FEASIBILITY OF LOCALIZATION AND DIAGNOSTICS OF DYNAMIC INHOMOGENEITIES IN STRONGLY SCATTERING TURBID MEDIA

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The feasibility of localization and diagnostics of dynamic inhomogeneities in strongly scattering turbid media is investigated based on an analysis of the temporal autocorrelation functions of the multiply scattered radiation measured at the boundary of the medium. Conditions are determined when localization is possible. The accuracy of the examined localization method is estimated.

Turbid media that strongly scatter light and have inhomogeneities of the dielectric constant whose dimensions are comparable with the wavelength λ of the propagating radiation are nontransparent even when the light absorption by these media on the examined wavelength is insignificant.[1] This is connected, first, with the multiple light scattering by these media, and second, with sufficiently high probability of scattering at large angles in each elementary scattering event. Therefore, traditional optical methods are ineffective for the construction of the image of an object located within the bulk of the turbid medium.

At the same time, one of the important applied problems now is the localization and diagnostics of pure dynamic inhomogeneities located within the bulk of the turbid medium.[2] The dynamic inhomogeneity has the same light absorption and scattering coefficients as the surrounding medium. However, the particles located inside this inhomogeneity move differently than the particles of the surrounding medium. This permits one to use the temporal autocorrelation function

$$G_1(\mathbf{r}, \tau) = \langle E(\mathbf{r}, t)\, E^*(\mathbf{r}, t + \tau)\rangle$$

of the depolarized multiply scattered light measured at different points $\mathbf{r}$ of the boundary to be used for localization of dynamic inhomogeneity within the bulk of the turbid medium and the character and intensity of motion of the light scattering particles to be estimated.[3–5]

In the present paper, we consider a theoretical approach to a calculation of the temporal correlation function of the radiation multiply scattered in dynamically inhomogeneous turbid media and analyze in detail the feasibility of localization of dynamic inhomogeneities in these media.

1. PRINCIPLES OF THE THEORETICAL APPROACH

One of the most widely known approaches to a solution of the light scattering problem in turbid media is the diffusion approximation of the radiative transfer equation.[1] In this case, as shown in Refs. 6–7, the temporal correlation function $G_1(\mathbf{r}, \tau)$ of the field measured at the point $\mathbf{r}$ of the specimen boundary in the stationary case can be found as a solution to the diffusion equation

$$[\nabla^2 - \alpha^2(\tau)]\, G_1(\mathbf{r}, \tau) = \frac{F(\mathbf{r})}{D_p}, \qquad (1)$$

where $D_p = cl^*/3$ is the light diffusion coefficient, l^* is the transport mean free path of photons, c is the light speed in the medium, $F(\mathbf{r})$ describes the distribution of light sources in the medium, and a specific form of the function $\alpha(\tau)$ depends on the character of motion of scatterers in the medium. As shown in Refs. 8 and 9, in case of Brownian motion of particles in the medium characterized by the diffusion coefficient D_B, $\alpha^2(\tau) = 3\tau/(2\tau_0 l^{*2})$, where $\tau_0 = (4k^2 D_B)^{-1}$ and $k = 2\pi/\lambda$. For a laminar flow of scatterers, $\alpha^2(\tau) = 6(\tau/(\tau_f l^*))^2$, where the characteristic time τ_f depends on the geometry of the flow.[10,11]

Let us consider a semi-infinite medium occupying the half-space $x > 0$ with a latent inclusion (an object) of volume V_1 bounded by the surface S_1. The transverse size of this object in any direction is much larger than l^*. Of interest for us is the case in which the dynamics of scatterers inside the volume V_1 differs from their dynamics in the surrounding medium. In order to describe this difference, we introduce additional spatial dependence of the term $\alpha^2(\tau)$ in equation (1) in the form

$$\alpha^2(\tau) = \begin{cases} \alpha^2_{\text{in}}(\tau), & \mathbf{r} \in V_1, \\ \alpha^2_{\text{out}}(\tau), & \mathbf{r} \notin V_1. \end{cases} \qquad (2)$$

This approach is correct only for sufficiently large dynamic inhomogeneities when the diffusion approximation is true inside the inclusion itself. In this case, inside the object far from its boundaries $G_1(\mathbf{r}, t)$ is described by equation (1) with $\alpha^2(\tau) = \alpha^2_{\text{in}}(\tau)$ and in the remaining medium it is described

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by equation (1) with $\alpha^2(\tau)=\alpha^2_{\mathrm{out}}(\tau)$. To obtain the solution for the entire medium, equation (1) should be solved inside and outside of the object with the following boundary conditions[1,2] on the boundary of the medium S and on the boundary of the inhomogeneity S_1:

$$G_1^{\mathrm{out}}(\mathbf{r},\tau) - \frac{2}{3}\, l^* \,(\mathbf{n}\,\nabla\, G_1^{\mathrm{out}}(\mathbf{r},\tau)) = 0, \quad \mathbf{r}\in S, \tag{3}$$

$$G_1^{\mathrm{in}}(\mathbf{r},\tau) = G_1^{\mathrm{out}}(\mathbf{r},\tau)\,, \quad \mathbf{r}\in S_1, \tag{4}$$

$$D_p^{\mathrm{in}}\,(\mathbf{n}\,\nabla\, G_1^{\mathrm{in}}(\mathbf{r},t)) = D_p^{\mathrm{out}}\,(\mathbf{n}\,\nabla\, G_1^{\mathrm{out}}(\mathbf{r},t)),\ \mathbf{r}\in S_1. \tag{5}$$

Here, $\mathbf{n}$ is the unit vector orthogonal to the corresponding surface and directed inward, $G_1^{\mathrm{in,out}}(\mathbf{r},\tau)$ and $D_p^{\mathrm{in,out}}$ are the solutions of Eq. (1) and the light diffusion coefficients inside and outside of the volume V_1, respectively. Because here we are interested only in dynamic inhomogeneities, $D_p^{\mathrm{in}} = G_p^{\mathrm{out}}$ and condition (5) is simplified.

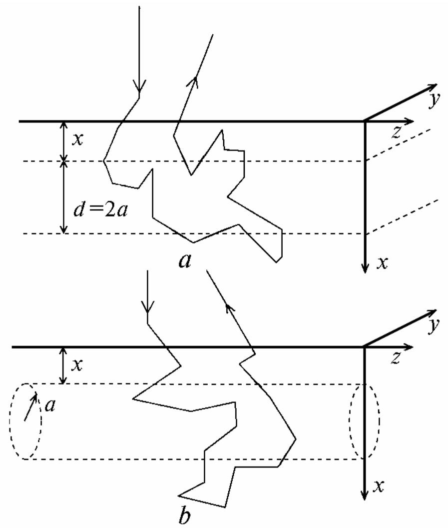

FIG. 1. Schemes of the examined dynamically inhomogeneous media.

Condition (3) can be approximated by the zero boundary condition for $G_1^{\mathrm{out}}(\mathbf{r},t)$ on the so-called extrapolated boundary[1,12] $x = -x_1 = -\Delta l^*$, where Δ depends on the conditions of scattering near the boundary.[1,11] For isotropic scattering and equal refractive indices of scattering and surrounding media from the Milne theory it was found that $\Delta = 0.7104$ (see Ref. 1). Different refractive indices can be considered by varying the position of the extrapolated boundary, that is, by changing Δ.

Let us restrict ourselves to the cases in which the dynamic inhomogeneity has the form of either a plane-parallel layer of thickness d (Fig. 1*a*), or a cylindrical capillary of diameter d (Fig. 1*b*) parallel to the surface of the semi-infinite medium $x = 0$ and consider a laminar flow of scattering particles with the Poiseuille profile of the velocity inside the inhomogeneity. In case of capillary, we direct the z axis along the capillary axis. Let us consider that the diffusion coefficients of the particles inside and outside of the volume of dynamic inhomogeneity are equal. In combination with the above-imposed requirements of constant light diffusion coefficient D_p in the entire volume this means that the same solution which fills the remaining volume of the specimen is pumped through the inhomogeneity. Different types of motion (translational and chaotic) inside the dynamic inhomogeneities can be considered independent: the velocity of an individual scatterer is taken to be equal to a sum of the velocities of its translational and chaotic motions. Then, because the scatterers in the medium surrounding the object take part only in the Brownian motion, we have[5]

$$\alpha^2_{\mathrm{in}}(\tau) = \frac{3\tau}{2\tau_0\, l^{*2}} + 6\left(\frac{\tau}{\tau_f\, l^*}\right)^2, \tag{6}$$

$$\alpha^2_{\mathrm{out}}(\tau) = \frac{3\tau}{2\tau_0\, l^{*2}}\,. \tag{7}$$

In accordance with Ref. 10–11, we consider that τ_f is determined by the rms value of the velocity gradient Γ_1 inside the inclusion

$$\tau_f = \sqrt{30}\,/\,(k\, l^*\, \Gamma_1). \tag{8}$$

For the Poiseuille profile of the flow velocity, Γ_1 is easily calculated.

Such problem formulation means that we consider additional loss of photon correlation in the region of flow localization depending only on the total length of photon trajectories inside the volume V_1 rather than on the specific forms of the photon trajectories. This is true only for $d \gg l^*$ and only for the photons that have sufficiently long trajectories. Therefore, in accordance with the results obtained by Bicout and Maynard,[11] the theory developed here will be true only for sufficiently short times τ, namely, for $\tau < \tau_0, \tau_f$.

Finally, let us assume that a plane monochromatic wave of unit amplitude is incident on the medium described above. Considering that the incident radiation is scattered for the first time after passage of the distance of the order of l^*, the source function in Eq. (1) is written as $F(\mathbf{r}) = \delta(x - x_0)$, where $x_0 \sim l^*$.

2. SOLUTIONS FOR THE SPECIFIC GEOMETRY

At first, we determine the expression for the normalized correlation function $g_1(\mathbf{r},\tau) = G_1(\mathbf{r},\tau)/G_1(\mathbf{r},0)$ of depolarized light scattered in the backward direction by a macrohomogeneous semi-infinite medium ($\tau_f \to \infty$). In this case, the solution of equation (1) with boundary condition (3) at $x = x_0$

(the plane where the scattered radiation leaves the medium) in the limit $\tau \ll \tau_0$ has the form

$$g_1^0(\tau) \equiv \frac{G_1(\tau)}{G_1(0)} = \exp\left\{-\gamma\sqrt{\frac{3\tau}{2\tau_0}}\right\}, \qquad (9)$$

where $\gamma = 1 + \Delta$ is the numerical constant of the order of two and the superscript “0B of the correlation function $g_1^0(\tau)$ is introduced to denote that it corresponds to macrohomogeneous medium. The result given by Eq. (9) was obtained elsewhere by other methods.[7,9] It agrees well with the experimental data reported in Refs. 6 and 7.

If the object located in the medium has the form of a plane-parallel layer of width d located at the depth x within the semi-infinite medium, the solution of equation (1) with boundary conditions (3)–(5) is[4]

$$g_1(\tau) \equiv \frac{P(\xi_1)}{P(\xi_2)}, \qquad (10)$$

where

$$P(\xi) = \alpha_{\text{in}}\,\alpha_{\text{out}}\exp(\alpha_{\text{out}}\,\xi) + \text{th}(\alpha_{\text{in}}\,d) \times$$
$$\times\,[\alpha_{\text{out}}^2\,\text{ch}(\alpha_{\text{out}}\,\xi) + \alpha_{\text{in}}^2\,\text{sh}(\alpha_{\text{out}}\,\xi)],$$
$$\xi_1 = x - x_0, \quad \xi_2 = x + x_1.$$

Finally, for the object of the form of cylindrical capillary of infinite length with the radius a located parallel to the z axis at the distance x from the surface of the semi-infinite medium, the solution can be written in the form $g_1(\mathbf{r}, \tau) = g_1^0(\tau) + g_1^{\text{scatt}}(\mathbf{r}, \tau)$, where $g_1^0(\tau)$ is given by Eq. (9) and the last term describes the influence of the object on the correlation function. Considering that $g = 0$ corresponds to the point located immediately in front of the capillary axis, we first solve equation (1) for the above-described geometry without boundary condition (3) at the boundary of the medium $x = 0$ (see Ref. 4)

$$g_1^{\text{scatt}}(x, y, \tau) = -\frac{h - x_0}{2\pi\, l^*}\sum_{n=1}^{\infty}\int_{-\pi/2}^{\pi/2}\frac{d\theta_s}{\cos\theta_s}\times$$
$$\times\, K_n\left(\alpha_{\text{out}}\frac{h - x_0}{\cos\theta_s}\right) K_n\left(\alpha_{\text{out}}\sqrt{h^2 + y^2}\right)\cos(n(\theta_s - \theta))\times$$
$$\times\left[\frac{\alpha_{\text{out}}\, I_n'(\alpha_{\text{out}}\, a)\, I_n(\alpha_{\text{in}}\, a) - \alpha_{\text{in}}\, I_n(\alpha_{\text{out}}\, a)\, I_n'(\alpha_{\text{in}}\, a)}{\alpha_{\text{out}}\, K_n'(\alpha_{\text{out}}\, a)\, I_n(\alpha_{\text{in}}\, a) - \alpha_{\text{in}}\, K_n(\alpha_{\text{out}}\, a)\, I_n'(\alpha_{\text{in}}\, a)}\right]. \qquad (11)$$

In this expression I_n and K_n are the modified Bessel functions, the prime denotes differentiation of corresponding function with respect to its argument, $h = x + a$ is the abscissa of the capillary axis, and $\theta = \arctan(y/h)$.

To satisfy the zero boundary condition in the plane $x = -x_1$ and thereby to obtain g_1^{scatt} for the geometry of interest to us, we use the method of images. To this end, we place the same capillary and radiation source with the opposite sign on the other side of the plane $x = -x_1$ so that the geometry of the problem becomes symmetric. Then the desired solution is written in the form of a sum of the terms (given by Eq. (11)) that correspond to two different capillaries and light sources. However, this result is approximate and describes satisfactorily the processes of light scattering only when the influence of the dynamic inhomogeneity on the temporal correlation function is weak.

To illustrate the influence of the object on the temporal correlation function more vividly and to estimate the feasibility of registration of this influence, we introduce the maximum deviation of the correlation function $g_1(\mathbf{r}, t)$, measured with the object, from $g_1^0(\tau)$ corresponding to the macrohomogeneous medium

$$\Delta g(\mathbf{r}) = \max_{0 < \tau < \infty} |g_1(\mathbf{r}, \tau) - g_1^0(\mathbf{r}, \tau)|. \qquad (12)$$

3. MAIN RESULTS AND THEIR DISCUSSION

Correlation functions of the backscattered light are shown in Fig. 2 for the cylindrical capillary at $g = 0$ (the solid lines) and for the plane-parallel layer (the dashed lines). These curves are shown for different depths x of the inhomogeneity inside the turbid medium.

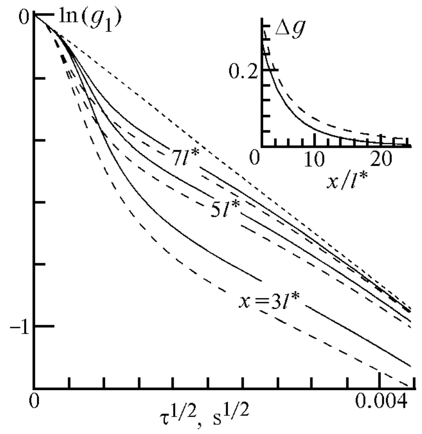

FIG. 2. Normalized temporal autocorrelation functions of the backscattered light for the Poiseuille flow in the plane-parallel layer (the dashed lines) and cylindrical capillary (the solid lines) located in a semi-infinite medium ($y = 0$, $d = 22l^*$, $\tau_0 = 2.66 \cdot 10^{-4}$ s, $\tau_f = 4.6 \cdot 10^{-6}$ s, $\gamma = 1 + \Delta = 2.8$). The depths x of objects are indicated in the breaks of corresponding curves. The dashed straight line corresponds to the macrohomogeneous semi-infinite medium. In the insert, Δg is shown as a function of x.

As can be seen from Fig. 2, the influence of the dynamic inhomogeneity is significant only for the finite range of variations of the time delay τ. For sufficiently small and large τ the correlation function is determined by the Brownian motion of particles which is the same inside and outside of the dynamic inhomogeneity. For small values of τ this is connected with the different temporal behavior of the terms describing decorrelation caused by the Brownian and translational motion of the scatterers. For the Brownian motion, the decorrelation is $\sim \sqrt{\tau/\tau_0}$ (see Eq. (9)), whereas for translational motion it is $\sim \tau/\tau_f$. It is evident that for sufficiently short times, namely, for $\tau < \tau_f^2/\tau_0$ the first contribution is larger than the second contribution, and the influence of the translational motion of the scatterers on the correlation function is insignificant.

For large values of τ the behavior of the correlation function is explaned on the basis of the correspondence between the long correlation time and the short lengths of trajectories of multiply scattered photons.[7] Really, because decorrelation due to a single scattering event decreases with the increase of τ for the Brownian and translational motions of scatterers, at small τ the noticeable contribution to radiation decorrelation comes only from the photons that undergo a large number of scattering events and hence have the longest trajectories. For long τ, these photons are completely decorrelated and the behavior of the correlation function is determined by the photons that undergo a small number of scattering events and therefore have comparatively short trajectories. Thus, with the increase of τ the behavior of $g_1(\tau)$ is determined by the photons that have increasingly shorter trajectories. Therefore, at long τ the photons that determine the behavior of $g_1(\tau)$ simply do not reach the object located at the finite depth inside the medium. As a consequence, its influence on the correlation function is small for long τ.

In Fig. 2 the maximum deviation Δg is also shown determined by formula (12) as a function of the depth x of the dynamic inhomogeneity. For small x this parameter is sufficiently large. However, with the increase of x it decreases fast and for $x > (15–20)l^*$ it becomes less than 1% of g_1.

Finally, when the dynamic inhomogeneity has the capillary form, Fig. 3 shows the results of calculation of Δg as a function of the coordinate y for fixed x. The width of the curve $\Delta g(y)$ at half-maximum is equal to the capillary diameter shown in this figure by the horizontal bar, to the accuracy of $(1–2)l^*$. Thus, the temporal correlation function can be used to estimate satisfactorily the object size inside the turbid medium.

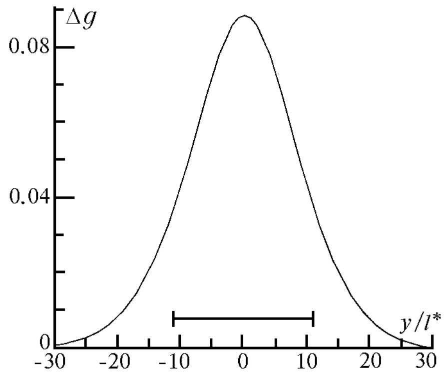

FIG. y. The dependence of Δg is shown on the coordinate y for fixed depth $x = 7l^*$ of the cylindrical capillary and the rest of the parameters being the same as in Fig. 2. The horizontal bar shows the diameter of the capillary.

4. CONCLUSION

In this paper, the feasibility is demonstrated of obtaining the information about dynamic inhomogeneities within the bulk of the medium by way of analysis of the correlation properties of the scattered radiation. On the basis of our analysis, the following conclusions can be drawn:

1. In turbid media the feasibility exists of determination of the character (Brownian or translational) of scatterers' motion inside the dynamic inhomogeneity as well as its intensity based on the form of temporal correlation function of multiply scattered light measured at the boundary of the medium.

2. An analysis of the temporal correlation functions allows one to determine the position and size of dynamic inhomogeneities to the accuracy of 1–2 transport mean free path's l^*.

3. The dynamic inhomogeneity has significant influence on the temporal autocorrelation function of backscattered light when the distance from it to the boundary of the medium does not exceed $x = (15–20)l^*$.

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