RELATION BETWEEN CONJUGATION ERROR AND INVERTED FIELD AMPLITUDE DEVIATION

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In this paper, we investigate the error transfer characteristic in an adaptive system operating in a turbulent reciprocal medium for different field modifications in the conjugation plane.

Adaptive optical systems harness the principle of reciprocity in the turbulent atmosphere.[1] Influence of the input error on the output error is one of the principal characteristics of the system. It is of great interest to find the relation between the conjugation error ε_c in the conjugation plane due to any field modification and the deviation ε_0 of the amplitude of inverted field from the amplitude of initial field in the plane of a source. To study this dependence, numerical experiments were carried out for the experimental configuration shown in Fig. 1.

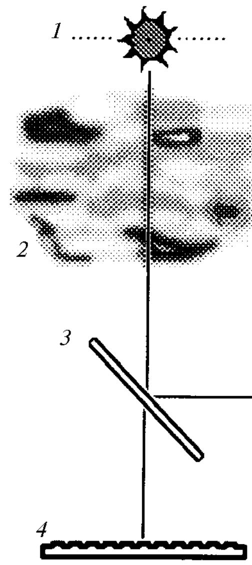

FIG. 1. Diagram showing the experimental configuration of optical system considered in the numerical experiment: 1) plane of a light source, 2) turbulent medium, 3) light-splitting plate, conjugation plane, 4) and 6) receivers, and 5) Fourier transformer.

Let us define the errors ε_c and ε_0 as ratios of the norms integrated with the square of functions in one-half powers

$$\varepsilon_c = \left\{ \frac{\sum\limits_{i=1}^{N} \sum\limits_{j=1}^{N} | W_c(i, j) - \tilde{W}_c(i, j) |^2}{\sum\limits_{i=1}^{N} \sum\limits_{j=1}^{N} | W_c(i, j) |^2} \right\}^{1/2},$$

$$\varepsilon_0 = \left\{ \frac{\sum\limits_{i=1}^{N} \sum\limits_{j=1}^{N} [A_0(i, j) - \tilde{A}_0(i, j)]^2}{\sum\limits_{i=1}^{N} \sum\limits_{j=1}^{N} A_0(i, j)^2} \right\}^{1/2}.$$

The error ε_c is defined for complex functions $W_c(i, j)$ and $\tilde{W}_c(i, j)$. The first function is the source field transmitted through the turbulent medium, and the second is the same field after its modification. Both of them are considered in the conjugation plane. The error ε_0 is defined for real functions, here $A_0(i,j)$ is the field amplitude of the source, and $\tilde{A}_0(i, j)$ is the inverted field amplitude in the source plane produced by conjugating $\tilde{W}_c(i, j)$ in the conjugation plane. The choice of ε_0 definition is explained by our desire to compare the quantities with identical dimensionalities. In this case, there are the field in the conjugation plane and the field amplitude in the source plane. In addition, adaptive system is usually destined for forming an image or concentration of energy, so it is not necessary to study the effect of the inverted field phase.

We studied the propagation of a Gaussian beam in the turbulent atmosphere modeled with a phase screen with different values of the Fried radius[2] r_0. The energy conservation law and the principle of reciprocity were fulfilled with computer accuracy. The order of the matrix of readings was 32 and 64. Spectral density of the Gaussian beam phase fluctuations and the other parameters have the following form:

$$F_s(\kappa) = 0.489 r_0^{-5/3} (\kappa^2 + \kappa_0^2)^{-11/6},$$

$$\kappa_0 = 2\pi / L_0,\ L_0 = 1\ \text{m},\ L = 6\ \text{km},$$

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$$r_0 = \left[0.423k^2 \int\limits_L C_n^2(l)\mathrm{d}l \right]^{-3/5},$$

$k = 2\pi/\lambda$, $\lambda = 0.6328\ \mu$m.

The errors ε_c and ε_0 were calculated for each realization of the numerical experiment for different field modifications enumerated below.

1. Addition of additive or multiplicative noise to the field. The noise was taken to be low-frequency and nonnegative, with nearly normal distribution.

2. Setting of a threshold for the field intensity. Then the field was taken to be equal to zero at the points of the conjugation plane where its intensity was lower than the preset threshold that was equal to a fraction of the field intensity maximum.

3. Setting of a threshold for the squared moduli of the field spatial frequency spectrum. Only such a portion of the Fourier transform of the function $W_c(i, j)$ that had the squared moduli greater than the preset threshold was conjugated.

4. The field and the spatial frequency spectrum were modified together. In this case, the field intensities lower than the preset threshold were replaced by the Fourier transform of the spatial frequency spectrum modified by item 3. Then the resultant field was conjugated.

The modification according to items 2–4 means, for example, elimination of discontinuities of the wavefront and phase dislocations which connected with low levels of field intensity or spectrum.

5. Narrowing the field spatial frequency spectrum. This modification means that only such part of the field whose spatial frequencies fall within a circle of the preset radius was conjugated. In different experiments the center of the circle was taken at the point of the global maximum of the moduli of the spatial frequency spectrum, at the spectrum centroid, and at the origin of coordinates of the spatial spectrum. A search for maximum energy of the spatial spectrum was also conducted within the circle of the preset radius.

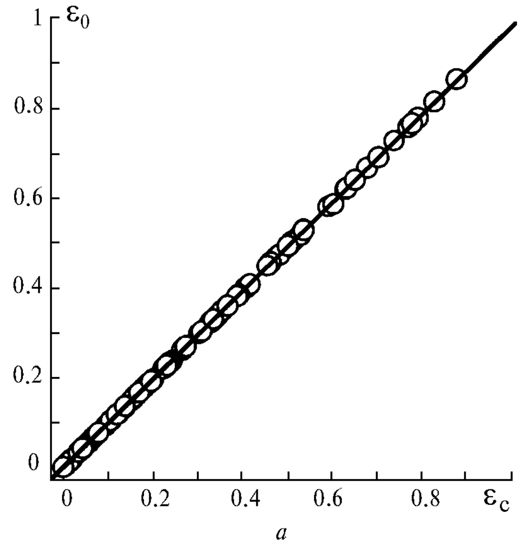
The noise variance, thresholds, and radius varied so that the range of ε_c variation from 0 to 1 was uniformly filled.

The results of numerical experiments with the use of modification by item 1 are shown in Fig. 2*a*, and with the use of modification by item 4 they are tabulated in Table I. The results of all other experiments (see items 2, 3, and 5) were identical for the entire range of error variation and are shown in Fig. 2*b*. It turned out that for the experiment by item 1 the random processes ε_c and ε_0 are highly correlated and are described by linear regression in the range of error variation from 0 to 1, Fig. 2*a*. These two processes practically coincide.

Another situation is shown in Fig. 2*b*. Linear regression well describes the processes only for a part of the range of error variation ($\varepsilon_c < 0.7$ and $\varepsilon_0 < 0.5$), and the regression line lies below the quadrant bisector. Then the rate of growth of ε_0 increases, and the correlation between the random errors decreases.

TABLE I.

r_0, m	0.03	0.1	0.3	1	3
ε_c (field)	0.40	0.16	0.12	0.10	0.10
ε_c (field + spectrum)	0.12	0.08	0.04	0.03	0.02
ε_0 (field)	0.50	0.18	0.14	0.12	0.12
ε_0 (field + spectrum)	0.28	0.11	0.06	0.05	0.06



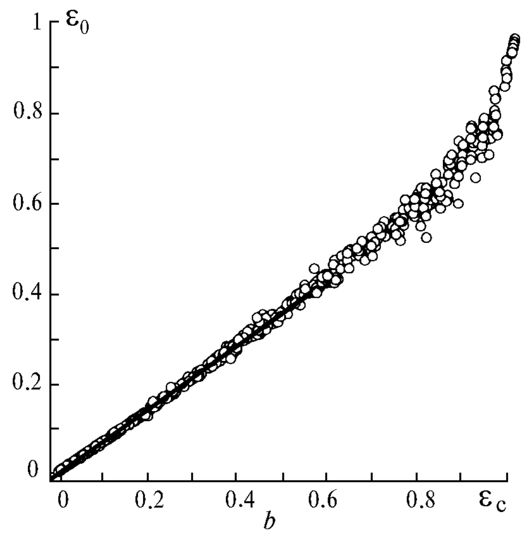


FIG. 2. Relation between ε_c and ε_0 at $r_0 = 0.05$ m: a) additive and multiplicative noise in the conjugation plane, b) setting of the threshold for the field to be conjugated, its spatial frequency spectrum, and narrowing the spatial frequency band of the field to be conjugated. Parameters of the regression line: constant: a) 0.0004 (0.0002), *b)* 0.002 (0.0004)*; slope: a)* 0.998 (0.0004), *b)* 0.727 (0.002)*; standard deviation of the points from the regression line: a)* 0.001, *b)* 0.0052*; number of points: a)* 106, *b)* 370*; correlation coefficient between ε_c and ε_0: a), b)* 0.999.

The case of joint conjugation (item 4), when the conjugation threshold was equal to 0.1 for the field and 0.05 for the spatial frequency spectrum, is of interest.

It is seen from Table I that ε_0 is smaller for the joint conjugation of field and spectrum than for independent conjugation of field with the same threshold.

CONCLUSION

It is revealed from the results of our numerical experiments that not only value of the error in conjugating the reciprocal field in the conjugation plane influences the amplitude of inverted field, but also way it is introduced into the field to be conjugated. We can explain this fact by nonlinear nature of the amplitude that is nonadditive function of the fields producing it.

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