in the considered layer; σ_0^2 is the square of the width of the Doppler spectrum, determined by the duration of the sensing pulse, and σ_s^2 is the Doppler spectrum broadening due to inhomogeneity of the mean wind.[6] For $\hat{\sigma}_{\hat{V}}^2(h) - \hat{\sigma}_e^2(h)$ and $\hat{\sigma}_t^2(h)$, the corresponding quantities are $\sigma_{\hat{V}}^2 = <[V'_D(R,\varphi)]^2>$ and

$$\sigma_t^2 = \int_{-\infty}^{+\infty} dz Q_s(z) <[V'_r(R+z,\varphi) - V'_D(R,\varphi)]^2> = \sigma_V^2 - \sigma_{\hat{V}}^2,$$

where σ_V^2 is the variance of wind velocity. Therefore, summing $\hat{\sigma}_{\hat{V}}^2(h) - \hat{\sigma}_e^2(h)$ and $\hat{\sigma}_t^2(h)$, we obtain an estimate of the variance of wind velocity $\hat{\sigma}_V^2(h)$. As shown in Ref. 6, from the ratio $\hat{\sigma}_t^2(h)/\hat{\sigma}_V^2(h)$ we obtain an estimate of the integral turbulence scale $\hat{L}_V(h)$, and then from formula (7) we can calculate the dissipation rate $\hat{\varepsilon}(h)$. Method of determination of $\hat{L}_V(h)$ and $\hat{\varepsilon}(h)$ estimates from the transversal structure function $\hat{D}_V(h,y_i)$ is given in the preceding section.

Results

From the measured arrays $<\hat{V}_D(R_k,\varphi_l)>/\cos(\varphi_l)$, representing the two-dimensional distribution of the projection of the average wind velocity onto the plane of scanning by the sensing beam, we reconstructed the altitude profiles of velocity projections up to height $h = 1$ km. The results of reconstruction of velocity profiles in the measurement days are presented in Figs. 1 and 2.

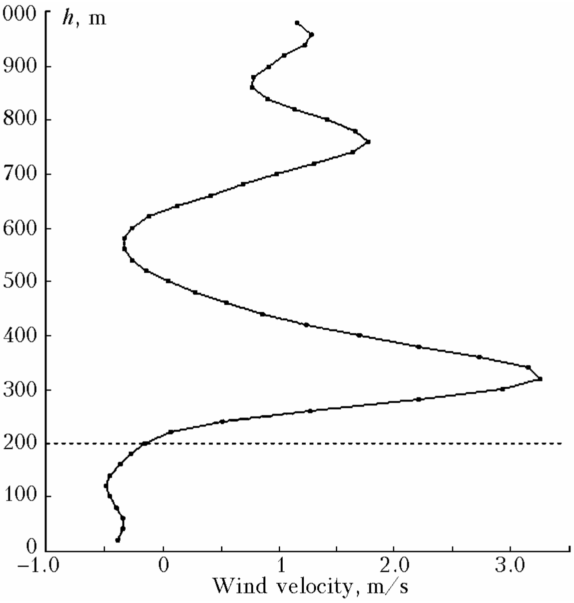

Fig. 1. The altitude profile of the projection of the average wind velocity onto the plane of scanning by the sensing beam, reconstructed from lidar data obtained on May 26, 2005.

Analysis of data shows that the average wind velocity field has layered structure. Lower part of the atmosphere up to heights $h \approx 200$ m (first day) and 400 m (second measurement day) represents the layer of turbulent mixing of the wind flow, above which there are the layers with large vertical gradients of wind velocity in passing from one layer to another.

As follows from Fig. 1, in the first measurement day above the mixing layer, there was quite narrow jet stream at height $h \approx 300$ m.

Because of averaging over the sensed volume, in the upper layers, with large gradients of the mean wind, the altitude profile of the measured velocity turns out to be smoothed. It seems to us that, without application of the special procedure of reconstruction of the average wind velocity field, taking into account the wind averaging over the sensed volume [see formula (2)], the turbulence characteristics for these layers cannot be estimated correctly. Therefore, in this paper we confine ourselves to consideration of reconstruction of altitude profiles of turbulence only in the mixing layer.

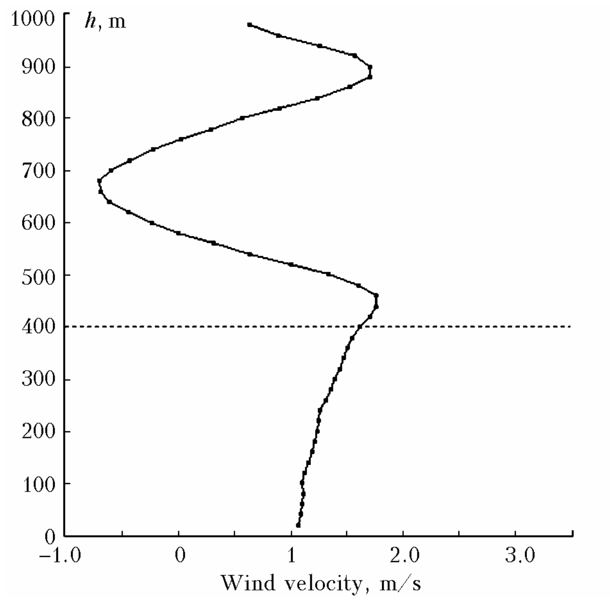

Fig. 2. Altitude profile of the projection of average wind velocity onto the plane of scanning by the sensing beam, reconstructed from lidar data obtained on May 27, 2005.

Figures 3 and 4 present the results of reconstruction of altitude profiles of turbulence characteristics.

From comparison of contributions of fluctuations of radial velocity $\hat{\sigma}_V^2(h)-\hat{\sigma}_e^2(h)$, averaged over the sensed volume, and turbulent broadening of the Doppler spectrum $\hat{\sigma}_t^2(h)$ to the variance of wind velocity $\hat{\sigma}_V^2(h)$ it is seen that, in the first measurement day (see Fig. 3*a*), $\hat{\sigma}_t^2(h)$ exceeds $\hat{\sigma}_V^2(h)-\hat{\sigma}_e^2(h)$ by approximately a factor of two. This situation is possible only when the integral turbulence scale L_V is much less than the longitudinal size of the sensed volume $\Delta z = 94$ m, i.e., when the fluctuations of radial velocity, measured by lidar, are considerably averaged out, and most of the turbulent energy goes to the broadening of the Doppler spectrum. The results of reconstruction of the profile of integral turbulence scale quite closely correspond to this statement (see Fig. 3*b*); they are obtained for L_V equaling 20–30 m. At the same time, the considered methods of L_V and ε estimation give close results.

On the contrary, in the second measurement day, the contribution of fluctuations of radial velocity, averaged over the sensed volume, to the estimated variance $\hat{\sigma}_V^2(h)$ is larger than the contribution of turbulent broadening of Doppler spectrum (Fig. 4*a*). In this case, the ratio $[\hat{\sigma}_V^2(h)-\hat{\sigma}_e^2(h)]/\hat{\sigma}_t^2(h)$ grows with altitude up to $h \approx 300$ m.

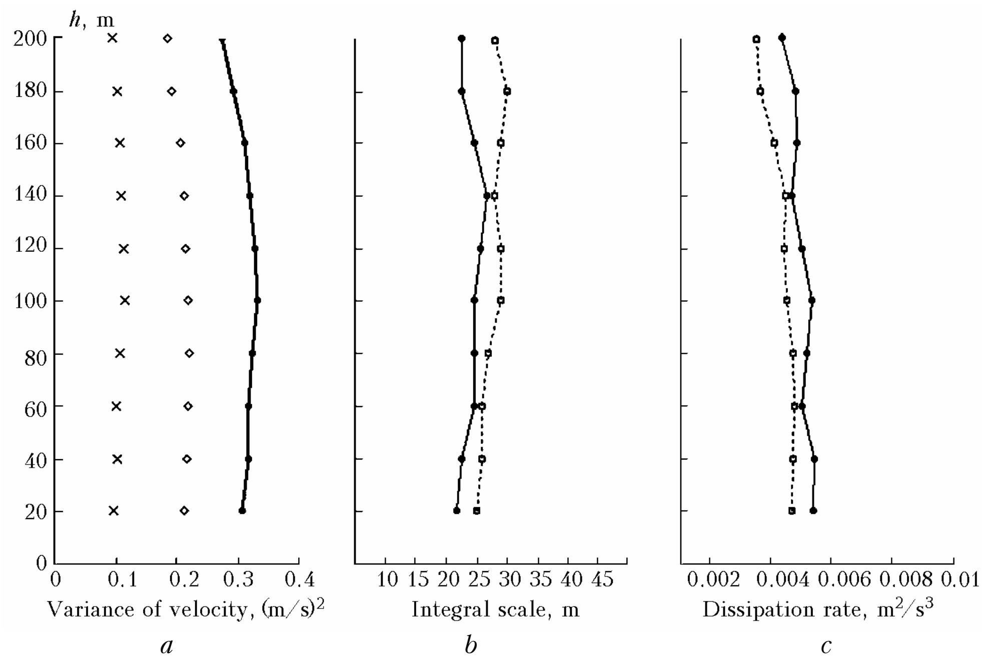

Fig. 3. Results of reconstruction of altitude profiles of turbulence characteristics from data obtained on May 26, 2005. Solid line shows the variance of wind velocity $\hat{\sigma}_V^2(h)$, and crosses and diamonds show $\hat{\sigma}_V^2(h)-\hat{\sigma}_e^2(h)$ and $\hat{\sigma}_t^2(h)$ respectively (*a*). Estimates of the integral turbulence scale (*b*) and turbulent energy dissipation rate (*c*), obtained from the measurements of Doppler spectrum width (dashed line) and transversal structure function (solid line).

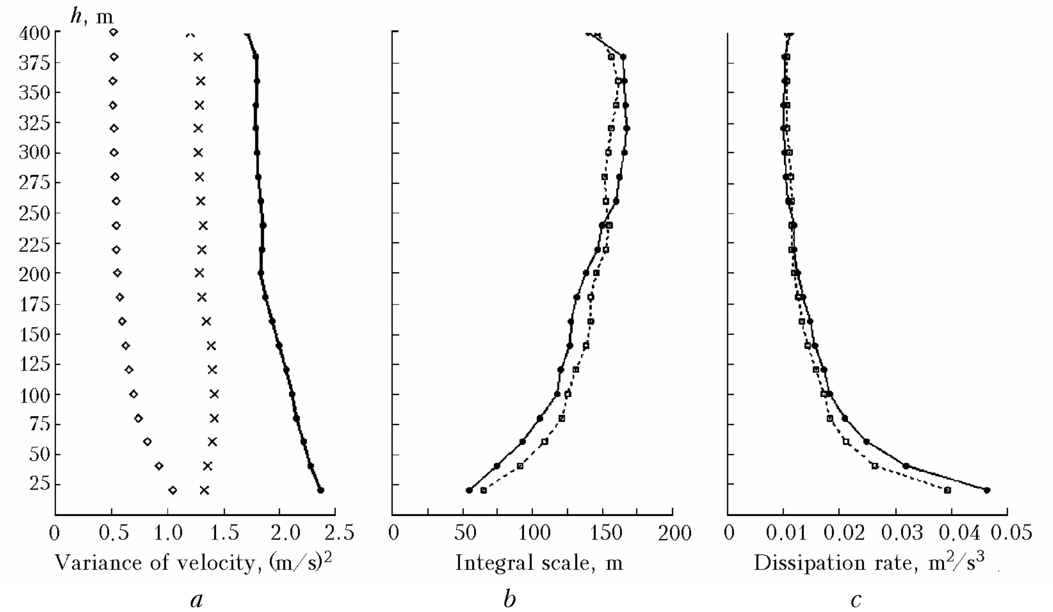

Fig. 4. The same as in Fig. 3, but for results of reconstruction of altitude profiles of turbulence characteristics from data obtained on May 27, 2005.

As expected, in this case the integral turbulence scale (Fig. 4*b*) considerably overestimates the data presented in Fig. 3*b*. From Fig. 4 it is seen that the methods of L_V and ε estimation from the Doppler spectrum width and from the transversal structure function of velocity give close results, as in the first measurement day.

Conclusion

We compared the measurements of integral turbulence scale and turbulent energy dissipation rate by the method (1) based on the estimation of turbulence characteristics from lidar-derived transversal structure function of velocity and (2) from the Doppler spectrum width. It is shown that the methods give close results. The data of Figs. 3*c* and 4*c* suggest that we performed the measurements in the mixing layer under conditions of moderate (on May 26, 2005) and strong (on May 27, 2005) turbulence.

Smalikho et al.[6] showed that the estimation of turbulent energy dissipation rate from the Doppler spectrum width, as compared with the method of ε estimation from the longitudinal structure function of velocity, may have large error under conditions of weak turbulence. In the case of statistically homogeneous wind field, the accuracy of estimation of turbulence characteristics from transversal structure function is, at lest, no worse than the estimation from the longitudinal counterpart. For the real atmosphere, the use of the transversal structure function should give higher (compared with longitudinal structure function) resolution of turbulence in altitude.

Thus, under weak-turbulence conditions the lidar measurements for estimation L_V and ε should preferably use the method, based on the application of the transversal structure function of velocity.

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