MODAL CORRECTION FOR DISTORTIONS OF A LASER BEAM PROPAGATING THROUGH A GAS CELL

I.E. Tel'pukhovskii and S.S. Chesnokov

M.V. Lomonosov State University, Moscow
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In this paper we consider a closed numerical model of thermal blooming of the single–mode, partially coherent radiation propagating under conditions of self–induced convection. Study of compensations for the laser beam phase distortions is carried out using optical modes of the first, second, and third orders. Efficiency of compensation is estimated by a spectral criterion characterizing angular divergence of the beam at the cell exit.

When a laser beam propagates in a closed volume containing a gas or liquid the convection flows appear in a medium, which affect the formation of the temperature field inducted by the beam. Since the analysis of thermal defocusing of the beam under conditions of self–induced convection is complicated, only a few papers (see, for example, Refs. 1–4) are devoted to the study of this problem in self–consistent statement. The prediction of the phase distortions, whose the laser beam undergoes in a cell with gas, and the possibility for their compensation procedure is of most important interest for applications.

This problem was previously analyzed in Ref. 4. The phase distortions of the first and second orders were found in Ref. 4 to predominate at the cell exit for the collimated beam. They can effectively be compensated by the elastic mirror controled by four drives. However, the analysis[4] of contribution of modes of a higher order than second one into the output phase of a beam seems to be insufficient. It is also important to elucidate the self–interaction peculiarities and to evaluate the limit possibilities of compensation for phase distortions of multimode radiation. The present paper is devoted to study of aforementioned problems.

1. A MODEL OF RADIATION INTERACTION WITH A MEDIUM

Let us assume that the light beam with an initial radius a_0 propagates along a longitudinal axis of a horizontal gas cell z_0 in length and l in transverse size. Based on the estimates made in Ref. 4, the influence of a boundary layer at front and back walls of the cell on distribution of the velocity and temperature over its volume can be neglected. As a result, a three–dimensional problem of hydrodynamics amounts to a set of plane (two–dimensional) problems, whose number coincides with the number of steps N_z of discretization over the longitudinal variable z. In each of the planes $z = \text{const}$ the motion of viscous heat–conducting gas is described by a set of Navier–Stokes equations in the Boussinesq approximation, which is commonly written in the variables "current function ψ – vorticity ω"

$$\frac{\partial \omega}{\partial t} + (\mathbf{V}\nabla_\perp)\,\omega = \frac{1}{\mathrm{Re}}\,\Delta_\perp\,\omega + \frac{q}{\mathrm{Re}^3}\,\frac{\partial T}{\partial x}\,; \quad (1)$$

$$\frac{\partial T}{\partial t} = (\mathbf{V}\nabla_\perp)\,T = \frac{1}{\mathrm{Pr\ Re}}\,\Delta_\perp\,T + f\,; \quad (2)$$

$$\Delta_\perp\,\psi = -\omega\,. \quad (3)$$

In this system of equations the operators $\nabla_\perp$ and $\Delta_\perp$ are taken with respect to the transverse coordinates x and y, while the gas velocity $\mathbf{V}$ consists of the two components V_x and V_y, related to the current function by the relationships $V_x = \partial\psi/\partial y$, $V_y = -\partial\psi/\partial x$. For dimensionless variables, appearing in Eqs. (1)–(3) the commonly accepted normalization was used (see, for example, Ref. 4). The basis for the above normalization is the characteristic velocity of motion of a medium in the regime of developed convection:

$$V_c = \nu\, q^{1/3} / a_0\,, \quad (4)$$

where $q = \alpha\, I_0\, a_0^5\, \beta\, g/(\nu^3\, \rho_0\, C_p)$ is the dimensionless thermal system. The remaining designations in Eqs. (1) and (2) are the following: $\mathrm{Re} = a_0\, V_c/\nu$ is the Reynolds number, $\mathrm{Pr} = \nu/\chi$ is the Prandtl number, and f is the function characterizing the profile of laser beam intensity.

In the quasioptical approximation of the diffraction theory the light beam propagation is described by the dimensionless equation with reference to a complex slowly varying amplitude of a light wave

$$2i\,\frac{\partial E}{\partial z} = \Delta_\perp E + RTE\,, \quad (5)$$

where R is the nonlinearity parameter,[5] proportional to the beam power P_0 and to the time of radiation interaction with the medium $\tau_\nu = a_0/V_c$ in the stationary convection regime. Thus, the beam and the medium influence each other by the perturbation field of temperature T and the profile of thermal source $f = |E|^2$ and such a mutual influence is determined by the four parameters of similarity: R, Re, Pr, and q.

At the cell entrance (at $z = 0$) a boundary condition is given for the complex amplitude E:

$$E(x,\, y,\, 0,\, t) = E_0(x,\, y)\, F(t)\, \exp\,[i\; U(x,\, y,\, t)]\,,$$

where E_0 is the profile of beam amplitude, F is the temporal envelope curve of a light pulse, and U is the phase profile. When analyzing the thermal blooming of single–mode radiation we prescribe E_0 in the form of Gaussian function

$$E_0 = \exp\,[-\,(x^2 + y^2)/2]\,, \quad (6)$$

while U we take to be equal to zero (a collimated beam). In the multimode regime the radiation is assumed to have

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Gaussian statistics with the function of mutual coherence of the form:

$$\Gamma_2(x, y, x', y', 0) =$$
$$= \exp\left\{-\frac{1}{2}(x^2 + y^2 + x'^2 + y'^2) + \frac{N_c}{2}[(x - x')^2 + (y - y')^2]\right\}, \quad (7)$$

where $N_c = a_0^2/r_{c0}^2$ is the number of inhomogeneities in the initial beam cross section determining the transverse modes and r_{c0} is the initial correlation radius. In the numerical experiment the random realizations of beam amplitude at the entrance into the medium are given by spatial distributions of real and imaginary parts

$$\tilde{E}(x, y, 0, t) = \tilde{u}(x, y, t) + i\,\tilde{v}(x, y, t) . \quad (8)$$

To simulate the random fields $\tilde{u}$ and $\tilde{v}$, the method of frequency sample[6] was used with subsequent averaging over 100 realizations at each temporal step.

2. ANALYSIS OF LIGHT FIELD AT THE CELL EXIT AND CORRECTION FOR PHASE DISTORTIONS

The spatial radiation structure in the plane of the cell exit window ($z = z_0$) should be characterized by the spectral criterion

$$J_\Omega(t) = \frac{1}{4\pi P_0} \int\int \Omega(k_x, k_y)\, |\tilde{\tilde{E}}(k_x, k_y, z_0, t)|^2\, dk_x\, dk_y, \quad (9)$$

playing the role of relative contribution of light power, concentrated at a fixed solid angle Ω. Here k_x and k_y are the projections of the wave vector onto the plane perpendicular to the direction of beam propagation and $\tilde{\tilde{E}}$ is the spectrum of a complex amplitude of a wave. Under conditions of strong fluctuations of light field it is also convenient to use the integral criterion:

$$J(t) = \int_0^t J_\Omega(\tau)\, d\tau , \quad (10)$$

where t is the observation time. In the numerical experiments the solid angle Ω was chosen in such a way that in the absence of nonlinear distortions the value of the criterion J_Ω limited by diffraction was 0.5.

To organize the control over wave front of output radiation $\phi(x, y, t)$ in order to decrease its angular divergence we shall use the expansion of ϕ minus the constant component using the system of lowest optical aberrations and separating out the residual distortions $\tilde{\phi}$ in an explicit form

$$\phi(x, y, t) = \phi_N(x, y, t) + \tilde{\phi}(x, y, t) , \quad (11)$$

where

$$\phi_N(x, y, t) = \sum_{k=1}^{N} a_k(t)\, Z_k(x, y) \quad (12)$$

is the phase low–mode components ($N \le 9$), $Z_1 = x$, $Z_2 = y$, $Z_3 = 2p^2 - 1$, $Z_4 = x^2 - y^2$, $Z_5 = xy$, $Z_6 = (3p^2 - 2)x$, $Z_7 = (3p^2 - 2)y$, $Z_8 = (x^2 - 3y^2)x$, $Z_9 = (y^2 - 3x^2)y$, and $\rho = \sqrt{x^2 + y^2}$, the coefficients of expansion (12) in basis are determined by the formulas

$$a_k(t) = \frac{1}{||Z_k||} \int\int \phi(x, y, t)\, Z_k(x, y)\, dx\, dy ,$$

where $||Z_k||$ is the norm of kth mode.

The nonlinear distortions obtained by a beam in the cell is proposed to be minimized in real time by subtracting of its low–mode component $\phi_N(x, y, t)$ from the running output phase $\phi(x, y, t)$.

Depending on the radiation conditions the quality of correction is conveniently estimated from improving the spectral criterion $J_\Omega(t)$ and from relative increasing the integral criterion

$$\eta(t) = J(t) / J_{\text{without}}(t) , \quad (13)$$

where $J_{\text{without}}(t)$ is the running value of the criterion without control.

3. RESULTS OF NUMERICAL SIMULATION

3.1 Dynamic structure of nonlinear distortions and their composition of modes. Phase nonlinear distortions of laser radiation under conditions of self–induced convection were calculated over a wide range of values of the heat release parameter ($q = 10^3 \ldots 10^5$) corresponding to the change of the nonlinearity parameter in the range of $200 \le |R| \le 4500$.

An investigation of composition of phase distortion modes has shown that at different values of the heat release parameter the behavior of lowest–order aberrations is similar. These aberrations differ mainly in the amplitude and time of determination of a stationary temperature field τ_{stat} (Table I). In this case maximum distortions are attained at $t = \tau_{\text{stat}}/2$. As an example, Fig. 1 shows the time dependences of expansion coefficients of beam phase in basis (12).

TABLE I. Stationary values of coefficients of expansion of output phase in basis (12) and times of determination of stationary temperature field at different values of q.

q	$\tau_{\text{stat}}/\tau_v$	Tilt, a_2	Defocusing, a_3	Astigmatism, a_4	Coma, a_7	Coma, a_9
10^3	30	1.56	1.1	–0.31	–0.29	0.25
10^4	15	3.39	2.37	–0.49	–0.75	0.75
10^5	7.8	7.88	5.74	–0.79	–1.77	1.59

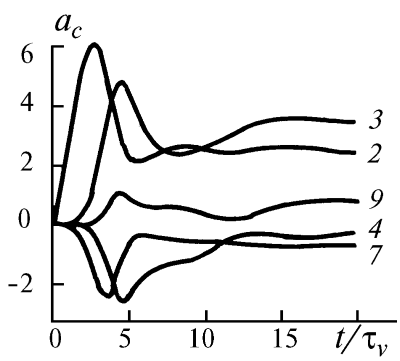

FIG. 1. Time dependences of expansion coefficients of a beam phase at the cell exit in basis of lowest aberrations at $q = 10^4$. Figures at the curves correspond to numbers of modes in (12).

The results of numerical experiments based on correction of mode of phase nonlinear distortions are presented in Fig. 2 for the heat release parameter $q = 10^4$.

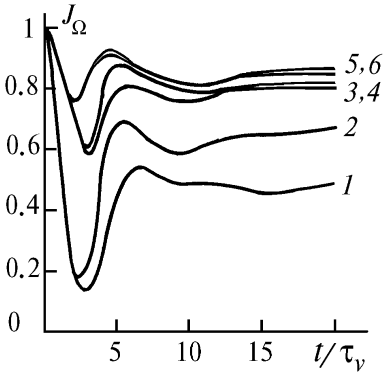

FIG. 2. Time dependences of the spectral criterion J_Ω at modal compensation for beam phase distortions. Without compensation (1), compensation along slant path (2), compensation for the modes of the first and second orders (3 and 4), and for the modes of the first, second, and third orders (5 and 6).

It can be seen that the main contribution to distortions is made by a lens distortion (in the stationary regime the elimination of this aberration results in the 40% improvement in spectral criterion (9)). The effect of aberrations of the second order is somewhat less (the 15% improvement of the criterion) and the elimination of coma leads to an increase of the spectral criterion by 10%. The regularity determined, namely, "saturation" of compensation quality with increase of order of controllable modes is confirmed by the calculations at other values of q (Table II). At the same time, the analysis of this table indicates that the spatial structure of phase distortions is complicated with increase of the heat release parameter. This results in the increase of relative contribution of highest aberrations, which, in principle, cannot be eliminated by means of modal corrector (12).

TABLE II. Relative contribution of different modes to the stationary phase of a beam at the cell exit determined by the spectral criterion J_Ω.

q	Tilt, %	Mode of the second order, %	Mode of the third order, %	Highest modes, %
10^3	45.8	31.7	5.8	16.7
10^4	36.1	28.2	8.7	27.0
10^5	7.2	20.6	10.3	61.9

3.2 Limiting possibilities of compensation for phase distortions of multimode radiation. Since a reliable analysis of thermal blooming of multimode radiation demands the laborious statistical processing the only value of the heat release parameter was taken in calculations, namely, $q = 10^4$. The quality of correction was estimated using the parameter $\eta(t)$ from Eq. (13) with averaging of output light field over 100 realizations. The ratio of the initial correlation radius to the initial beam radius r_{c0}/a_0 being characteristic of degree of input radiation coherence varies from 1/3 to 2. Figure 3 shows the dynamic dependences of the parameter characterizing the relative improvement in the integral criterion $\eta(t)$ for different values of the ratio r_{c0}/a_0 at a control over complete basis (12) composed of nine modes. It is clear that the efficiency of phase compensation decreases with decrease of the initial radius of coherence and at $r_{c0}/a_0 = 1/3$ the parameter η does not exceed 1.2 over entire time interval. It should be noted that in the case of single–mode radiation under the same conditions of propagation we succeeded in improving of the integral criterion almost by a factor of two.

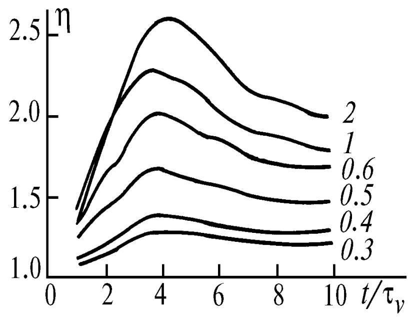

FIG. 3. Time dependences of the quality parameter η of compensation for thermal blooming of multimode radiation at different values of the ratio r_{c0}/a_0 indicated near the curves.

The calculations made at different quantity of controllable modes indicate that in the case of thermal blooming of multimode radiation the saturation effect of correction quality is also observed with increase of dimensionality of a control basis (Fig. 4). The parameter r_{c0}/a_0 is the main factor determining in this case the relative contribution of irremovable phase distortions.

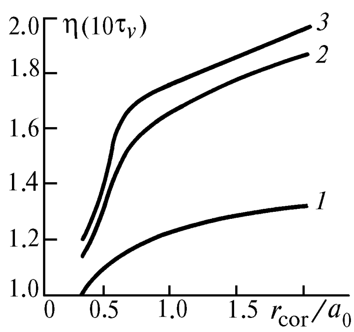

FIG. 4. The parameter of compensation quality $\eta(10\tau_v)$ depending on r_{c0}/a_0. Compensation along slant path (1), compensation for the modes of the first and second orders (2), and for the modes of the first, second, and third orders (3).

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