ACCURACY OF COMPENSATION FOR THE WAVE–FRONT PHASE DISTORTIONS USING AN ADAPTIVE MIRROR WITH DIFFERENT RESPONSE FUNCTIONS

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Accuracy of compensation is estimated for the wave–front distortions using a solid adaptive mirror with three response functions. Some recommendations on the choice of parameters of the considered response functions are given which allow one to improve the efficiency of the adaptive optical system functioning.

The efficiency of adaptive optical systems (AOS) is governed by a variety of factors one of which is the accuracy of compensation for phase distortions by an actuator of the system, i.e., an adaptive mirror. In this case the attainment of a required quality of an optical system depends on a spatiotemporal structure of the phase distortions to be compensated and a bandwidth of actual deformations of the adaptive mirror surface. In this connection the development of the theory of adaptive mirrors is an important problem.

Vast literature material is devoted to different aspects of constructing adaptive mirrors.[1–4] The most important of them is the choice of a type and parameters of the response function of a solid deformable adaptive mirror since it is closely related to the problem of correct approximation of a wave front and, consequently, the possibility of selecting minimum number of independent channels of mirror control required for reaching the desired accuracy of correction.

The goal of this paper is to find the dependence of the error of phase distortions approximation with a solid deformable adaptive mirror on statistical characteristics of wave–front fluctuations and parameters of different response functions.

In the calculations it was assumed that the errors in the wave–front approximation dominated over the errors of measurements and dynamic errors of tracking and therefore they are in fact the errors of compensation for the wave–front distortions with an adaptive mirror.

The distribution of the adaptive–mirror actuators is assumed to be within the aperture of the diameter D, at the nodes of a square grid with a step $\rho = D/10$. It should be noted that the choice of geometry of the actuator positions is an important and complicated problem and is not considered in this paper. The proposed distribution of actuators can be supported by the fact that the use of an adaptive mirror with a "square–nest" packing of actuators, in contrast to, e.g., a hexagonal one, requires simpler algorithms for data processing and control and, hence, allows one to simplify the construction of the wave–front sensors as well as of special calculators of an AOS and, therefore, of the entire system.

Three types of response functions of an adaptive mirror are studied:

– a pyramidal

$$f_i(\mathbf{r})=\begin{cases}\left(1-\dfrac{|x-x_i|}{\rho}\right)\left(1-\dfrac{|y-y_i|}{\rho}\right), & |x-x_i|<\rho,\ |y-y_i|<\rho,\\ 0\ , & |x-x_i|\ge\rho,\ |y-y_i|\ge\rho,\end{cases}$$

where $\mathbf{r} = \{x, y\}$ is the radius–vector in the mirror plane, x_i and y_i are the coordinates of the ith actuator;

– Gaussian

$$f_i(\mathbf{r}) = \exp\left[-\frac{(x-x_i)^2+(y-y_i)^2}{S_0^2}\right]$$

where S_0 is the radius of a deformed surface;

– and a function of the type

$$f_i(\mathbf{r}) = \exp\left[-\frac{(x-x_i)^2+(y-y_i)^2}{f_0^2(x, y)}\right],$$

where

$$f_0(x, y) = \begin{cases} S_0/\cos\arctan\left|\dfrac{x-x_i}{y-y_i}\right|, & \left|\dfrac{x-x_i}{y-y_i}\right| \le 1\ ,\\ S_0/\cos\operatorname{arccot}\left|\dfrac{x-x_i}{y-y_i}\right|, & \left|\dfrac{x-x_i}{y-y_i}\right| > 1\ ,\end{cases}$$

below called "anisotropic".

Let the distribution of a light wave phase incident onto an adaptive mirror be $\Phi(\mathbf{r})$. Then applying a single control action to the ith actuator the correction in the plane of coordinates $\mathbf{r}$ is accomplished which is determined by the response function of this actuator $f_i(\mathbf{r})$. For a linear mirror, when the control action is applied to all of the actuators the resulting function of the phase correction is

$$\tilde{\Phi}(\mathbf{r}) = \sum_{i=1}^{N} a_i\, f_i(\mathbf{r})\ , \qquad (1)$$

where a_i is the amplitude of a signal at the ith actuator control.

Then the phase distribution in the reflected wave, i.e., the error of compensation, can be defined as

$$\Delta\Phi(\mathbf{r}) = \Phi(\mathbf{r}) - \tilde{\Phi}(\mathbf{r})\ . \qquad (2)$$

For the best system of response functions the rms error of the phase approximation $\Phi(\mathbf{r})$ over the aperture S should be minimum, i.e., it is necessary to satisfy the condition

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$$\varepsilon = \left\{ \frac{1}{S} \iint_{\Omega} \left[\Phi(\mathbf{r}) - \sum_{i=1}^{N} a_i f_i(\mathbf{r}) \right]^2 \mathrm{d}^2 r \right\}^{1/2} \to \min .$$

Since the wave–front distortions are often described by a set of known phase distributions, in particular, Zernike polynomials,[5] first the quality of functioning of the adaptive mirror with different response functions has been studied in statistical correction of the phase distortions represented by Zernike polynomials. For this purpose the problem of the best, from the rms viewpoint, approximation of the first four Zernike polynomials $Z_j(\mathbf{r})$ was solved. The error of approximation was calculated using numerical methods by the formula

$$\varepsilon_j = \left(\frac{1}{S} \iint_{\Omega} \left(Z_j(\mathbf{r}) - \sum_{i=1}^{N} a_i f_i(\mathbf{r}) \right)^2 \mathrm{d}^2 r \right)^{1/2} . \tag{3}$$

The results of calculating the correction errors of the first four Zernike polynomials ε_j using an adaptive mirror with the above–considered response functions are listed in Table I (for a Gaussian and "anisotropic" response functions the values of ε_j were calculated as a function of the ratio of the radius of surface deformation S_0 to the distance ρ between actuators).

TABLE I.

Type of Zernike polynomial	Error of correction ε_j								
	Pyramidal response function	Gaussian response function S_0/ρ				Anisotropic response function S_0/ρ			
		0.4	0.5	0.6	0.7	0.4	0.5	0.6	0.7
Tilt	0.15	0.45	0.24	0.10	0.38	0.25	0.13	0.42	0.80
Defocusing	0.32	1.0	0.50	0.25	0.70	0.51	0.28	0.81	1.31
Astigmatism	0.25	0.64	0.36	0.15	0.48	0.38	0.20	0.55	0.83
Coma	0.29	0.8	0.44	0.19	0.56	0.48	0.24	0.67	0.96

The quality of approximation of several types of wave fronts (plane, slant, and spherical) with the Gaussian and "anisotropic" response functions vs the ratio S_0/ρ was also studied using relation (3). Depicted in Fig. 1 are the plots of approximation errors ε of the results of wave fronts of the Gaussian (solid lines) and "anisotropic" (dashed lines) response functions vs the ratio S_0/ρ.

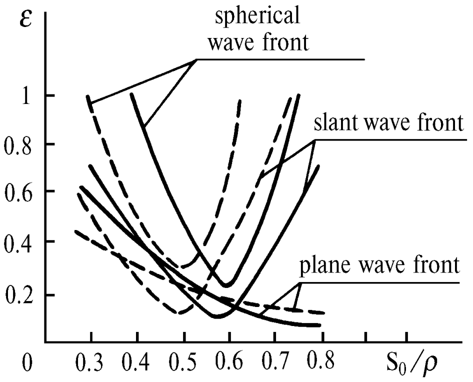

FIG. 1.

As can be seen from the analysis of this figure, the errors of approximation when $S_0/\rho < 0.6$ are smaller for "anisotropic" response function and when $S_0/\rho > 0.6$ for a Gaussian one. Moreover it is possible to conclude that the minimum error of approximation of the slant and spherical wave fronts for the Gaussian response function is for $S_0/\rho = 0.6$ and that for the "anisotropic" response function – for $S_0/\rho = 0.5$. In the general case the Gaussian response function enables one to obtain minimum error of approximation.

Under real conditions the wave front of a light wave passed through the atmosphere is a random field since it is affected by different random factors, e.g., atmospheric turbulence, therefore, the approximation of such a field with the help of a limited number of Zernike polynomials or some other system of orthogonal functions does not provide an adequate profile of an adaptive mirror. Therefore it is also expedient to elucidate the ability of the considered response functions of the corrector to compensate for random phase distortions. Since in this case we deal with an infinity of random functions and it is necessary to compensate for different distortions, a statistical approach must be used in the analysis.

In the studies the adaptive mirror is assumed to be a filter of spatial frequencies.[2,4] In this case the residual phase error of correction is accounted for by a limited bandwidth of the transmission of this filter and has the form

$$\Delta\Phi(\mathbf{r}) = \Phi(\mathbf{r}) - \frac{\int_{-\infty}^{\infty} \Phi(\mathbf{r}') f(\mathbf{r} - \mathbf{r}') \, \mathrm{d}^2\mathbf{r}'}{\int_{-\infty}^{\infty} f(\mathbf{r}) \, \mathrm{d}^2\mathbf{r}} . \tag{4}$$

Then the variance of the residual phase error can be found by integrating over spatial frequencies

$$d = \int_{-\infty}^{\infty} \Phi(\mathrm{k}) \left| 1 - \frac{(2\pi)^2 f_j(\mathrm{k})}{\int_{-\infty}^{\infty} f(\mathbf{r}) \mathrm{d}^2 r} \right|^2 \mathrm{d}^2\mathrm{k} , \tag{5}$$

where $\Phi(\kappa)$ is the spectral density of phase fluctuations, $f_\kappa(\kappa)$ is the Fourier transform of the response function, and κ is the vector of spatial frequencies.

Equation (5) was integrated using numerical methods for a plane wave where spectral density of fluctuations caused by wave propagation in a turbulent atmosphere is[6]

$$\Phi(\mathrm{k}) \approx 0.123 \, r_0^{-5/3} \mathrm{k}^{-11/3} ,$$

where r_0 is the radius of Fried coherence.

Based on the results of integration the following relations for variance of the residual phase error were derived:

– for a pyramidal response function

$$d = 2\pi \int\limits_0^\infty \Phi(k)\,[1 - J_0(k\rho) - 3\,J_2(k\rho)]^2 k\,dk = 0.31(\rho/r_0)^{5/3}\,,$$

where $J_0(\kappa\rho)$ and $J_2(\kappa\rho)$ are the Bessel functions of the corresponding order;

– for a Gaussian response function

$$d = 2\pi \int\limits_0^\infty \Phi(k)\left(1 - \exp\left(\frac{k^2 S_0^2}{4}\right)\right)^2 k\,dk = \alpha(S_0/r_0)^{5/3}\,;$$

– for an "anisotropic" response function

$$d = 2\pi \int\limits_0^\infty \Phi(k)\left(1 - \exp\left(\frac{k^2 f_0^2(k)}{4}\right)\right)^2 k\,dk = \beta(S_0/r_0)^{5/3}\,.$$

The values of the coefficients α and β for different ratios S_0/ρ are given in Table II.

TABLE II.

S_0/ρ	0.4	0.5	0.6	0.7
α	1.78	1.11	0.54	0.70
β	2.01	0.86	0.96	1.06

Typical family of curves describing variances of the residual phase error as a function of the relation $(\rho/r_0)^{5/3}$ is shown in Fig. 2. Solid lines are for a Gaussian response function of the adaptive mirror, dashed lines stand for an "anisotropic" function and a dashed–dotted line is for a pyramidal one. It can be seen that for $S_0/\rho = 0.5$ the "anisotropic" and for $S_0/\rho = 0.6$ the Gaussian response functions enable one to approximate a random wave front more accurately than the pyramidal.

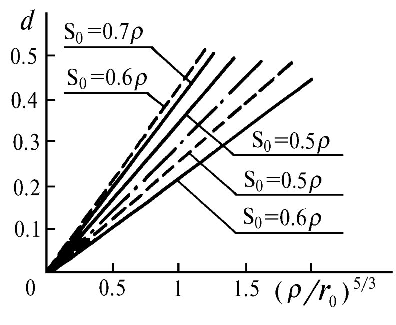

FIG. 2.

It should also be noted that the control with the help of an adaptive mirror and, hence, the quality of correction of the wave–front distortions depend on the degree of mutual effect of actuators working at different portions of the corrector. If the interference is significant, the system turns out to have severe cross relations and the loss of convergence of the iteration process in one channel inevitably influences the rest channels what naturally leads to worsening of the spatial resolution of the correction process with an adaptive mirror. When the interference of actuators is insignificant the correction is more stable therefore there is a real possibility of performing a parallel control of all the actuators and reaching the required quality of correction of wave–front distortions. Following Ref. 7 we define the coefficient of relation between the channels of the adaptive mirror C_q as the ratio of a signal of error in a given channel caused by displacements of a neighbor actuator to a signal of the error produced by a comparable in magnitude displacement in the channel under study

$$C_\rho = \exp\left[\frac{1}{2}(\rho/S_0)^2\right]. \qquad (6)$$

Then for a Gaussian response function for $S_0/\rho = 0.6$ the coefficient of cross relation is $C_\rho = 0.24$ and for an "anisotropic" one $C_\rho = 0.14$ for $S_0/\rho = 0.5$.

Thus, in constructing the adaptive mirrors it is necessary to take into account the fact that the maximum accuracy of approximation of different wave–front distortions is provided with the Gaussian response function of the corrector with the ratio of the radius of deformation of the surface portion S_0 to the distance between the actuators ρ equal to 0.6. However, the "anisotropic" response function with the ratio $S_0/\rho = 0.5$ enables one to obtain a system with weaker cross relations (overlapping of action of individual actuators decreases by approximately 40%) and to increase stability of the correction system functioning accompanied only by insignificant decrease of the accuracy (by 25% on the average) of different wave–front approximations compared to the Gaussian one for $S_0/\rho = 0.6$.

In conclusion it should be noted that despite of the fact that the results of this paper were obtained by mathematical simulations of functioning of a solid adaptive mirror with different response functions which is naturally characterized by some limitations and simplifications and taking into account those complications which can arise in practice of adaptive mirrors engineering with the required types of response functions I think that the results presented here can be useful for designers of adaptive mirrors and specialists in the field of developing adaptive optical systems.

REFERENCES

1. R. Hadgin, J. Opt. Soc. Am. **67**, No. 4, 393–397 (1977).
2. V.G. Taranenko, Radiotekh. Elektron. **29**, No. 8, 1510–1519 (1984).
3. S.V. Butsev, Atm. Opt. **3**, No. 2, 178–180 (1990).
4. V.G. Taranenko, Yu.G. Gorokhov, and N.S. Romanyuk, Zarubezh. Radioelektron., No. 8, 19–43 (1982).
5. S.V. Butsev and V.Sh. Khismatulin, Opt. Atm. **2**, No. 2, 174–178 (1989).
6. V.I. Tatarskii, *Wave Propagation through a Turbulent Atmosphere* (Nauka, Moscow, 1967), 548 p.
7. T.R. O'Meara, J. Opt. Soc. Am. **67**, No. 4, 318–324 (1977).