THE EFFECT OF THE NUMBER OF ELEMENTS IN A HARTMANN SENSOR ON THE QUALITY OF ESTIMATING THE TOTAL WAVEFRONT TILT

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We consider the effect of the number of elements in a Hartmann sensor on the signal–to–noise ratio of photomultipliers (PMT) and analize the dependence of the quality of estimating the total wavefront tilt on that number.

Astrophysical objects are single–point (unresolved with a telescope) and extended sources of incoherent radiation. The Hartmann sensors are demonstrated[1] to be preferable for adaptive optical system operating with such incoherent signals.

In the Hartmann method the atmospheric perturbatons are imaged at an array of lenses. If the wavefront of the incident wave is distorted, the image will be displaced with respect to its ideal position. The error variance Δ of displacement measured with the Hartmann sensor is well known[1] to be given by the relation:

$$d_\Delta = \frac{1}{q_1^2} r_s ,$$

where $r_s = \dfrac{0.6\lambda F}{D_s} + \dfrac{\Theta F}{2}$ is the radius of the light spot, D_s is the diameter of the input subaperture of the sensor, F is the focal distance of the wavefront sensor, q_1^2 is the signal–to–noise ratio within the subaperture, and Θ is the angular size of the object.

The measurement error variance of the wavefront tilt within one of the subapertures will than be determined in the following way:

$$d_{t1} = d_\Delta \frac{\kappa^2}{F^2} = \frac{\pi^2}{q_1^2}\left(\frac{1.22}{D_s} + \frac{\Theta}{\lambda}\right)^2 , \qquad (1)$$

where $k = 2\pi/\lambda$ is the wave number.

At low signal intensities the possible signal–to–noise ratios of photomultiplier are determined by the quantum character of signal. The numbers of signal and background photoelectrons obey the Poisson law, so that the signal–to–noise ratio has the following form:

$$q_1^2 = \frac{\bar{n}_s^2}{\bar{n}_s + \bar{n}_{bg}} , \qquad (2)$$

where $\bar{n}_s$ is the average number of signal photoelectrons, and $\bar{n}_{bg}$ is the average number of background photoelectrons.

The average number of signal photoelectrons is related to the brightness of the space object, expressed in star magnitudes, by the formula:

$$\bar{n}_s = \frac{S_1 A \, \tau_a \tau_o \gamma \, T_{acc}}{h\bar{\nu}} \cdot 10^{\frac{m_0 - m_{st}}{2.5}} , \qquad (3)$$

where $m_0 = -14^m$, 18 ± 0.05 is the star magnitude corresponding to illumination of 1 lux, m_{st} is the star magnitude of the space object, S_1 is the area of the receiving subaperture, $A = 1/683$ W/l is the mechanical equivalent of light, τ_a is the atmospheric transmittance, τ_o is the transmittance of the optics, γ is the quantum efficiency of the photodetector, T_{acc} is accumulation time, $h = 6.62 \cdot 10^{-34}$ J·s is Planck's constant, and $\bar{\nu}$ is the average frequency of the optical radiation being recorded.

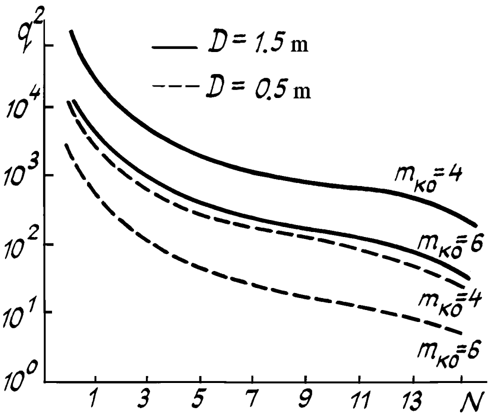

FIG. 1.

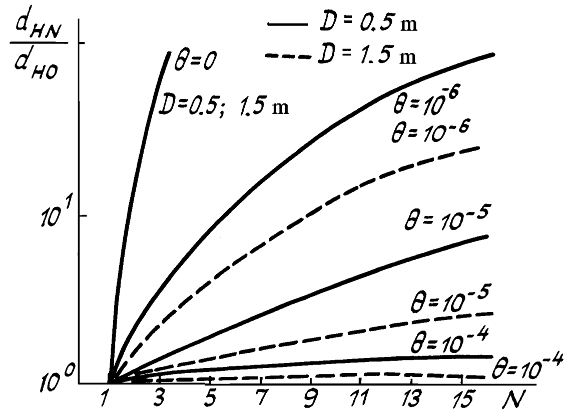

FIG. 2.

The increase of the number of sensor elements results in the decrease of the subaperture diameter, so the number of signal photoelectrons incident on the PMT's correspondingly decreases.

The average number of background photoelectrons is related to sky brightness B

$$\bar{n}_{bg} = \frac{BS_1\Delta\lambda\tau_a\gamma T_{acc}\Delta\Omega}{h\bar{\nu}}, \quad (4)$$

where $\Delta\Omega = \frac{\pi}{4}\left(\frac{\lambda}{D}\right)^2$ is the solid angle corresponding to the field–of–view angle of the entire optical system, and D is the diameter of the aperture of the telescope.

The signal–to–noise ratio is mainly determined by the number of signal photoelectrons and is practically independent of the angular size of the object. Figure 1 shows the dependence of the number of elements in the sensor $N^2 = (D/D_s)^2$ on the signal–to–noise ratio for the following conditions: D = 0.5 and 1.5 m, $\tau_a = 0.7$, $\tau_o = 0.5$, $T_{acc} = 5$ ms, $\bar{\nu} = 5.4\cdot 10^{14}$ Hz, $B = 2\cdot 10^{-11}$ W/(sr cm^2 μm), and $\gamma = 0.15$. As can be seen, the signal–to–noise ratio falls down with increase of the number of sensors, which is explained by decrease in the number of signal photoelectrons.

Let us analyze the effect of the number of elements in the sensor on the quality of estimating the total wavefront tilt. Figure 2 shows the variations of the relative value d_{tN}/d_{t0} with the number N of such elements. Here d_{t0} is the error variance of the wavefront tilt measured over the entire aperture of the telescope in the case of the Hartmann sensor consisting of a single element only, d_{tN} is the error variance of the wavefront tilt measured over the aperture in the case of the sensor consisting of N elements. In the case in which the components of the total tilts within the individual subapertures are completely correlated, it is given by the relation

$$d_{tN} = \frac{d_{t1}}{N^2}. \quad (5)$$

When observing an ideal single–point object ($\Theta = 0$) the multielement Hartmann sensor sharply deteriorate the quality of tilt measurements regardless of the size of the receiving aperture. In the case of the extended object, the deterioration of quality is less pronounced for larger apertures and large angular size of the object. The multielement Hartmann sensor should only be considered acceptable for $d_{tN}/d_{tilt0} < 2...4$.

REFERENCES

1. J.W. Hardy, Proc. IEEE **66**, No. 6, 31–85 (1978).