ALGORITHM FOR THE ADAPTIVE CONTROL OVER A WAVE–FRONT CORRECTOR

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This paper concerns with the problem on functioning of an adaptive optical system under conditions of the wave–front corrector parameters uncertainty and influence of an external stochastic disturbance. An algorithm for the adaptive control over the wave–front corrector which minimizes uncertainty of its parameters and the external stochastic disturbance is synthesized using methods of the parametric identification and optimum control.

Synthesis of algorithms for functioning of the adaptive optical systems (AOS) has been performed in Refs. 1–4, which consists of the synthesis of algorithms for processing filtration of the AOS incoming signals and the synthesis of algorithms for control over its active elements – the wavefront corrector (WFC).

The algorithm for optimal control was developed in Ref. 4, however, some adverse factors, such as vibrations, noise of drives, weight and thermal disadjustments and the like were not taken into account. Although they have negative effect on the functioning of an optical–mechanical channel of the optical system. Thus, it is of interest to develop the adaptive control algorithm of the AOS active element minimizing the disturbances to reduce these negative effects.

In Ref. 4 the process of the WFC AOS functioning was described by the stochastic differential equations of the form:

$$\dot{\mathbf{z}}(t) = A(t)\,\mathbf{z}(t) + B(t)\,u_\alpha(t)\,, \tag{1}$$

$$\tilde{\alpha}(t) = C\,\mathbf{z}(t)\,, \tag{2}$$

where $z(t)$ is the n–dimensional vector of state of the object to be controlled; $A(t)$ is the matrix of $n\times n$ dimensionality; $B(t)$ is the column–vector of $n\times 1$ dimensionality; $C(t)$ is the vector–row of $1\times n$ dimensionality; $\tilde{\alpha}(t)$ is the controllable value corresponding to slope angles of corrector relative to the input pupil plane of an optical system in two orthogonal directions; and $u_\alpha(t)$ is the voltage applied to the drives of the corrector.

The elements of A, B, C matrices are suggested to be known and assigned, they are selected in accordance with properties of the WFC AOS drives.

However, models (1)–(2) do not take into account the disturbances appearing under the real conditions. Therefore in experience, some individual elements of A and B matrices can be unknown. Besides, the noise of disturbances should be also accounted for. In this case the process of functioning of the AOS active element should be described by stochastic differential equations of the form

$$\dot{\mathbf{z}}_1(t) = A'(t)\,\mathbf{z}_1(t) + B'(t)\,u_{\alpha_1}(t) + \boldsymbol{\omega}(t),$$

$$\tilde{\alpha}_1(t) = C\,\mathbf{z}_1(t)\,, \tag{3}$$

where $z_1(t)$, $A'(t)$, $B'(t)$, $u_{\alpha_1}(t)$, C, and $\tilde{\alpha}_1(t)$ have the same dimensionalities and sense as $z(t)$, $A(t)$, $B(t)$, $u_\alpha(t)$, C, and $\tilde{\alpha}(t)$ in Eqs. (1)–(2), but a set of elements of $A\tilde{\alpha}_1(t)(t)$, $B'(t)$ matrices is unknown; $\omega(t)$ is the random process with zero expectation and spectral density matrix S_ω.

To synthesize the adaptive control algorithm, let us apply the identification method, which allows one, under conditions of uncertainty, to obtain estimations of unknown parameters of the system and disturbances as well as to use them to compensate for the negative effects of these uncertainties.

By substructing Eq. (2) from Eq. (3) one can derive a linearized model of the disturbances for controls under consideration

$$\delta\dot{\mathbf{z}}(t) = \Delta A(t)\,\delta\mathbf{z}(t) + \Delta B(t)\,\delta u_\alpha(t) + \boldsymbol{\omega}(t),$$

$$\varepsilon = \tilde{\alpha}(t) - \tilde{\alpha}_1(t) = C\,\delta\mathbf{z}(t)\,, \tag{4}$$

where

$$\Delta A(t) = A'(t) - A(t)\,;\ \Delta B(t) = B'(t) - B(t)\,;$$

$$\delta\mathbf{z}(t) = \mathbf{z}(t) - \mathbf{z}_1(t)\,;\ \ \delta u_\alpha(t) = u_\alpha(t) - u_{\alpha_1}(t)\,.$$

The determination of the control law with feedback for the equations describing the disturbance, requires that the system parameters from Eq. (4) be known at every instant of time in controlling over the WFC AOS. To determine the elements of ΔA and ΔB matrices one make use of the parametric identification method, in a frame of definition of which the problem on control over the corrector is reduced to the problem on determination of $\delta u_\alpha(t)$ that approaches $\delta z(t)$ to zero over the time of control over the WFC. To determine $\delta u_\alpha(t)$ for compensating disturbances, we can use the one–step law of the optimal control.[5]

Let us solve the problem in the discrete time. For that, let us take sampling of Eq. (4) to obtain the discrete equations for determination of the parameters required

$$\delta\mathbf{z}(kT + T) = F(kT)\,\delta\mathbf{z}(kT) + G(kT)\,\delta u_\alpha(kT) + \boldsymbol{\omega}(kT)\,, \tag{5}$$

where T is the period of sampling; $k = 0,\ 1,\ldots,\ m - 1$; $\delta u_\alpha(kT)$ is a piece–constant incoming vector of the control

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within the time interval between any two sequential moments of sampling; $\delta z(kT)$ is the n–dimensional disturbed vector of a state, which is determined by the following expression:

$$\delta \mathbf{z}(kT) = \Gamma(kT, t_0)\, \mathbf{z}(t_0) + \int_{t_0}^{kT} \Gamma(kT, t)\, \Delta B(t)\, \delta u_a(t)\, dt + \boldsymbol{\omega}(kT).$$

where $\Gamma(kT, t_0)$ is the transient matrix of a system state; $F(kT)$ and $G(kT)$ are the matrices of $(n\times n)$ and $(n\times 1)$ dimensions, respectively, which are determined as follows:

$$F(kT) = \Gamma[(k + 1)\, T,\ kT]\ ;$$

$$G(kT)\, \delta u_\alpha(kT) + \int_{kT}^{k+1} \Gamma[(k + 1)\, T,\ t]\, \Delta B(t)\, \delta u_\alpha(t)\, dt\ \ ;$$

$\omega(kT)$ is the n–dimensional vector of the discrete "white" sequences with zero mean and dispersion matrix D_ω.

The problem on the system parameter determination can be solved using the different algorithms such as the rms method, method of maximum as well as variational and stochastic approximation methods. To determine the system parameters in $F(kT)$ and $G(kT)$ matrices, the recurrent scheme can be best used owing its simplicity. In this case the following assumptions should be accepted:

1) The system parameters change slowly, and the velocity of their change is much as fast as velocity of adaptation.

2) Measurement errors are negligible.

3) The variable states $z(kT)$ in Eq. 5 can be measured.

In order to apply the recurrent identification algorithm to Eq. 5 by the rms–method, one should transform the system of equations to the form being convenient for calculation. Having written the ith row of unknown parameters of Θ_i^{T}–adaptive system in terms of $(n + 1)$–dimensional vector in the kth instant of time we have

$$\Theta_i^{\mathrm{T}}(kT) = [\ f_{i1}(kT),\ \dots,\ f_{in}(kT),\ g_i(kT)]\ ,\quad i = 1,\ \dots,\ n\ . \tag{6}$$

Let us determine in the same way the output and the input of disturbed system from Eq. 5 by $(n + 1)$D vector at the kth instant of time

$$\mathrm{x}^{\mathrm{T}}(kT) = [\delta \mathbf{z}_1(kT),\ \dots,\ \delta \mathbf{z}_n(kT),\ \delta u_\alpha(kT)]\ , \tag{7}$$

as well as the state at the kth instant of time by the n–dimensional vector

$$\delta \mathbf{z}^{\mathrm{T}}(kT) = [\delta \mathbf{z}_1(kT),\ \dots,\ \delta \mathbf{z}_n(kT)]\ . \tag{8}$$

Finally, the system according to Eq. 3 can be rewritten as follows:

$$\delta \mathbf{z}_i(kT + T) = x^{\mathrm{T}}(kT)\, \Theta_i(kT) + \boldsymbol{\omega}_i(kT)\ , \tag{9}$$

$i = 1,\ \dots,\ n$.

In this terms the parameters in each column $\Theta_i(kT)$ are required to be determined based on measurements $\mathbf{x}^{\mathrm{T}}(kT)$. The best estimation can be obtained by minimizing the prediction criterion

$$I_{\mathrm{pr}}(\Theta_i) = M\left\{ \sum_{k=0}^{m-1} [\delta \mathbf{z}_i(kT + T) - \mathbf{x}^{\mathrm{T}}(kT)\, \hat{\Theta}_i(kT)]^2 \right\} \to \min\nolimits_\theta, \tag{10}$$

where M is the expectation operator.

Minimizing the prediction criterion (10) based on the adaptive approach, relative to unknown parameters of the vector Θ_i we obtain the recurrent scheme of identification using the rms–method in the real time.[6]

$$\hat{\Theta}_i(kT + T) = \hat{\Theta}_i(kT) + P(kT)\, \mathbf{x}(kT)\ \times$$

$$\times\ [\delta \mathbf{z}_i(kT + T) - \mathbf{x}^{\mathrm{T}}(kT)\, \hat{\Theta}_i(kT)]\ ;$$

$$\hat{\Theta}(0) = \mathbf{Q}_0\ ;$$

$$P(kT + T) = P(kT) - \gamma(kT)\, P(kT)\, \mathbf{x}(kT)\, \mathbf{x}^{\mathrm{T}}(kT)\, \mathrm{P}(kT)\ ; \tag{11}$$

$$\gamma(kT) = [\mathbf{x}^{\mathrm{T}}(kT)\, P(kT)\, \mathbf{x}(kT) + 1]^{-1}\ ;$$

$$P(0) = \beta I\ ,\quad \beta \gg 1\ .$$

As is seen from recurrent equations (11), the estimation of $\hat{\Theta}_i(kT + T)$ at $(kT + T)$th time period is equal to the previous estimation $\hat{\Theta}_i(kT)$ corrected by the value proportional to $[\delta z_i(kT + T) - \mathbf{x}^{\mathrm{T}}(kT)\hat{\Theta}_i(kT)]$. The term $\mathbf{x}^{\mathrm{T}}(kT)\hat{\Theta}_i(kT)$ is the value $\delta z_i(kT + T)$ to be predicted, which is based on estimating the $\hat{\Theta}_i(kT)$ parameters and measurement vector $\mathbf{x}(kT)$. The $P(kT)\mathbf{x}(kT)$ elements of the vector are the weight coefficients determining a value of correction of the previous estimation for obtaining the new estimate $\hat{\Theta}_i(kT + T)$.

By defining the estimation parameters of the $F(kT)$, $G(kT)$ matrices, the law of optimal control over the AOS corrector can be formulated that takes into account the disturbances, satisfies the conditions of Eqs. (3) and (5), and also minimizes the criterion of the form

$$I(kT) = 1/2\ M\ \times$$

$$\times\left\{ \sum_{k=0}^{m-1} [\delta\alpha_1^{\mathrm{T}}(kT + T)\, \Theta \delta\alpha_1(kT + T) + u_{\alpha_1}^{\mathrm{T}}(kT)\, \mathrm{R}\, u_{\alpha_1}(kT)] \right\}, \tag{12}$$

where $\delta\alpha_1(kT) = \alpha(kT) - \tilde{\alpha}_1(kT)$ is the error of correction for wavefront distortion with account of disturbances; Q is the weight matrix determined as semipositive, and R is the weight matrix determined as positive.

By using the Pontryagin principle[5] of a maximum, one can show that the optimal control minimizing the functional (12) will satisfy the condition

$$u_{\text{opt}}(kT) = u^*(kT) + \delta u^*(kT) =$$

$$= -[\mathbf{R} + B^{\mathrm{T}}(kT)]^{-1} B(kT)\, Q\, A(kT)\, \hat{\mathbf{z}}(kT) -$$

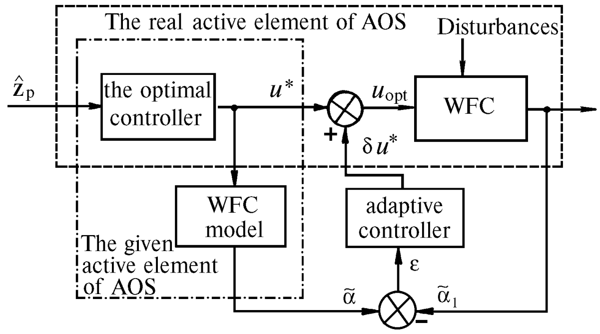
$$- [R + \hat{G}^{\mathrm{T}}(kT)\, Q\, \hat{G}(kT)]^{-1}\, \hat{G}(kT)\, Q\, \hat{F}(kT)\, \delta\mathbf{z}(kT)\ ,$$

where $u^*(kT)$ is the optimal control without disturbances, which has been synthesized in Ref. 4; $\hat{\mathbf{z}}(kT) = \begin{bmatrix} \hat{\alpha}(kT) \\ \mathbf{z}(kT) \end{bmatrix}$;

$\hat{\alpha}(kT)$ is the wavefront state estimation; $\delta u^*(kT)$ is the optimal control minimizing the correction errors over disturbances. $\hat{F}(kT)$ and $\hat{G}(kT)$ are the system parameters obtained using the identification algorithm (11).

The structural scheme of the WFC adaptive control is demonstrated in Fig. 1.

Under conditions of uncertainty and influenced stochastic external disturbances, the suggested approach allows one to reduce the AOS functioning problem to a set of equations. In a frame of such a description of the quality determined by functionals (10) and (12), the WFC adaptive control algorithm, that has been developed, provides the best functioning of the AOS.



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