A SEMI-LAGRANGIAN SCHEME OF TRACER TRANSFER IN THE CLIMATOLOGY MODEL ECSIB

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A semi-Lagrangian, semi-implicit scheme of transfer in the climatic model ECSib is described. The semi-Lagrangian method has some advantages as compared with the Eulerian approach. The numerical scheme of the model is described for the equations of momentum, temperature, near-surface pressure, humidity, and chemical tracers.

1. INTRODUCTION

A project on simulation of distribution greenhouse gases and their effect upon the climate is developed within the frameworks of the world climate program (WCRP). The study of greenhouse gases in the atmosphere is conducted jointly by specialists on atmosphere dynamics and atmospheric chemistry. Development of global models for chemical tracers' transfer was the first stage of this project (the description of these models and comparison of the obtained simulation results can be found, for instance, in Refs. 1–3).

In this paper, we propose a description of a semi-Lagrangian version of the climatic model ECSib[5–7] where the semi-Lagrangian scheme of transfer is used jointly with the semi-implicit scheme of integration over time.[4] Formulation of the semi-Lagrangian method does not depend on the way of spatial adaptation of the system's equations. The results of test computations for a passive admixture transfer on a sphere through its pole are discussed.

2. THE SEMI-LAGRANGIAN VERSION OF THE CLIMATIC MODEL ECSIB. NUMERICAL SCHEME

Consider a system of equations of the atmospheric dynamics in the σ-coordinate system on a sphere.[5] The rates of the momentum variation which are caused by vertical small-scale diffusion and surface friction stress are described by the equation of diffusion and by equations of the Monin-Obukhov theory. The processes of convection and small-scale diffusion of heat and humidity, processes of condensation and humidity vaporization, as well as of the radiation transfer are taken into account parametrically. The conditions of periodicity along the longitude and the condition that the solution is bounded at the poles are taken as boundary conditions for the dynamic operator. The following boundary conditions are put along the vertical: $\dot{\sigma} = 0$ for $\sigma = 0.1$; $\Phi_s = gz_s$ for $\sigma = 1$, where z_s is the height of the Earth's surface above sea level; $\dot{\sigma}$ is the vertical component of the rate in the σ-system of coordinates; g is the acceleration of gravity. At the Earth's surface, one assigns geographical average-climatic distributions of ice, ocean surface temperature, temperature and humidity of soil at the depth of 2 m, latitude distribution of the declination angle of the Sun, and the ozone concentration. Moisture content of the soil and thickness of snow cover vary in time.

Let us write the system of equations in the vector form, which is more convenient to apply the semi-Lagrangian method.

The equation for the momentum is

$$\frac{\mathrm{d}(\mathbf{v} + 2\boldsymbol{\Omega} \times \mathbf{r})}{\mathrm{d}t} = -\nabla\Phi - RT\nabla(\ln p) + F_{\mathbf{v}}, \qquad (1)$$

where $\mathbf{v}$ is the vector of horizontal velocity, T is the temperature, p is the pressure, Φ is the geopotential, $\boldsymbol{\Omega}$ is the angular velocity of the Earth's rotation, $\mathbf{r}$ is the radius vector equal to the Earth's radius, R is the gas constant of dry air, and $F_{\mathbf{v}}$ are the rates of angular momentum that are caused by the Reynolds stresses.

The equations for temperature, vapor, liquid fraction, ice, and the passive admixture are as follows:

$$\frac{\mathrm{d}T}{\mathrm{d}t} = \frac{RT}{c_p}\frac{\omega}{p} + F_T, \qquad (2)$$

$$\frac{\mathrm{d}q}{\mathrm{d}t} = F_q, \qquad (3)$$

$$\frac{\mathrm{d}q_l}{\mathrm{d}t} = F_{ql}, \qquad (4)$$

$$\frac{\mathrm{d}q_i}{\mathrm{d}t} = F_{qi}, \qquad (5)$$

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$$\frac{d\chi}{dt} = F_\chi. \quad (6)$$

Here c_p is the specific heat of air at constant volume; ω is the vertical component of the velocity in the p-system of coordinates; F_q, F_{ql}, F_{qi} are non-adiabatic sources (sinks); F_χ is the term describing the source (sink) and diffusion of the admixture.

2.1. The semi-Lagrangian scheme with vertical interpolation. General case.

When numerically solving the equations of atmospheric dynamics in the Eulerian form, one have to calculate the value of a sought variable $X(t + \Delta t)$ at time $t + \Delta t$ at the point F of the domain by use of known value $X(t - \Delta t)$ at the same point but at time $t - \Delta t$. In turn, solving equations in the Lagrangian form, one calculates $X(t + \Delta t)$ at the point F while using $X(t - \Delta t)$ known at some other point O. The trajectory connecting the points O and F is a part of a great circle of a sphere. The middle point of the trajectory is denoted by the letter M. The points O and M can be obtained as a solution of the non-linear system of equations of characteristics by iteration method. In the general case, every equation from the system (1)–(6) can be written in the form

$$\frac{dX}{dt} = \mathcal{A} + \mathcal{F},$$

where $\mathcal{A}$ is the total contribution coming from the dynamic sources, $\mathcal{F}$ is the total contribution from the physical sources, the part $\mathcal{B}$ which is correspondingly resolved here in the dynamic source $\mathcal{A}$ is approximated by the semi-implicit scheme. Let the subscripts (F, O, M) denote the geometric position of a point and the superscripts (+, –, 0) denote the time ($t + \Delta t$, $t - \Delta t$, t). Then the approximation of the above-mentioned equation has the form

$$(X - (1 + \varepsilon_\mathcal{B})\,\Delta t\beta\mathcal{B})^+_F = \{X^- + [(1 - \varepsilon_\mathcal{A})\,\Delta t\mathcal{A} -$$

$$- (1 - \varepsilon_\mathcal{B})\,\Delta t\beta\mathcal{B}]^{\pm 0} + [(1 - \varepsilon_\mathcal{B})\,\Delta t\beta\,\mathcal{B} +$$

$$+ 2\Delta t\,\mathcal{F}\,]^-\}_{F,O,M} + \{[(1 + \varepsilon_\mathcal{A})\,\Delta t\,\mathcal{A} -$$

$$- (1 + \varepsilon_\mathcal{B})\,\Delta t\beta\,\mathcal{B})^{\pm 0}\}_F,$$

if the explicit terms at time t are calculated as the mean value at the end and initial points F and O. The values of the corresponding parameters at these points are obtained by use of isogeometric interpolation procedures.[2]

If the explicit terms at time t are calculated for the middle point M (the values at this point are also obtained by interpolation), we have

$$(X - (1 + \varepsilon_\mathcal{B})\,\Delta t\beta\mathcal{B})^+_F = \{X^- + [(1 - \varepsilon_\mathcal{B})\,\Delta\beta\mathcal{B} +$$

$$+ 2\Delta t\,\mathcal{F}\,]^- - [(1 - \varepsilon_\mathcal{B})\,\Delta t\beta\mathcal{B}]^{\pm 0}\}_{F,O,M} +$$

$$+ \{[2\Delta t\,\mathcal{A}]^{\pm 0}\}_M + \{-[(1 + \varepsilon_\mathcal{B})\,\Delta t\beta\mathcal{B}]^{\pm 0}\}_F,$$

$\varepsilon_\mathcal{A}$ and $\varepsilon_\mathcal{B}$ are the parameters of averaging to be done over the trajectory in the semi-implicit scheme. The operation of averaging permits one to eliminate noise (gravitation waves). The semi-implicit terms enter the equations with the weight β.

2.2. The equations for momentum, temperature, humidity, and passive admixture

Further all the designations correspond to those accepted in Refs. 5–7.

The terms at time t are calculated by the following way:

$$[..]^{\pm 0} = \frac{(1 - \varepsilon_\mathcal{A})\,[..]^{\pm 0}_{F,O,M} + (1 + \varepsilon_\mathcal{A})\,[..]^{\pm 0}_F}{2}$$

for explicit terms and

$$[..]^{\pm 0} = \frac{(1 - \varepsilon_\mathcal{B})\,[..]^{\pm 0}_{F,O,M} + (1 + \varepsilon_\mathcal{B})\,[..]^{\pm 0}_F}{2}$$

for semi-implicit terms.

The equation for momentum

Definitions of X, $\mathcal{A}$, $\mathcal{B}$, and $\mathcal{F}$ and conditions at the Xpper and lower boXndaries are as follows:

$$X = \mathbf{V} + \delta_\mathbf{v}(2\boldsymbol{\Omega} \times \mathbf{r}),$$

$$\mathcal{A} = -2(1 - \delta_\mathbf{v})\,(\boldsymbol{\Omega} \times \mathbf{V}) - \nabla t \quad - RT\nabla(\ln(p)),$$

$$\mathcal{B} = -\nabla\left[\gamma T + \frac{R_a\bar{T}}{\bar{\Pi}}\,\Pi\right], \quad \mathcal{F} = \mathbf{F}_\mathbf{v},$$

$$\mathbf{V}_{\eta=0} = \mathbf{V}_{l=1}, \quad \mathbf{V}_{\eta=1} = \mathbf{V}_{l=L};$$

$\Phi = \gamma T$ is the quasistatics relation; $\eta = f(p, p_s)$ is the generalized (hybrid) vertical coordinate (for instance, σ); pressure at the levels η is determined in the following way: $p = A + b\,\Pi$; $\Pi = p_s$; A and b are the functions of η; L is the number of layers along vertical in the model.

A comprehensive description of the approximation of the equation for momentum in the ECSib model is presented in Ref. 7.

The equation for temperature

Definitions of X, $\mathcal{A}$, $\mathcal{B}$, and $\mathcal{F}$ and the conditions at the Xpper and lower boXndaries are as follows:

$$X = T, \quad \mathcal{F} = F_T,$$

$$\mathcal{A} = \frac{RT}{c_p}\,\frac{\omega}{p}, \quad \mathcal{B} = -\frac{m'^2}{m^2}\,\tau D,$$

τ is the matrix describing contribution of divergence to the trend of temperature; m, m' are metric coefficients in the cartographic coordinate system.

At the upper boundary

$T_{\eta=0} = T_{l=1}$.

At the lower boundary

$T_{\eta=1} = T_{l=L}$.

The semi-Lagrangian approximation of the equation for temperature in the interpolation to the middle point of the trajectory is

$$\left\{T - (1 + \varepsilon_{\mathscr{B}})\ \beta\Delta t \left(-\frac{m'^2}{m^2}\tau D\right)\right\}_F^+ =$$

$$= \left\{\left[T + (1 - \varepsilon_{\mathscr{B}})\ \beta\Delta t \left(-\frac{m'^2}{m^2}\tau D\right) + 2\Delta t F_T\right]^- + \right.$$

$$+ (1 - \varepsilon_{\mathscr{B}})\ \Delta t \left[-\beta\left(-\frac{m'^2}{m^2}\tau D\right)\right]^{\pm 0}\Big\}_{F,O,M} +$$

$$+ \left\{\left[2\Delta t \frac{RT}{c_p}\frac{\omega}{p}\right]^{\pm 0}\right\}_M +$$

$$+ \left\{(1 + \varepsilon_{\mathscr{B}})\ \Delta t \left[-\beta\left(-\frac{m'^2}{m^2}\tau D\right)\right]^{\pm 0}\right\}_F .$$

The equation for humidity q and passive admixture χ

Definitions of X, $\mathscr{A}$, $\mathscr{B}$, and $\mathscr{F}$ and the conditions at the upper and lower boundaries are as follows:

$X = (q, \chi)$, $\mathscr{A} = 0$,

$\mathscr{B} = 0$, $\mathscr{F} = F_q$.

Upper boundary:

$(q_{\eta=0}, \chi_{\eta=0}) = (q_{l=1}, \chi_{l=1})$.

Lower boundary:

$(q_{\eta=1}, \chi_{\eta=1}) = (q_{l=L}, \chi_{l=1})$.

The semi-Lagrangian approximation of the equation for water vapor is

$$\{q\}_F = \{[q + 2\Delta t\ F_q]^-\}_{F,O,M}.$$

Liquid water, ice, and passive admixture χ. The approximation scheme is the same as for water vapor:

$$\{q_l\}_F = \{[q_l + 2\Delta t\ F_{ql}]^-\}_{F,O,M}$$

for liquid water,

$$\{q_i\}_F = \{[q_i + 2\Delta t\ Fq_i]^-\}_{F,O,M}$$

for ice,

$$\{\chi\}_F = \{[\chi + 2\Delta t\ F_\chi]^-\}_{F,O,M}$$

for the admixture.

2.3. The equation of discontinuity with $\Pi = p_s$ as a prognostic variable

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial p}{\partial\eta}\right) = -\frac{\partial p}{\partial\eta}\left(D + \frac{\partial\dot{\eta}}{\partial\eta}\right) + F'_m, \qquad (7)$$

F'_m is the contribution of physical processes to variability of the near surface pressure. We have the following equalities:

$$p = A(\eta) + b(\eta)\ \Pi, \quad \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial A}{\partial\eta}\right) = 0,$$

$$\nabla\left(\frac{\partial A}{\partial\eta}\right) = 0, \quad \frac{\partial\Pi}{\partial\eta} = 0, \qquad (8)$$

where p is the pressure at the corresponding level; η is the generalized vertical coordinate (may be σ, like in the ECSib model). The equalities yield

$$\frac{\partial b}{\partial\eta}\frac{\partial\Pi}{\partial t} + \frac{\partial p}{\partial\eta}D + \frac{\partial}{\partial\eta}\left(\dot{\eta}\frac{\partial p}{\partial\eta}\right) = F'_m. \qquad (9)$$

Approximation. Let us denote $F_m = F'_m\,\Delta\eta$. Now let us consider vertical approximation for each layer l (the layer l is between the intermediate layers $\bar{l}$ and $\bar{l} - 1$). The equation (9) takes the form

$$\Delta B_l \frac{\partial\Pi}{\partial t} + \Delta p_l\, D_l + \left(\dot{\eta}\frac{\partial p}{\partial\eta}\right)_{\bar{l}} - \left(\dot{\eta}\frac{\partial p}{\partial\eta}\right)_{\bar{l}-1} = F_m. \qquad (10)$$

Discrete values for $\dot{\eta}\dfrac{\partial p}{\partial\eta}$ are determined at the intermediate levels $\bar{l}$:

$$\left(\dot{\eta}\frac{\partial p}{\partial\eta}\right)_{\bar{l}} = -\left[B_{\bar{l}}\frac{\partial\Pi}{\partial t} + \sum_{j=1}^{l}\{D_j\Delta p_j + (\mathbf{V}_j\nabla\Pi)\,\Delta B_j\}\right] + \mathscr{C}_{\bar{l}}, \qquad (11)$$

where

$$\frac{\partial\Pi}{\partial t} = -\sum_{l=1}^{L}\{D_l\,\Delta p_l + (\mathbf{V}_l\,\nabla\Pi)\,\Delta B_l\} \qquad (12)$$

and

$$\mathscr{C}_{\bar{l}} = 0,$$

if we suppose that the volume of air occupied by rain droplets is not replaced by dry air when the droplets fall. If the volume is replaced, $\mathscr{C}_{\bar{l}} = gB_{\bar{l}}(P+E) - gF_{p\,\bar{l}}$, $\mathscr{C}_{\eta=0} = 0$, $\mathscr{C}_{\eta=1} = gE$,

Substituting Eq. (11) into Eq. (12), we obtain

$$\Delta B_l\frac{\partial\Pi}{\partial t} - \Delta B_l\left\{\frac{\partial\Pi}{\partial t} + \mathbf{V}_l\,\nabla\Pi\right\} + \Delta\mathscr{C}_l = F_m, \qquad (13)$$

where $\dfrac{\partial\Pi}{\partial t}$ is taken from Eq. (12).

Integrating Eq. (13) over the vertical and using $\sum_{l=1}^{L}\Delta B_l = 1$, we obtain

$$\Pi^+ = \sum_{l=1}^{L}\Delta B_l\left\{\Pi^- + 2\Delta t\left(\frac{\partial\Pi}{\partial t} + \mathbf{V}_l\,\nabla\Pi\right)^{\pm 0}\right\} -$$

$$- 2\Delta t \mathscr{e}^{\pm 0}_{\eta=1} + 2\Delta t \sum_{l=1}^{L} F_m^- . \qquad (14)$$

Using $\sum_{l=1}^{L} \Delta B_l = 1$, one can write F_m as $\sum_{l=1}^{L} \Delta B_l (\sum_{l=1}^{L} F_m)$ and $\mathscr{e}^{\pm 0}_{\eta=1}$ as $\sum_{l=1}^{L} \Delta B_l (\mathscr{e}^{\pm 0}_{\eta=1})$.

In applying the semi-implicit scheme, the t-terms are either interpolated to the middle point or are taken as mean values at the initial and end points.

3. THE CASE OF ADVECTION THROUGH A POLE FOR A SCALAR FIELD WITH LOCAL STRUCTURE

To verify the transfer scheme and some procedures of isogeometric interpolation, we have chosen the test proposed in Ref. 2. In this test, the rate of horizontal advection is put by the formulas

$$u = U[\cos\beta \cos\phi + \sin\beta \sin\phi \cos\lambda],$$
$$v = -U \sin\beta \sin\lambda,$$

where β is the angle between the axis of hard rotation and polar axis; $U = (\pi/46)/2$ radians for a one temporal step Δt at the spherical 72×46 grid in the ECSib model, Δt = 35 min.

A scalar field with the local structure that within the neighborhood centered at the point ($\lambda = 3\pi/2$, $\phi = 0$) is considered as the transferable substance:

$$f(\lambda, \phi) = \begin{cases} 0.5\,(1 + \cos(\pi r/R)), & \text{if } r < R, \\ 0 & \text{in other cases,} \end{cases}$$

where $r = \arccos[\cos(\lambda - 3\pi/2) \cos\phi]$ radians for $|\lambda - 3\pi/2| < \pi/2$; $R = 7(2\pi)/72$ radians.

Figures 1–3 present the distributions of the field f at the moment preceding the transfer of the local structure across the pole, at the moment of structure's transfer through the pole, and after passing the pole. As seen from the figures, the transfer is realized without any changes in the shape.

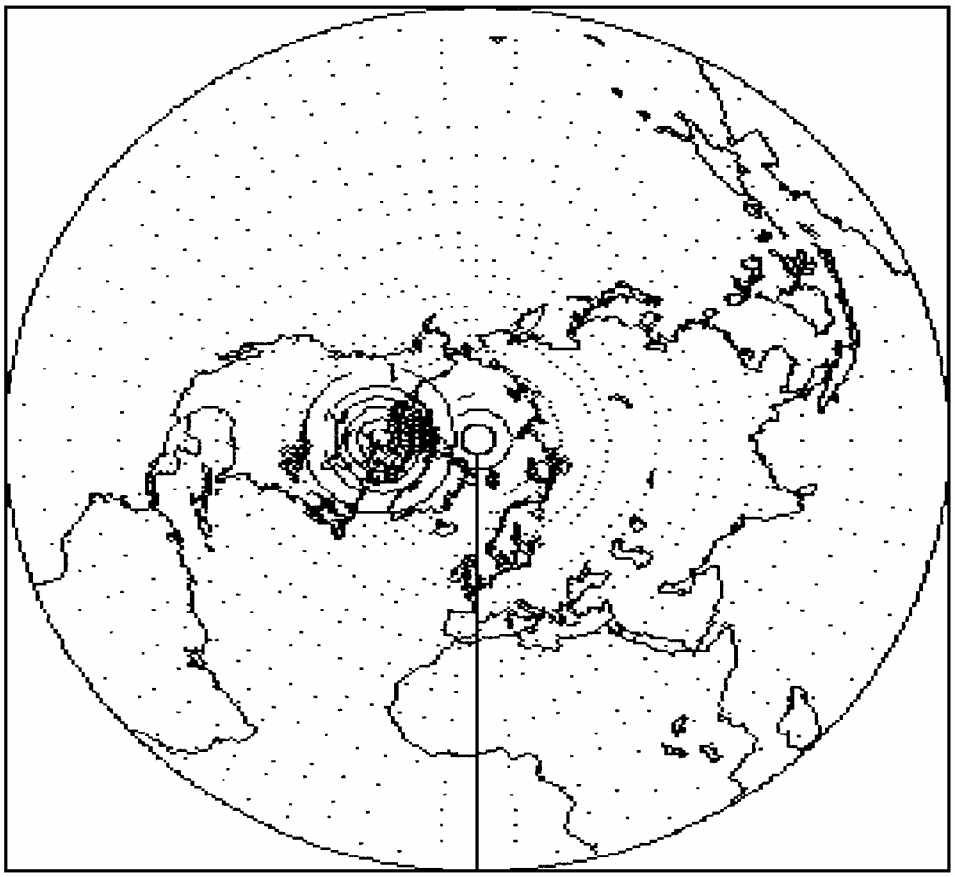

FIG. 1. Distribution of the field f at the time moment preceding the transfer of the local structure across the pole.

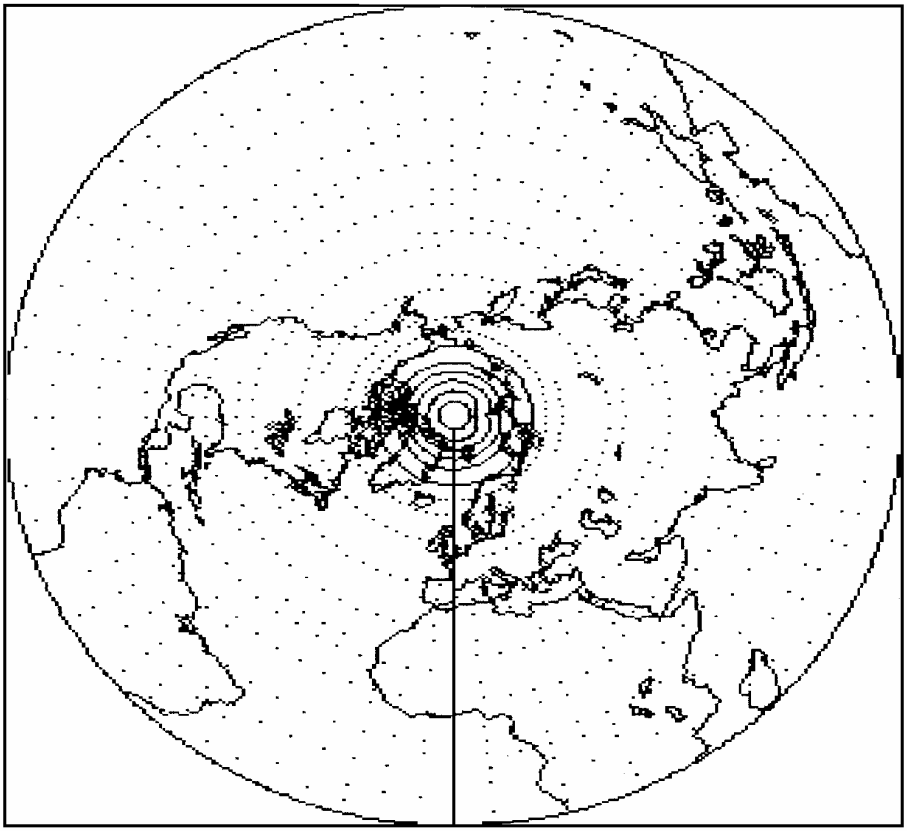

FIG. 2. Distribution of the field f at the moment of structure's transfer across the pole.

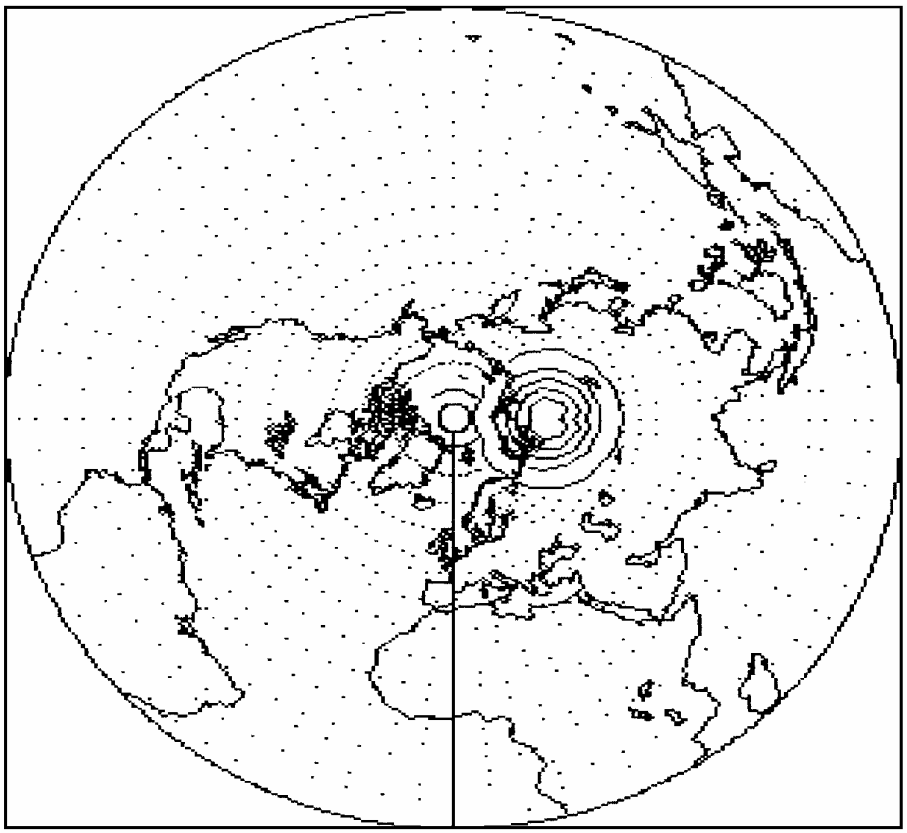

FIG. 3. Distribution of the field f after the local structure passage across the pole.

CONCLUSION

The three-dimensional models of chemical tracers transfer are necessary when estimating global balance of greenhouse and other gases, and, what is important, they make it possible to interpret series of measurements at different points. This paper proposes a description of a three-dimensional semi-Lagrangian version of the climatic model ECSib developed at the Institute of Computational Mathematics and Mathematical Geophysics, Siberian Branch of the Russian Academy of Sciences. Besides, the model permits one to simulate distributions of chemical tracers in the atmosphere.

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