SOME ASPECTS OF THE THEORY OF TOMOGRAPHIC SOUNDING OF SCATTERING MEDIA

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The problem is considered of reconstructing spatial distribution of the extinction coefficient of a medium when using the bistatic tomographic optical arrangement of sounding from moving and stationary platforms. An analytical solution is obtained for the extinction coefficient profiles and the transmission. This solution does not require calculation of the logarithmic derivatives of scattering signals recorded with some error. It is shown that the error in estimates of these optical characteristics is almost solely determined by the signal measurement error, because the method is tolerant to the destabilizing factors like, for example, multiple scattering. Implementing this method into practice could allow an essential change in the metrological and operation capabilities of different tomographic devices.

The tomographic methods are now widely used in many areas of science, engineering, and technology. The tomographic sounding of scattering media (the atmosphere, water, clouds, and others) is among the possible applications.

The tomographic approach relying on the possibility of extracting information about the objects under study from the echo signals coming from different directions has been proposed in Ref. 1 as applied to observations with an airborne lidar. The finite-difference algorithm used for processing echo signals required *a priori* boundary conditions for the parameters to be retrieved. Later on analytical solutions have been derived of the integral equations of the airborne lidar tomographic sounding of the atmosphere for two- and three-beam schemes.[2,3] The analytical solutions to the problem of tomographic sounding have also been obtained for the schemes not involving a moving measuring system.[4,5] Those use angular scanning over some domain with two or three lidars.

In contrast to traditional schemes of laser sounding of the atmosphere and water media, the tomographic technique of reconstructing spatial distribution of the extinction coefficient does not require *a priori* information on the functional relationships among the optical properties or about their spatial structure. However, the basic point of the schemes proposed for tomographic sounding[2–6] is calculation of the logarithmic derivatives of the return signals. Taking into account discrete nature of actual experimental data, as well as the fact that the problem of numerical differentiation of recorded signals is ill-posed, practical implementation of this approach can be considered as inefficient, because small measurement errors may lead to large errors in the solution, i.e., in the extinction coefficient.[6] This paper presents analytical solution to the problem of tomographic sounding of scattering media (the atmosphere, water medium) both for moving measuring platform and angular scanning from a stationary one. This solution does not require calculation of the logarithmic derivatives of the recorded signals.

Let us consider solution to the problem within the framework of the bistatic optical arrangement of laser detection and ranging using the tomographic sounding with angular scanning as an example. Block-diagram of sounding is shown in the Fig. 1.

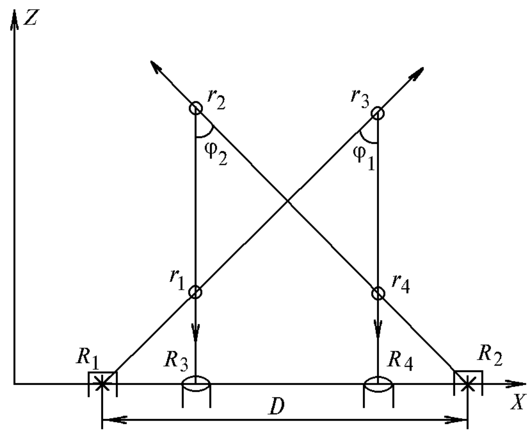

FIG. 1. Block-diagram of the bistatic tomographic sounding.

The radiation sources *1* and *2* are located at the points R_1 and R_2 at a distance D between them. The

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sources *1* and *2* emit sounding signals along the paths that cross at some point. The receivers *3* and *4* are at the points R_3 and R_4 on the line passing through the points R_1 and R_2. The optical axes of the receivers intersect the directions of sounding signals from the sources *1* and *2* at the points r_i, $i = 1, \ldots, 4$.

Echo signals at the points R_3 and R_4 from the scattering volumes with coordinates r_1 and r_3 (sounding signal comes from the source *1*) are determined by the expression[7]:

$$S(R_1, r_1, R_3) = A_1 P_{01}\,\sigma_{\varphi_1}(r_1)\,T(R_1, r_1)\,T(R_3, r_1),$$

$$S(R_1, r_3, R_4) = A_2 P_{01}\,\sigma_{\varphi_1}(r_3)\,T(R_1, r_3)\,T(R_4, r_3), \qquad (1)$$

where $S(R_1, r_1, R_3) = P(R_1, r_1, R_3)\,|r_1 - R_3|^2$; $S(R_1, r_3, R_4) = P(R_1, r_3, R_4)\,|r_3 - R_4|^2$; $|r_1 - R_3|$, $|r_3 - R_4|$ are the distances from the receivers to the scattering volumes; A_1 and A_2 are the instrumental constants of the receivers *3* and *4*; P_{01} is the power of a signal emitted by the source *1*; $P(R_1, r_i, R_j)$ is the power of signals received by the receivers *3* and *4*; $\sigma_{\varphi_1}(r_i)$ are the scattering coefficients at an angle φ_1 at the points r_1 and r_3:

$$T(R_1, r_3) = T(R_1, r_1)\,T(r_1, r_3)\,,$$

$$T(R_4, r_3) = T(R_4, r_4)\,T(r_4, r_3)\,,$$

$$T(R_1, r_1) = \exp\left\{-\int_{R_1}^{r_1} \varepsilon(r)\,\mathrm{d}r\right\},$$

$$T(r_1, r_3) = \exp\left\{-\int_{r_1}^{r_3} \varepsilon(r)\,\mathrm{d}r\right\},$$

$$T(R_4, r_4) = \exp\left\{-\int_{R_4}^{r_4} \varepsilon(r)\,\mathrm{d}r\right\},$$

$$T(r_4, r_3) = \exp\left\{-\int_{r_4}^{r_3} \varepsilon(r)\,\mathrm{d}r\right\};$$

$\varepsilon(r)$ is the extinction coefficient at the point r.

Similar expressions can be written for the echo signals recorded at the points R_4 and R_3 from the scattering volumes with the coordinates r_4 and r_2: initiated by the sounding radiation from source *2*

$$S(R_2, r_4, R_4) = A_2 P_{02}\,\sigma_{\varphi_2}(r_4)\,T(R_2, r_4)\,T(R_4, r_4),$$

$$S(R_2, r_2, R_3) = A_1 P_{02}\,\sigma_{\varphi_2}(r_2)\,T(R_2, r_2)\,T(R_3, r_2), \qquad (2)$$

where P_{02} is the power of a signal from source *2*; $\sigma_{\varphi_2}(r_i)$ are the scattering coefficients at an angle φ_2 at the points r_4 and r_2:

$$T(R_2, r_2) = T(R_2, r_4)\,T(r_4, r_2)\,,$$

$$T(R_3, r_2) = T(R_3, r_1)\,T(r_1, r_2)\,,$$

$$T(R_2, r_4) = \exp\left\{-\int_{R_2}^{r_4} \varepsilon(r)\,\mathrm{d}r\right\},$$

$$T(r_4, r_2) = \exp\left\{-\int_{r_4}^{r_2} \varepsilon(r)\,\mathrm{d}r\right\},$$

$$T(R_3, r_1) = \exp\left\{-\int_{R_3}^{r_1} \varepsilon(r)\,\mathrm{d}r\right\},$$

$$T(r_1, r_2) = \exp\left\{-\int_{r_1}^{r_2} \varepsilon(r)\,\mathrm{d}r\right\}.$$

The common solution of the system of equations (1) and (2) gives the following expression:

$$\frac{S(R_1, r_3, R_4)\,S(R_2, r_2, R_3)}{S(R_1, r_1, R_3)\,S(R_2, r_4, R_4)} = \frac{\sigma_{\varphi_1}(r_3)\,\sigma_{\varphi_2}(r_2)}{\sigma_{\varphi_1}(r_1)\,\sigma_{\varphi_2}(r_4)} \times$$

$$\times\, T(r_4, r_3)\,T(r_1, r_2)\,T(r_1, r_3)\,T(r_4, r_2). \qquad (3)$$

If the scattering volume at the points r_i, $i = 1, \ldots, 4$, is small, then the condition of homogeneity usually holds within it. In this case, Eq. (3) can be reduced to the following form:

$$\bar{\varepsilon}(r) = -\frac{1}{\alpha}\ln\frac{S(R_1, r_3, R_4)\,S(R_2, r_2, R_3)}{S(R_1, r_1, R_3)\,S(R_2, r_4, R_4)}\,, \qquad (4)$$

where $\alpha = |r_2 - r_1| + |r_3 - r_1| + |r_2 - r_4| + |r_3 - r_4|$.

As seen from Eq. (4), the expression for the extinction coefficient contains neither instrumental functions of emitting and receiving units nor energy of the radiation sources. It also contains no parameters describing the radiation extinction along a path from a source to a receiver. This means that the proposed method is tolerant to instabilities in the optical and electronic channels of the system, cleanness of the optical elements, and changes in the state of a medium between the volume sounded and the measuring system. Besides, the above-said means that this tomographic sounding scheme does not require calibration.

The proposed algorithm of tomographic sounding is also tolerant to the contribution from multiple scattering, because measured echo signals come from points that are spaced not so far from each other. Contributions from multiple scattering C_i for such points (differing by $\Delta r \to 0$) are almost the same. Thus, the algorithm of the form (4) can be written as

$$\varepsilon(r) = -\frac{1}{\alpha}\ln\frac{C_1\,S_1\,C_2\,S_2}{C_3\,S_3\,C_4\,S_4} = -\frac{1}{\alpha}\ln\frac{S_1\,S_2}{S_3\,S_4}\,,$$

because $C_1 \approx C_2 \approx C_3 \approx C_4$. This means that it is tolerant to the contribution from multiple scattering.

The analytical expression for the error of the proposed scheme of tomographic sounding can be easily derived. Having applied the method of finite increments[8] to Eq. (4), we obtain (when $\delta S_1 = \delta S_2 = \delta S_3 = \delta S_4$)

$$\delta\varepsilon = \Delta\varepsilon/\varepsilon = \frac{4}{\varepsilon}\,\delta S + \delta\alpha\,. \qquad (5)$$

It follows from Eq. (5) that the error in estimation of the extinction coefficient for homogeneous areas of a medium (at $\delta\alpha = 0$, i.e., if $\Delta r = r_J - r_i$ is taken as known precisely) is only determined by the measurement error in the scattered signals. For inhomogeneous areas of a medium under study, the expression for the error (derived from Eq. (3) using the method of finite increments)

$$\delta\varepsilon = 4(\delta S/\varepsilon + \delta\sigma_\varphi) + \delta\alpha \quad (6)$$

includes the components $\delta\sigma_\varphi$ due to a spread in the scattering coefficient at the points r_i within the studied volume (it is taken that $\delta\sigma_\varphi(r_1) = \delta\sigma_\varphi(r_2) = \delta\sigma_\varphi(r_3) = \delta\sigma_\varphi(r_4)$).

The main advantage of the proposed scheme is that there is no need to calculate the logarithmic derivatives of scattering signals. This circumstance makes the problem of bistatic tomographic sounding well-posed and excludes the necessity of using regularizing algorithms and the apparatus of the method of splines. Studies with the use of this method practically do not need a development of computational algorithms for solution of inverse problems with regard for discrete character of actual measured data, which is needed in lidar tomographic sounding by return signals.

If the direction of sounding signals is changed at the points R_1 and R_2, then other scattering volume (r'_1, r'_2, r'_3, r'_4) is studied. Another way to change the point to be sounded is to change the angles of optical axes of the receivers at the points R_3 and R_4.

Bistatic tomographic sounding using the approach proposed can also be conducted from moving platforms. However, taking into account that parameter D is limited for known carriers (airplanes, helicopters, etc.), it is necessary to use narrow beams of the sounding and scattered radiation. The most suitable object for investigation with a moving carrier is the water medium. Because of high density of water, the parameter D for it can be from several tens of centimeters to one meter, and it can be easily realized at any object both stationary and moving along a water surface. Actually, creation of a bistatic tomographic remote meter of transparency will eliminate the need to use towed meters of transparency.

Thus, we have considered the analytical solution to the problem of bistatic tomographic sounding for both stationary and moving emitting and receiving devices. In contrast to traditional tomographic schemes for sounding of scattering media, this scheme does not require calculation of logarithmic derivatives of scattering signals recorded with some error. For this method, the error in estimation of the extinction coefficients and transparency practically depends only on the error in the signal recording. This is achieved due to tolerance of the method to destabilizing factors occurring in emitting and receiving devices, as well as in the ambient medium. The method is also tolerant to contributions from multiple scattering. Implementation of this method in practice could allow a considerable change in metrological and operation capabilities of different tomographic devices.

REFERENCES

1. J.A. Weinman, Appl. Opt. **23**, No. 21, 3882 (1984).

2. M.M. Kugeiko, Inventor's Certificate No. 1798664, Bull. No. 8, 1993.

3. V.V. Veretennikov, Atm. Opt. **4**, No. 6, 466–473 (1991).

4. V.V. Veretennikov, Atm. Opt. **4**, No. 7, 489–492 (1991).

5. N.M. Sergeev, M.M. Kugeiko, and D.A. Ashkinadze, Inventor's Certificate No. 38, 1982.

6. Yu.E. Voskoboinikov, N.G. Preobrazhenskii, and A.I. Sedel'nikov, *Mathematical Processing of Experimental Results in the Molecular Gas Dynamics* (Nauka, Novosibirsk, 1984), 240 pp.

7. Yu.S. Balin, I.V. Samokhvalov, G.G. Matvienko, et al., in: *Problems of Remote Sounding of the Atmosphere* (Nauka, Novosibirsk, 1975), pp. 23–29.

8. B.M. Shchigolev, *Mathematical Processing of Observations* (Nauka, Moscow, 1969), 344 pp.