INFLUENCE OF FLUCTUATIONS OF THE REFRACTIVE INDEX OF AN ACTIVE MEDIUM ON THE CHARACTERISTICS OF THE X-RAY LASER EMISSION

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The coherence characteristics of the X-ray laser emission are studied as functions of the distribution of amplifying and refracting properties of an active medium. Analytical and numerical solutions have been obtained considering the effect of the dielectric constant fluctuations on output radiation parameters. Calculations have been performed on the basis of combination of the ray-tracing techniques: method of characteristics used for a solution of the radiative transfer equation and the phase approximation of the Huygens-Kirchhoff method.

A number of approaches can be used to describe theoretically the X-ray laser emission. All are based on the paraxial approximation of wave optics for a solution of the parabolic equation[1–3] or of the equation for the coherence function.[4,5] Within the scope of the last approach, the effect of the dielectric constant fluctuations on the output radiation parameters was estimated. However, these calculations were performed for the uniform distribution of amplifying and refracting properties of an active medium. Simultaneous account of inhomogeneous properties and dielectric constant fluctuations within the framework of the above-mentioned approaches is highly conjectural.

This paper describes an approach that allows us to take into account both regular inhomogeneities and fluctuations in the active medium. The approach is based on combination of the ray tracing technique for a solution of the radiative transfer equation being the Fourier transform of the equation for the coherence function[4,5] and on the use of the phase approximation of the Huygens-Kirchhoff method for taking account of the effect of the turbulent fluctuations of the dielectric constant.

1. Let us first consider application of the approach to computation of output radiation parameters for an active medium without fluctuations. As an initial equation, we will consider the parabolic wave equation

$$2\, ik \frac{\partial E}{\partial z} + \nabla_\perp^2 E + k^2\, \Delta\varepsilon(z,\, \rho)\, E(z,\, \rho) = P_{\rm sp}(z,\, \rho), \qquad (1)$$

where k is the wave number, $\Delta\varepsilon$ is the relative perturbation of the complex dielectric constant, $P_{\rm sp}$ is the term caused by the spontaneous polarization in a medium, and $\mathbf{r} = (z,\, \rho)$.

We consider the active medium with the following spatial distribution of the dielectric constant:

$$\Delta\varepsilon(z,\, \rho) = \varepsilon(z,\, \rho) + i\, \sigma(z,\, \rho), \qquad (2)$$

where ε is the real part of the dielectric constant and σ is its imaginary part connected with the amplification coefficient of the medium g by the following relation:

$$\sigma(z,\, \rho) = -\, k^{-1}\, g(z,\, \rho).$$

The form of functions ε and σ is determined by the spatial distribution of the density of population inversion in the medium. Spontaneous radiation is caused by the random polarization in the medium, which is considered to obey Gaussian statistics and to satisfy the condition

$$\langle P_{\rm sp}(\mathbf{r})\, P^*_{\rm sp}(\mathbf{r}')\rangle = W_{\rm ef}(\mathbf{r})\, g_0\, \delta(\mathbf{r} - \mathbf{r}'), \qquad (3)$$

where $W_{\rm ef}$ is the effective intensity of spontaneous emission, and g_0 is the amplification coefficient at the origin of coordinates. Then we can write the equation for the coherence function in the approximation of paraxial optics

$$\frac{\partial \Gamma_2}{\partial z} + \left[\frac{1}{ik}\nabla_\rho \nabla_{\mathbf{R}} + \frac{k}{2i}\rho\, \nabla_{\mathbf{R}}\varepsilon(z,\mathbf{R}) + k\sigma(z,\mathbf{R})\right]\Gamma_2(z,\mathbf{R},\rho) =$$
$$= \frac{g_0}{2\, k^2}\, W_{\rm ef}(z,\, \mathbf{R})\, \delta(\rho), \qquad (4)$$

where the sum $\mathbf{R} = (\rho_1 + \rho_2)/2$ and difference $\rho = \rho_1 - \rho_2$ transverse coordinates were introduced and the approximate Taylor series expansion[6]

$$\Delta\varepsilon(z,\, \mathbf{R} + \rho/2) - \Delta\varepsilon^*(z,\, \mathbf{R} - \rho/2) \approx$$
$$\approx \rho\, \nabla_{\mathbf{R}}\, \varepsilon(z,\mathbf{R}) + 2\, i\sigma(z,\mathbf{R}) \qquad (5)$$

was used. Next, taking the Fourier transform in ρ, we obtain the equation

$$\frac{\partial J}{\partial z} + \left[\frac{\mathbf{n}_\perp}{k}\nabla_{\mathbf{R}} + \frac{k}{2}\nabla_{\mathbf{R}}\, \varepsilon\, \nabla_{\mathbf{n}_\perp} + k\, \sigma(z,\mathbf{R})\right] J(z,\mathbf{R},\mathbf{n}_\perp) =$$
$$= \frac{g_0}{8\, \pi^2\, k^2}\, W_{\rm ef}(z,\, \mathbf{R}), \qquad (6)$$

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where J is the radiation brightness, which is defined as the Fourier transform of the coherence function

$$J(z, \mathbf{R}, \mathbf{n}_\perp) = (2\pi)^{-2} \int\int_{-\infty}^{\infty} \Gamma_2(z, \mathbf{R}, \rho) \exp(-i\,\mathbf{n}_\perp \rho)\, d\rho. \quad (7)$$

Solution of Eq. (6) can be represented in the form

$$J(z, \mathbf{R}, \mathbf{n}_\perp) = \frac{g_0}{8\,\pi^2\,k^2} \int_0^z dz'\, W_{\mathrm{ef}}(z', \mathbf{R}(z')) \times$$

$$\times \exp\left[\int_{z'}^{z} dz''\, g(z'', \mathbf{R}(z''))\right], \quad (8)$$

where the characteristic $\mathbf{R} = \mathbf{R}(z')$ obeys the equation

$$\frac{d^2\mathbf{R}}{dz^2} = \frac{1}{2} \nabla_{\mathbf{R}}\, \varepsilon(z, \mathbf{R}(z)) \quad (9)$$

with the initial conditions $\mathbf{R}(z' = z) = \mathbf{R}$ and $d\mathbf{R}(z' = z)/dz' = \mathbf{n}_\perp$.

Analytic solution for the parabolic distribution of the population inversion density

$$\varepsilon(\mathbf{R}) = 1 + (R^2 - a^2)/L_R^2,\ |\mathbf{R}| < a,\ \varepsilon(\mathbf{R}) = 1,\ |\mathbf{R}| > a,$$
$$g(\mathbf{R}) = g_0(1 - R^2/a^2),\quad |\mathbf{R}| < a,\quad g(\mathbf{R}) = 0,\quad |\mathbf{R}| > a, \quad (10)$$

was obtained in Ref. 6. This solution was found with the use of approximations

$$W_{\mathrm{ef}}(z, \mathbf{R}) = W_{\delta\mathrm{ef}}(\mathbf{R})\, \delta(z), \quad (11)$$

$$W_{\delta\mathrm{ef}}(\mathbf{R}_0) = W_{\delta 0}(\mathbf{R}) \exp(-R_0^2/a^2). \quad (12)$$

Condition (11) means that we consider only the contribution from an infinitely thin layer of radiating elements, located at the end of the active medium (approximation of incoherent disk), to the output radiation. Zemlyanov and Kolosov[6] pointed out that this approximation may yield more than twice overestimated values of the coherence radius of the output radiation of an X-ray laser. Condition (12) also should be considered as an approximation because in rigorous formulation of the problem, the distribution of the intensity of sources must copy the distribution of the amplification coefficient, i.e., it must have the parabolic profile.

2. Let us consider results of numerical solution of this problem for a two-dimensional medium. We assume that functions ε and g have the forms

$$\varepsilon(z, x) = 1 - (a^2/L_R^2)\, \mathrm{ch}^{-2}(x/a), \quad (13)$$

$$g(z, x) = g_0 \cosh^{-2}(x/a), \quad (14)$$

respectively, and that the sources of spontaneous radiation are distributed by the law

$$W_{\mathrm{ef}}(z, x) = W_0 \cosh^{-2}(x/a). \quad (15)$$

By numerical integration of the system of equations (8) and (9) and subsequent numerical Fourier transform

$$\Gamma_2(z, x, \rho) = \int_{-\infty}^{\infty} dn_\perp\, J(z, x, n_\perp) \exp(i\, n_\perp \rho), \quad (16)$$

we find the coherence function and hence the modulus of the degree of coherence

$$\mu(\rho) = \frac{|\Gamma_2(z, x = 0, \rho)|}{W(z, x = 0)} = \frac{|\Gamma_2(z, x = 0, \rho)|}{\Gamma_2(z, x = 0, \rho = 0)}. \quad (17)$$

This numerical solution considers the contribution of spontaneous sources throughout the volume of the active medium. When we were interested in the contribution of the end region solely, calculations were made with condition (11). Results of calculations of the modulus of the coherence degree in the given approximation are shown in Fig. 1. Analytic solution for parabolic profiles of the functions ε and g given by Eq. (10) with the Gaussian distribution of $W_{\delta\mathrm{ef}}$ given by Eq. (12) and numerical solution for ε, g, and $W_{\delta\mathrm{ef}}$ obeying the parabolic distribution are also shown in Fig. 1. It is seen that the last case corresponds to the highest degree of coherence of the output radiation. Numerical measure of output radiation coherence may be the coherence radius ρ_c, for which we will use two definitions. First, we define the coherence radius ρ_c as the transverse distance at which μ decreases from 1 (at $\rho = 0$) to $\sin(1) \approx 0.84$. Second, we will use the definition

$$\rho_c(z) = \frac{0.83}{2\,\pi^{1/2}} \int_{-\infty}^{\infty} d\rho\, \mu(\rho). \quad (18)$$

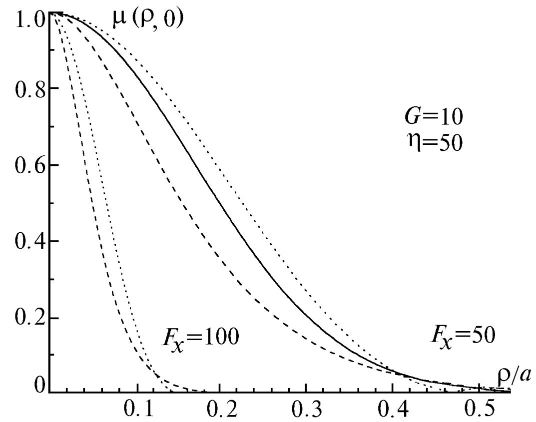

FIG. 1. Modulus of the coherence degree at the axis of an active medium with parabolic (...) and $\cosh^{-2}$(- - -) *profiles of population inversion for* $G = 10$, $\eta = \varepsilon_0/\sigma_0 = GF_x/(g_0L_R)^2 = 50$, *and* $F_x = 50$, 100. *Solid curve shows the corresponding analytic solution given by Eqs. (32) and (33).*

It is clear that both definitions give identical values of ρ_c for the modulus of the degree of coherence obeying the Gaussian distribution. For non-Gaussian distribution, these definitions yield different values of the coherence radius. Corresponding calculations of the coherence radius for parabolic and $\cosh^{-2}$ profiles are shown in Fig. 2.

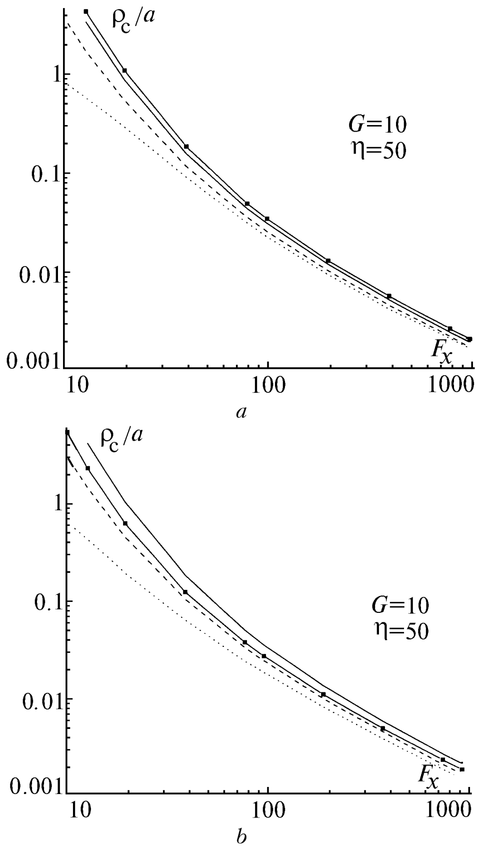

FIG. 2. The coherence radius as a function of the Fresnel number for a medium with parabolic (a) and $\cosh^{-2}$ *(b) profiles of the population inversion distribution for* $G = 10$ *and* $\eta = \varepsilon_0/\sigma_0 = GF_x/(g_0L_R)^2 = 50$: —•— *first definition and the incoherent disc approximation;* ···· *first definition and sources through on the volume;* - - - *second definition given by Eq. (18) and distributed sources;* -------- *analytic solution given by Eq. (33).*

3. The results discussed above were obtained neglecting fluctuations of the population inversion density in the active medium. In this section, we estimate the effect of fluctuations of the dielectric constant on the parameters of output radiation.

As previously, we will consider the parabolic approximation. The distribution of the relative dielectric constant is represented in the form

$$\Delta\varepsilon(z, \rho) = \varepsilon(z, \rho) + \delta\varepsilon(z, \rho) + i\,\sigma(z, \rho), \qquad (19)$$

where the fluctuations of dielectric constant $\delta\varepsilon$ are characterized by the structure function

$$D_\varepsilon(\mathbf{r}, \mathbf{r}') = \langle[\delta\varepsilon(\mathbf{r} + \mathbf{r}') - \delta\varepsilon(\mathbf{r})]^2\rangle \qquad (20)$$

and the mean $\langle\delta\varepsilon(\mathbf{r})\rangle = 0$, and angular brackets denote an ensemble averaging. Formal solution of Eq. (1) can be represented in the form

$$E(z, \rho) = \int_0^z dz_0 \iint_{-\infty}^{\infty} d\rho_0\, P_{sp}(z_0, \rho_0)\, \tilde{G}(\mathbf{r}, \mathbf{r}_0), \qquad (21)$$

where $\tilde{G}$ is the Green's function and $\mathbf{r} = (z, \rho)$. Consequently,

$$\Gamma_2(z, \rho_1, \rho_2) = \iint_0^z dz_{01}dz_{02}\times$$

$$\times\iint_{-\infty}^{\infty} d\rho_{01}d\rho_{02}\langle P_{sp}(z_{01}, \rho_{01})\, P^*_{sp}(z_{02}, \rho_{02})\rangle\langle\tilde{G}_1\tilde{G}^*_2\rangle =$$

$$= g_0\iint_{-\infty}^{\infty} d\rho_{01}\; W_{\delta ef}(\rho_{01})\,\langle\tilde{G}(\mathbf{r}_1, \mathbf{r}_{01})\tilde{G}^*(\mathbf{r}_2, \mathbf{r}_{01})\rangle,$$

$$\tilde{G}_i = \tilde{G}(\mathbf{r}_i, \mathbf{r}_{0i}),$$

where conditions (3) and (11) where taken into consideration. Using the phase approximation of the Huygens–Kirchhoff method,[7] we can write

$$\tilde{G}(z, \rho; 0, \rho_0) = G(z, \rho; 0, \rho_0)\exp[ik\,\tilde{S}(z, \rho; 0, \rho_0)], \qquad (22)$$

where G is the Green's function for the case $\delta\varepsilon(\mathbf{r}) \equiv 0$, and $\tilde{S}$ is determined by the expression

$$\tilde{S}(z, \rho; 0, \rho_0) = \frac{1}{2}\int_0^z dz'\, \delta\varepsilon(z', \rho(z'))$$

and is equal to a random run-on of the phase along unperturbed geometric ray connecting points $(0, \rho_0)$ and (z, ρ). This geometric ray $\rho(z')$ satisfies Eq. (9) with boundary conditions $\rho(z' = 0) = \rho_0$ and $\rho(z' = z) = \rho$. In this case,

$$\langle\tilde{G}_1\tilde{G}^*_2\rangle = G(\mathbf{r}_1, \mathbf{r}_{01})\, G^*(\mathbf{r}_2, \mathbf{r}_{02})\exp\Big[-\frac{k^2}{4}\iint_0^z dz'\,dz''\times$$

$$\times\frac{1}{2}[\psi_\varepsilon(z', z'', \rho_1(z') - \rho_1(z'')) + \psi_\varepsilon(z', z'', \rho_2(z') -$$

$$-\rho_2(z'')) - 2\psi_\varepsilon(z', z'', \rho_1(z') - \rho_2(z''))]\Big],$$

where

$$\psi_\varepsilon(z', z'', \rho_i(z') - \rho_j(z'')) = \langle\delta\varepsilon(z', \rho_i(z'))\,\delta\varepsilon(z'', \rho_j(z''))\rangle$$

for the homogeneous fluctuations in the transverse plane. Then we can finally write the solution in the form

$$\Gamma_2(z,\rho_1,\rho_2)=\exp\left[-\frac{k^2}{4\pi}\int_0^z \mathrm{d}z'\, H(z',\rho(z'))\right] g_0 \times$$

$$\times \int\int_{-\infty}^{\infty} \mathrm{d}\rho_{01} W_{\delta\mathrm{ef}}(\rho_{01})\, G(z,\rho_1;0,\rho_{01})\, G^*(z,\rho_2;0,\rho_{02}) =$$

$$=\exp\left[-\frac{k^2}{4\pi}\int_0^z \mathrm{d}z'\, H(z',\rho(z'))\right]\Gamma_{2\mathrm{R}}(z,\rho_1;\rho_2), \quad (23)$$

where $\rho(z')=\rho_1(z')-\rho_2(z')$, $\Gamma_{2\mathrm{R}}$ is the coherence function for a regular medium, and

$$H(z,\rho_1-\rho_2)=\frac{1}{\pi}\,[A(z,0)-A(z,\rho_1-\rho_2)];$$

$$H(z,\rho_1-\rho_2)\,\delta(z')=\left\langle\delta\varepsilon\left(z+\frac{z'}{2},\rho_1\right)\delta\varepsilon\left(z+\frac{z'}{2},\rho_2\right)\right\rangle, \quad (24)$$

$$2\pi\, H(z,\rho')\,\delta(z')=D_\varepsilon(z,\rho,z',\rho').$$

Solution (23) can be rewritten in the form

$$\Gamma_2(z,\mathbf{R},\rho)=\Gamma_{2\mathrm{R}}(z,\mathbf{R},\rho)\,\gamma(\rho), \quad (25)$$

where $\gamma(\rho)$ is the factor describing the contribution of the turbulent fluctuations

$$\gamma(\rho)=\exp\left[-\frac{k^2}{4\pi}\int_0^z \mathrm{d}z'\, H(z',\rho(z'))\right]. \quad (26)$$

We assume that the amplification coefficient and dielectric constant have parabolic distribution given by Eq. (10). Then for the homogeneous fluctuations of the dielectric constant with the Gaussian correlation function

$$\psi_\varepsilon(\mathbf{r})=\langle\delta\varepsilon(\mathbf{r}'+\mathbf{r})\,\delta\varepsilon(\mathbf{r})\rangle=\sigma_\varepsilon^2\exp[-r^2/r_\mathrm{c}^2] \quad (27)$$

we obtain

$$H(\rho)=\frac{\sigma_\varepsilon^2\, r_\mathrm{c}}{\sqrt{\pi}}\,[1-\exp(-\rho^2/r_\mathrm{c}^2)], \qquad \mathbf{r}=\{z,\rho\}. \quad (28)$$

Using the approximation (for $\rho < r_\mathrm{c}$)

$$H(\rho)\approx\sigma_\varepsilon^2\,\rho^2/\sqrt{\pi}\, r_\mathrm{c} \quad (29)$$

and taking into account that $\rho(z')=\rho\,\sinh(z'/L_\mathrm{R})/\sinh(z/L_\mathrm{R})$, we obtain

$$\gamma(\rho)=\exp\left[-\frac{\sqrt{\pi}}{8\, r_\mathrm{c}}\,\sigma_\varepsilon^2\, L_\mathrm{R}\, k^2\,\rho^2\,\frac{\sinh\bar z\,\cosh\bar z-\bar z}{\sinh^2\bar z}\right]=$$

$$=\exp\left[-\frac{\sqrt{\pi}}{8}\frac{\bar\sigma_\varepsilon^2}{\bar r_\mathrm{c}}\frac{L_\mathrm{D}^2}{L_\mathrm{R}^2}\frac{a}{L_\mathrm{R}}\bar\rho^2\,\frac{\sinh\bar z\,\cosh\bar z-\bar z}{\sinh^2\bar z}\right]=\exp\left[-\frac{\bar\rho^2}{4\,\bar a_\mathrm{t}^2}\right], \quad (30)$$

where

$$\bar a_\mathrm{t}^2=\bar r_\mathrm{c}^2\,\frac{2}{\sqrt{\pi}}\frac{L_\mathrm{R}^2}{L_\mathrm{D}^2}\frac{L_\mathrm{R}}{a}\frac{1}{\bar\sigma_\varepsilon^2\,\bar r_\mathrm{c}}\,\frac{\sinh^2\bar z}{\sinh\bar z\,\cosh\bar z-\bar z} \quad (31)$$

and $\bar\sigma_\varepsilon=\sigma_\varepsilon/\varepsilon_0$, $\varepsilon_0=a^2/L_\mathrm{R}^2$, $\bar r_\mathrm{c}=r_\mathrm{c}/a$, $\bar\rho=\rho/a$, $\bar z=z/L_R$.

In the approximation of incoherent disk, analytic solution was obtained in Ref. 6 for $\Gamma_{2\mathrm{R}}$. Then it follows from Eqs. (25) and (30) that

$$\mu(\rho)=\mu_\mathrm{R}(\rho)\,\gamma(\rho)=\exp[-\bar\rho^2/(4\,\bar a_\rho^2)-\bar\rho^2/(4\,\bar a_\mathrm{t}^2)], \quad (32)$$

where μ_R is the modulus of the complex degree of coherence for regular medium,[6]

$$\bar a_\rho^2=\frac{1}{F_x\,\bar z}\sqrt{\sinh(\bar z)+\frac{G}{2\,\bar z}(\sinh(\bar z)\cosh(\bar z)-\bar z)}\,, \quad (33)$$

$G=g_0\, z$; and $F_x=k\,a^2/z$ is the Fresnel number. Consequently, we obtain for the transverse coherence radius

$$\rho_\mathrm{c}=0.83\,\bar a_\rho/(\sqrt{1+\bar a_\rho^2/\bar a_\mathrm{t}^2}).$$

For homogeneous fluctuations with the correlation function

$$\psi_\varepsilon(\mathbf{r})=\sigma_\varepsilon^2\exp[-|\mathbf{r}|/r_\mathrm{c}] \quad (34)$$

we obtain in a similar manner

$$\mu(\rho)=\mu_\mathrm{R}(\rho)\,\gamma(\rho)=\exp[-\bar\rho^2/(4\,\bar a_\rho^2)-|\bar\rho|/(2\,\bar a_\mathrm{t}^2)], \quad (35)$$

where

$$\bar a_\mathrm{t}=\bar r_\mathrm{c}\,\frac{L_\mathrm{R}^2}{L_\mathrm{D}^2}\frac{L_\mathrm{R}}{a}\frac{1}{\bar\sigma_\varepsilon^2\,\bar r_\mathrm{c}}\,\frac{\sinh\bar z}{\cosh\bar z-1}\,. \quad (36)$$

Equation (35) was obtained with the use of the approximation

$$H(\rho)=2\,\sigma_\varepsilon^2\, r_\mathrm{c}/\pi\,[1-\exp(-|\rho|/r_\mathrm{c})]\approx 2\,\sigma_\varepsilon^2/\pi\,|\rho|. \quad (37)$$

Approximations (29) and (37) hold true under condition

$$\bar a_\mathrm{t}<\bar r_\mathrm{c}\,. \quad (38)$$

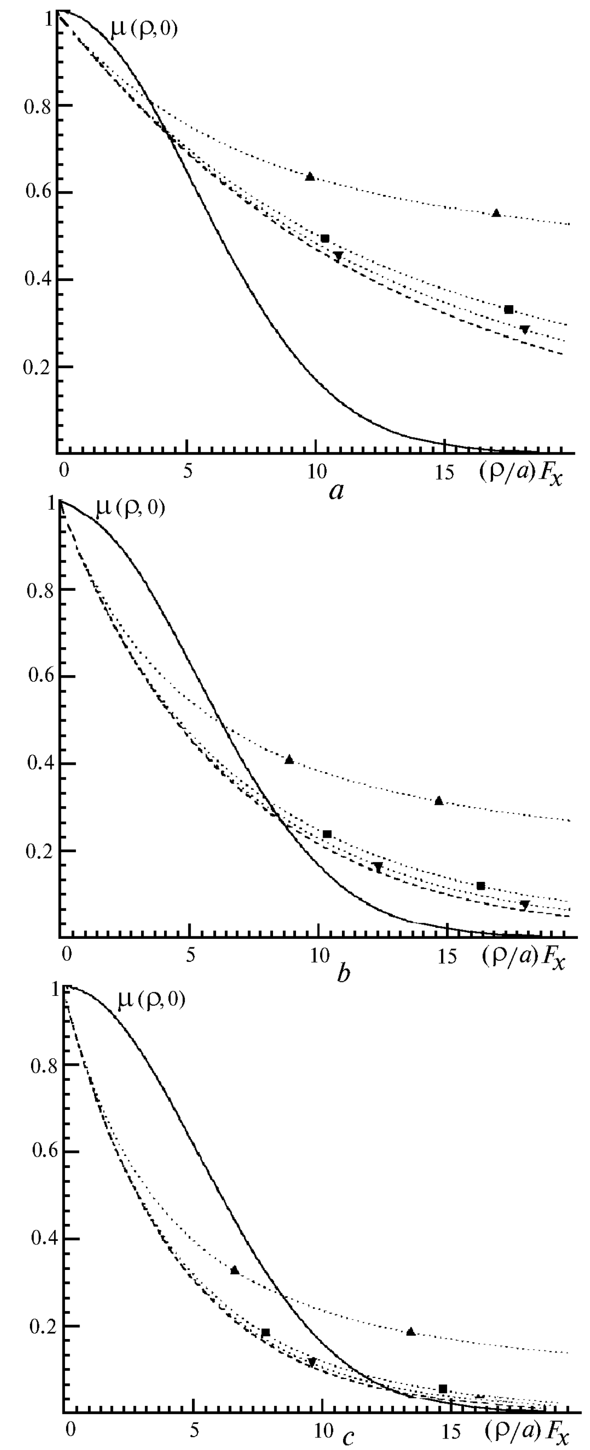

FIG. 3. Modulus of the degree of coherence μ_R and the parameter γ for the parabolic profile of the population inversion distribution in the incoherent disk approximation: — analytic solution for μ_R given by Eqs. (32) and (33); - - - analytic solution for γ given by Eqs. (35) and (36). Dashed curves show the results of numerical calculations of the parameter γ for $r_c = 0.1$ (▲); 0.5 (■), 1.0 (▼), *and* $G = 15$, $g_0 L_R = 7.5$, $L_D/L_R = 100$, $a/L_R = 10^{-2}$, $\sigma_\varepsilon^{-2} = 0.1$ *(a)*, 0.2 (*b*), and 0.3 *(c)*.

We point out that for $L_R/L_D \approx 10^{-3}$ and $a/L_R \approx 10^{-2}$ condition (38) is valid for the following dimensionless parameters: $\bar{\sigma}_\varepsilon^2 = 0.1$, 0.2, 0.3; $\bar{r}_c = 0.1$, 0.5, 1.0; $\bar{z} \geq 1$. We also notice that we can write $\mu(\rho) \approx \mu_R(\rho)$ under condition

$$a_\rho \ll a_t \qquad (39)$$

and violation of condition (38) has no effect on determination of ρ_c. It follows from Eqs. (31) and (36) that turbulent coherence radius does not vanish as in homogeneous medium and saturated at a fixed level.

In the case $L_R/L_D \approx 10^{-2}$ conditions (38) and (39) are violated for the above-indicated dimensionless parameters. Results of numerical calculations differ from analytical one given by Eq. (35). However, for $r_c = 0.5$ and 1.0 this difference is not great in the most important region $\rho < a_q$.

CONCLUSION

Thus, the coherent characteristics of the X-ray laser emission have been described theoretically with simultaneous consideration of nonuniform distribution of amplification and refraction profiles and fluctuations of dielectric constant of an active medium. This approach is based on combination of the ray tracing technique for a solution of the radiative transfer equation and phase approximation of the Huygens–Kirchhoff method.

The same characteristics [geometric rays given by Eqs. (9)] provide the basis for both methods. This circumstance essentially simplifies the problem from a mathematical point of view. An account of nonuniform distribution of amplification and refraction in an active medium and dielectric constant fluctuations reduce to the simple integration of corresponding Eqs. (8) and (26) along the given characteristics.

Within the framework of this approach, effective numerical algorithms have been constructed and accurate analytical solutions have been obtained. For an inhomogeneous medium the coherence radius is found to saturate with the increase of the distance of radiation propagation in the active medium (this effect is not observed in the homogeneous medium).

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