DYNAMICALLY BALANCED X–Y–Z ACTUATOR OF A SEGMENT OF A CONTROLLABLE MIRROR

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We consider in this paper a design of the X–Y–Z actuator of a segment of the a controllable mirror that provides the dynamic separation between movements of a case and mobile elements of the actuator. Such a design excludes the interference among segment actuators installed on a common base.

The composed (segmented) mirror, each segment of which can move in three directions being driven by individual actuators,[1] is one of the promising versions of a controllable mirror design for adaptive–optics systems.

The possibility of using the same construction of actuators for all segments, that simplifies the design and fabrication of a controllable mirror and electronic devices for generation of controlling actions, is among the advantages of the segmented mirror.

All actuators of segments are installed on a common base. In this case, the identity of actuators may cause unwanted effect due to interaction between actuators that could transfer dynamic actions to other segments via a base which is not absolutely rigid. This interaction will be especially significant in the case when each actuator, being a mechanic system, possesses a large Q–factor, i.e. small internal losses. As a result, we have a mechanic system with a number of vibrational elements offering the same resonance frequency.

Traditional methods for providing a high–quality operation of such a system have a lot of drawbacks:

(i) reduction in the Q–factor of a separate actuator may result in reduction of the accuracy of operation due to increasing internal losses;

(ii) increase in the actuator vibration damping with the help of control complicates the construction of an actuator and electronics;

(iii) increase in the base rigidity results in its mass increase;

(iiii) shift in resonance frequencies of separate actuators is in conflict with their unification and complicates the construction.

For radical solution of the problem of diminishing the actuators' interaction, we propose to use such a construction in which there are practically no dynamic interactions between each actuator and a common base.

Let us consider an X–Y–Z actuator of a hexagonal segment of a segmented mirror. The construction comprises three actuators of linear displacements, which fall within the projection of a segment. A peculiar feature of the construction is in the fact that the actuator of a controlled displacement is indirectly joined with a segment by a lever whose axis is fastened to a case.

Shown in Fig. 1 is the design model of an X–Y–Z actuator comprising the case *1*, a hexagonal mirror segment *2*, three identical linear actuators *3* joined with the segment *2* at three points by levers *4*. The actuator case is installed on a common base *5*.

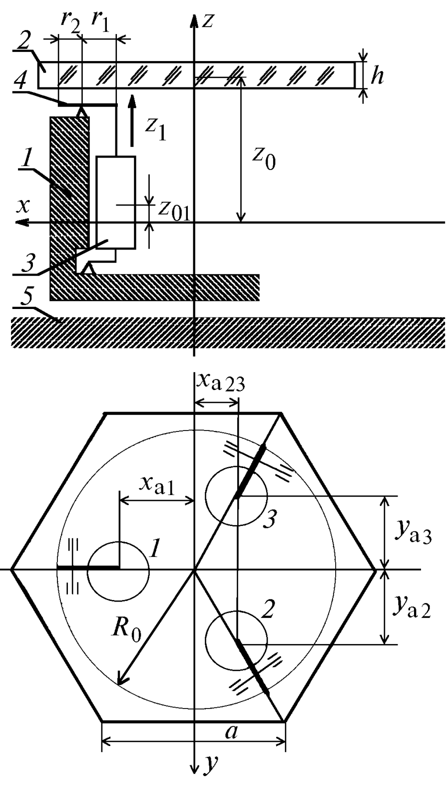

FIG. 1.

The mechanical model of the actuator is described by the following parameters:

(i) the case – M_c, J_{cx}, J_{cy}, J_{cz}, C_{cx}, C_{cy}, C_{cz}, $C_{c\alpha}$, $C_{c\beta}$, $C_{c\gamma}$ (mass, moments of inertia, and rigidity of fastening to the mirror base);

(ii) the mirror – M_0, J_{0x}, J_{0y}, J_{0z}, C_{0x}, C_{0y}, z_0 (mass, moments of inertia, rigidity of fastening to the

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case along the x and y coordinates, and coordinate of the mirror center of mass);

(iii) actuators – M_a, J_{ax}, J_{ay}, J_{az}, C_a, x_{ai}, y_{ai}, z_{ai} (mass, moments of inertia, rigidity in working displacements, coordinates of the center of mass of a separate actuator);

(iiii) transfer levers – R_0, r_2/r_1 (distance from the mirror axis to the point of lever fastening to the mirror, ratio of lever arms fastened to the mirror and to the actuator).

As the generalized coordinates of the model, we took

– x_c, y_c, z_c, α_c, β_c, γ_c – case displacements about the common base,

– z_1, z_2, z_3 – working displacements of actuators about the case,

– x_0, y_0 – mirror displacements about the case in nonworking directions.

Having expressed the displacements of all bodies composing the model of the actuator in terms of the generalized coordinates, we can write down the kinetic energy of the system. Following the Lagrange method, we find the equation of motion in the generalized coordinates:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\mathrm{d}T}{\mathrm{d}\dot{\varphi}_i}\right) = Q_i. \tag{1}$$

The generalized forces result from the rigidity in the corresponding displacements, excluding actuators where the controlling forces exerted by actuator are also added.

We have derived and analyzed a complete expression for the system (1). It has been revealed that in the model chosen it is possible to separate out the motion along the coordinate γ_c. The motions in the coordinates x_c, y_c, x_0, y_0 are related only slightly to other motions, and in the first approximation they can be neglected.

Most interesting are the interactions of the model along coordinates z_1, z_2, z_3, z_c, α_c, β_c, which govern the dynamic interactions between actuators installed on a common base. Having separated this part from Eq. (1), we obtain

$$D(p)X = F, \tag{2}$$

where $D(p)$ is 6×6 polynomial matrix, $X^{\mathrm{T}} = (\alpha_c, \beta_c, z_c, z_1, z_2, z_3)$; $F^{\mathrm{T}} = (0, 0, 0, U_1, U_2, U_3)$ (U_i are the controlling forces).

In derivation of Eqs. (1) and (2) we have made the following assumptions:

– the mirror is a homogeneous hexahedral prism of height h and side a (see Fig. 1);

– the arrangement of actuators' axes is as in Fig. 1:

$$x_{a1} = a/2,\ y_{a1} = 0,\ z_{a1} = z_{a2} = z_{a3} = z_a,$$

$$x_{a2} = -a/4,\ y_{a2} = s/4,$$

$$x_{a3} = -a/4,\ y_{a3} = -s/4,\ (s = a\sqrt{3}).$$

For a homogeneous hexahedral prism

$$J_{0x} = J_{0y} = M_0\left(\frac{5}{2}a^2 + h^2\right) / 12 = J_0, \tag{3}$$

where M_0 is the prism mass.

Taking into account these relationships, we derived the expressions for elements of the matrix D, Eq. (2):

$$\begin{aligned}
&D_{11} = [J_{cx} + 3J_{ax} + J_0 + M_a\,(3z_a^2 + 3a^2/8) + \\
&+ M_0\,z_a^2]\,p^2 + C_{c\alpha};\\
&D_{22} = [J_{cy} + 3J_{ay} + J_0 + M_a(3z_a^2 + 3a^2/8) + \\
&+ M_0\,z_a^2]p^2 + C_{c\beta};\\
&D_{33} = (M_c + 3M_a + M_0)p^2 + C_{cz};\\
&D_{44} = D_{55} = D_{66} = \left[M_a + M_0\,r_2^2/\left(9r_1^2\right) + \right.\\
&\left.+ J_0\,4r_2^2/\left(9r_1^2\,R_0^2\right)\right]p^2 + C_a;\\
&D_{12} = D_{13} = D_{14} = 0;\\
&D_{15} = -D_{16} = \left[M_a a\sqrt{3} - J_0\,2r_2/\left(\sqrt{3}\,r_1 R_0\right)\right]p^2;\\
&D_{23} = 0,\\
&D_{24} = -2/\sqrt{3}\,D_{15},\ D_{25} = D_{26} = -D_{24}/2;\\
&D_{34} = D_{35} = D_{36} = [(M_a - M_0\,r_2/(3r_1)]\,p^2;\\
&D_{45} = D_{46} = D_{56} = \left[M_0\,r_2^2/\left(9r_1^2\,R_0^2\right)\right]p^2.
\end{aligned} \tag{4}$$

It should be noted that $D_{ij} = D_{ji}$ for the matrix $D(p)$.

Diagonal elements of the matrix $D(p)$ characterize the dynamics of each isolated motion. Let us note that in the model under consideration all isolated motions are taken as undamped that is acceptable with a low level of internal losses.

Elements D_{34}, D_{35}, and D_{36} characterize the effect of the working displacements of actuators z_i on the vibrations of the case z_c that can be transferred to the other actuators through the common base.

To exclude this effect, the following relation

$$r_2/r_1 = 3M_a/M_0. \tag{5}$$

should be chosen.

Elements D_{15}, D_{16}, D_{24}, D_{25}, and D_{26} reflect the effect of actuator motions on rotations of the case. If the condition

$$M_a(a/2) - J_0[2r_2/(3r_1\,R_0)] = 0 \tag{6}$$

is satisfied, these motions can be separated.

Elements D_{45}, D_{46}, D_{56} characterize the dynamic interconnection between the separate channels of the X–Y–Z actuator. To exclude this interconnection, it is necessary to provide the fulfilment of the following condition:

$$M_0 - 2J_0/R_0^2 = 0. \tag{7}$$

The condition (5) should be considered as most important since the interaction between actuators through the common base is mainly due to vibrations normal to the plane of the base.

If the condition (5) is satisfied, the condition (6) takes the form

$$M_a(a/2) - J_0[2M_a/(M_0R_0)] = 0,$$
$$M_0aR_0 = 4J_0. \qquad (8)$$

Taking into account Eq. (3), the condition (7) can be converted into the form

$$R_0^2 = (5a^2 + 2h^2)/12, \qquad (9)$$

while the condition (8) can be written as

$$R_0 = (5a^2 + 2h^2)/6a. \qquad (10)$$

If the ratio $h/a = 0.1$ is taken, then we derive $R_0 = 0.6468a$ for Eq. (9) and $R_0 = 0.8367a$ for Eq. (10).

For $h/a = 0.2$ $R_0 = 0.6506a$ for Eq. (9) and $R_0 = 0.8467a$ for Eq. (10).

The conditions (7) and (8) cannot be satisfied at the same time, therefore we propose two approaches:

– to provide the dynamic separations of the actuator motions and the case rotations following Eqs. (8) and (10), whereas the separation of motions of actuators will be provided by generation of controlling forces in the form written by Eq. (2);

– to provide the dynamic separations of actuator channels following Eqs. (7) and (9), ignoring the case motions along coordinates a_c and β_c and assuming that this effect on other actuators installed on the common base will be transformed to the case vibrations in a coordinate, the dynamic separation in which is already provided by the fulfilment of the condition (5).

Additional effect from the introduction of a lever into the construction of segment actuator is in the following. In the case when

$$r_2/r_1 < 1$$

the inertial load of mirror applied to the actuator rod is reduced by the factor of $(r_2/r_1)^2$, that increases the rate of the system operation. When designing a concrete construction of the $X–Y–Z$ actuator, having refined the parameters x_{ai}, y_{ai} and relation (3), one will have a need to recalculate the coefficients of the matrix $D(p)$ that may yield some corrections to the relations (9) and (10).

The change in the actuator construction proposed here allows the operation of a large number of identical actuators installed on the common base to be essentially improved by sufficiently simple means.

REFERENCES

1. *Adaptive Optics* [Russian translation ed. by E.A. Vitrichenko] (Mir, Moscow, 1980), 456 pp.