SIMPLEX METHOD FOR OPTIMIZING THE PHASE OF THE BEAM IN A NONLINEAR MEDIUM

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The efficiency of the simplex method of compensation for the stationary wind refraction of the beam is numerically investigated. Control of the beam is performed based on the class of the first– and second–order wavefront aberrations. The search for the maximum illumination from the object being focused with the use of the simplex method is found to reduce the number of measurements of the goal function by the factor of 1.5–2 *in comparison with the gradient procedure.*

The problem of transmission of the light power to a given point is of great practical importance for the applied problems in atmospheric optics. Both adaptive and programmable methods of beam control are used to compensate for the distortions of the light wave produced by a nonlinear refraction and turbulent fluctuations of the refractive index of the medium. The aim of control in problems of atmospheric optics usually is the search for the phase of the light wave for which the criterion of radiation quality in the image plane reaches its maximum value. One of the methods widely used in optical systems with feedback loops is cross–aperture sensing, which makes it possible to optimize any measurable criteria describing the goal of such control under quasistationary conditions. However, gradient procedures forming the basis of the method of cross–aperture sensing often yield only the local extremum of the criterion of quality, strongly depend on the initial conditions, and may even be inefficient in the presence of interference in a feedback loop. For these reasons the development of the methods of phase control of the beams, based on the procedures, which do not require the calculation of the gradient of the goal function, is of great interest.

The present paper is devoted to the study of the efficiency of adaptive compensation for stationary wind refraction of the beam based on the simplex method. The simplest optical aberrations, such as wavefront tilt in the plane of the medium movement and cylindrical focusing in the planes parallel and perpendicular to the direction of medium movements are chosen as the base control modes. The goal function of control is the criterion of focusing, which characterizes concentration of light field within the given region in the image plane.

CONTROL WITH THE HELP OF ADAPTIVE SYSTEM

Here we give the main points of the idea of the simplex method,[1] intended to find the extremum of the goal function in a k–dimensional space of the controllable variables x_i. The sequential simplex method consists in approaching the optimum by repeated reflections of a certain figure (which is called simplex), having $k + 1$ tops, which never belong simultaneously to any space of lesser dimensionality. For example, the simplex is straight–line segment in a one–dimesional case, triangle in a two–dimensional case, and tetrahedron in a three–dimensional case. Regular simplexes, whose tops are spaced equidistantly, are usually used for practical purposes. To determine the direction toward the extremum, the value of the goal function is measured at the tops of the simplex. Searching for the maximum we move from the top with the minimum value of the goal function to the opposite side of the simplex. The step in that search consists in proceeding from the "old" simplex to the "new" one by excluding the worst top and plotting its mirror reflection with respect to the side common for both simplexes. Multiple reflections of the worst tops result in a step–by–step movement of the simplex center toward optimum along some broken line. With the exception of the initial moment when we must calculate (or measure) $k + 1$ values of the goal function, each step needs only one calculation.

The simplest algorithm based on the simplex of a constant size simultaneously provides for neither a high speed of movement at the beginning of the procedure of search nor an accuracy of finding the extremum at the final refining stage. Therefore, to achieve the extremum fast and accurately, algorithms were developed in which the size of the simplex changes in the course of search. One of the tops is retained at each step while the spacing of the tops is either decreased or increased. One may choose as a final top either a newly plotted top or a top with the maximum value of the goal function. The size of the simplex is usually changed following a power–law or exponential dependence. A simple expression relates the accuracy of finding the optimum, the initial size of the simplex, and the number of steps in the procedure. Indeed, as it was demonstrated in Ref. 1, the length of the simplex edge L_N is related to the maximum error in finding the extremum ε_0 and to the dimensionality of the control space k by the formula

$$L_N = \varepsilon_0 \sqrt{\frac{2\kappa}{\kappa + 1}} .$$

For example, prescribing the law of change of the edge length L_n as a function of the step number n in the form $L_n = L_0 \varepsilon^{-\eta n}$, where $\eta = \frac{1}{N} \ln \frac{L_0}{L_N}$, it becomes easy to find the number of optimization steps N, if the initial length of the edge L_0 is estimated in any possible way. To have an *a priori* estimate of L_0, it is natural to use the relation[1]

$$L_0 \le \frac{X}{2 + \kappa} \sqrt{\frac{\kappa(\kappa + 1)}{2}} ,$$

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where X is the characteristic size of the expected range of variation of the controllable coordinate. The empirically estimated value of L_0 may be updated by solving the test problems of optimization.

In this study we control the beam in a three–dimensional space of the lowest–order optical aberrations. Here $x_1 = h$ is the tilt of the wavefront and $x_2 = S_x$ and $x_3 = S_y$ are the reciprocal radii of focusing in the orthogonal directions. Upon entering the nonlinear medium the beam phase is prescribed in the form

$$u(x, y) = \kappa \left(\theta x + S_x \frac{x^2}{2} + S_y \frac{y^2}{2} \right) , \quad (1)$$

where κ is the wave number and x and y are the coordinates in the plane of the medium movement.

The beam propagation is described by the system of dimensionless equations for the complex amplitude of light field and for the perturbations of the temperature T

$$2i \frac{\partial E}{\partial z} = \Delta_\perp E + R_V TE , \quad (2)$$

$$\frac{\partial T}{\partial x} = EE^* , \quad (3)$$

in which the standard normalization is used for the variables.[2] Here we employ the quasistationary approximation of the of heat transfer equation (2). It is valid for a regime of rare enough corrections of a continuous radiation wavefront so that the field is completely established for the time intervals between the subsequent corrections.

We consider the case of optimizing the wavefront of a Gaussian beam for the path of length $z_0 = 0.5$. We used the following criteria for the goal function of control in the image plane:

peak intensity

$$J_m = \frac{1}{I_0} \max_{x,\, y} |I(x, y, z_0)| , \ I = EE^* ; \quad (4)$$

focusing criterion

$$J_f = \frac{1}{P_0} \int \int \exp(-(x^2 + y^2)) \, I(x, y, z) \, dxdy ; \quad (5)$$

integral of the squared intensity

$$J_2 = \frac{1}{I_0 P_0} \int \int I^2(x, y, z) \, dxdy , \quad (6)$$

where I_0 is peak intensity upon entering the medium and P_0 is the total beam power.

NUMERICAL RESULTS

As applied to the problem of optimization of the beam focusing into the nonlinear medium the simplex method can be most vividly illustrated in the case of control in the plane of two variables: S_x and S_y. In this case the trajectory of search can be easily plotted and analized. Since variations of the wavefront curvature result in changing the wind–induced displacement of the beam from its optical axis in a moving medium, the goal function should be chosen in the form of the criterion, insensitive to the position of the energy centroid of the beam, i.e., J_m or J_2. For illustration, Fig. 1 shows the procedure of search for the extremum of the peak intensity J_m on the basis of the simplex method (the best top is retained in each step). The range of variation of the controllable variables was $0 \le S_x$ and $S_y \le 4$, and the accuracy of finding the extremum ε was equal to 10%. In accordance with this, the initial size of the simplex was $L_0 = 2.5$, the increment of the step size $\eta = 0.55$, and the number of steps $N = 5$.

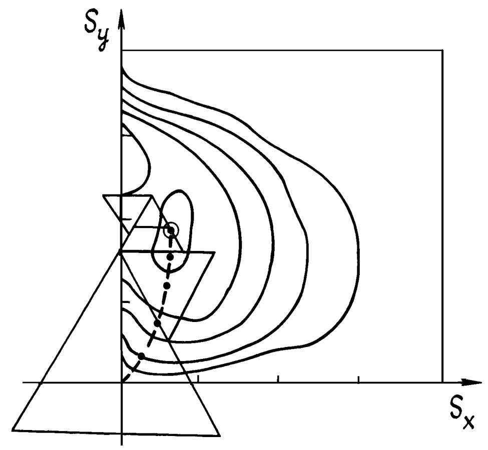

FIG. 1. Contour lines of peak intensity J_m in the plane of the variables S_x and S_y and the trajectories of search for the extremum: solid lines denote the simplex method and dashed lines – gradient methods (iteration steps are indicated by dots). Path is of length $z_0 = 0.5$ and the parameter of nonlinearity $R_V = -28$.

For comparison, the same figure shows the trajectory of search for the extremum by the gradient method with a variable step size.[3] It can be seen that the number of steps needed to reach the extremum with the prescribed accuracy is approximately identical for both methods, but the number of measurements of the goal function in the simplex method is 2–2.5 times less than that in the gradient method (each gradient step is accompanied by test measurements of the criterion of quality along every controllable coordinate).

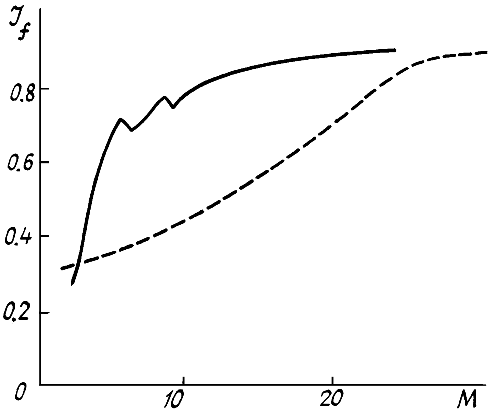

FIG. 2. Criterion of focusing J_f vs the number of its measurements M in the process of optimization . Solid line denote the simplex method, dashed line – gradient method. Path is of length $z_0 = 0.5$ and the parameter of nonlinearity $R_V = -28$.

When control is based on the focusing criterion J_f, which characterizes field localization within the given region in the image plane, the third controllable coordinate must be added – the angle of tilt of the beam θ. The dependence of J_f on the number of its measurements M in the process of optimization is shown in Fig. 2 for both the simplex and gradient methods. It can be clearly seen that the simplex method ensures faster rate of search for the extremum, which can be further increased improving the strategy of the search, e.g., it seems expedient starting from a certain step, to reflect the top, which has not yet been specularly reflected for the last $\kappa + 1$ steps instead of reflecting the worst top of the simplex.[1]

The final results of optimizing the beam phase are listed in Table I. For brevity, the above–described algorithm is called "Simplex–1".

As can be seen from the table the adaptive system of cross–aperture sensing intended to compensate for the stationary wind refraction on the simplex method ensures reliable finding of the maximum of the goal function with the prescribed accuracy. The main advantage of this method over the gradient one is a reduction of the number of measurements of the goal function by a factor of 1.5–2, so that the faster response rates can be achieved without any additional instruments.

TABLE I. Results of comparison of correction of the stationary wind refraction

Method of control	Nonlinearity parameter, R_V	The number of measurements of the goal function M	Parameters of radiation at the object	
			J_f	J_m
Simplex	–14	22	0.50	1.12
	–28	24	0.31	0.61
Simplex–1	–14	15	0.49	1.11
	–28	16	0.30	0.56
Gradient	–14	36	0.49	1.14
	–28	36	0.30	0.60

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