INSTABILITIES OF THERMAL BLOOMING

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Some effects resulting from an unstable behavior of intense wave beams under conditions of thermal blooming in a weakly absorbing medium are considered. Numerical simulation of the problem is carried out by solving a self–consistent problem using the splitting method. To provide a calculational stability a control procedure based on a spectral criterion of stability is used during the solution process. It is shown that in a stationary regime of a limited beam thermal blooming in a moving medium there can occur the decay instability as well as the occurrence of soliton–like solutions is possible. Instability of a phase conjugation that can appear during an adaptive control of an intense beam is analyzed.

In recent years researchers and designers of adaptive optical systems[1,2] concentrated their attention on the problems of a thermal blooming instability, in particular, on the stable behavior of a laser beam when its adaptive control is performed by the phase conjugation method. In spite of the fact that the qualitative aspect of such processes is clarified for some cases, it is difficult to describe them quantitatively, especially in the case of dynamic control of a beam.

Numerical simulation is the basic method intended for studying the instabilities which occur in the powerful laser beams propagating through the atmosphere as well as for solving applied problems of thermal blooming. It is necessary to apply the nonlinear wave theory and hydrodynamics of continuous media[3,4] to the mathematical description of thermal blooming of a beam

$$\begin{cases} \dfrac{\partial E}{\partial z} = \dfrac{i}{2}\left(\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2} + T\right) E, & (1) \\ \dfrac{\partial T}{\partial t} + \left(V_x \dfrac{\partial}{\partial x} + V_y \dfrac{\partial}{\partial y}\right) T = EE^* . & (2) \end{cases}$$

Equations (1) and (2) are the evolution quasilinear differential equations in partial derivatives of the first and second orders. At the present time the system of Eqs. (1) and (2) can be solved only by numerical methods. In their turn, calculational algorithms developed in terms of difference schemes and discrete series can be the cause of instabilities themselves, even for linear equations.[5]

Recently, a wide class of stable difference schemes for the homogeneous equations of the hyperbolic and parabolic types has been studied. The spectral and energy characteristics of stability were developed for both stationary and nonstationary problems. Absolute stability of the component–by–component splitting method for solving the quasilinear evolution equations has been proved theoretically.[6]

In this paper numerical simulation of some effects of the unstable behavior of intense wave beams is presented based on the modified splitting method proposed earlier.[8] In order to provide the calculational stability and obtain reliable results we use the procedure of a continuous control through the whole algorithm performance which is based on the spectral criterion of stability.

1. COMPUTATIONAL INSTABILITY

The splitting method is the indisputable leader among numerical methods intended for solving evolution equations (1) and (2). It has the second order of approximation with respect to z in a symmetric computational scheme and it is absolutely stable for sufficiently smooth functions E and T even for nonhomogeneous quasilinear equations.[6] However, as the experimental calculations show, the smoothness condition of functions E and T can be violated in the process of numerical simulation of the problem in the case of large inhomogeneities of a medium.

Figure 1 shows evolution of the spatial spectrum of the field at the stage of the numerical solution of the stationary problem of thermal blooming. The spectral analysis of the solution convincingly shows that nonlinearity can lead to a fast increase in the intensity of high frequency harmonics.

To remove instabilities in calculations it is necessary to introduce certain procedures of the solution control. Let us write Eq. (1) in the operator form

$$\frac{\partial E}{\partial z} = (L_D + L_R)\, E , \tag{3}$$

where $L_D = \dfrac{i}{2}\left(\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}\right)$ is the diffraction operator and $L_R = \dfrac{i}{2}\, T(x, y, z)$ is the refraction operator.

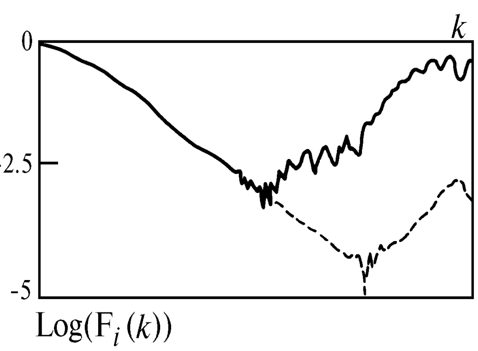

FIG. 1. Increase of the spatial spectrum harmonics of the field on the step Δz: $z = z_j$ (dashed curve) and z_{j+1} (solid curve).

We apply the splitting procedure to Eq. (3), i.e., divide the axis z of the evolution variable z to $N = z/\Delta z$ steps and write down the system of equivalent equations at the jth step in a symmetric form

$$\begin{cases} \dfrac{\partial E_D}{\partial z} = L_D E_D, & E_D(\Delta z) = E(0) ; \\ \dfrac{\partial E_R}{\partial z} = L_R E_R, & E_R(0) = E_D(\Delta z/2) ; \\ \dfrac{\partial E_D}{\partial z} = L_D E_D, & E_D(\Delta z/2) = E_R(\Delta z) . \end{cases}$$

Such a system is known to have the second–order approximation

$$||\tilde{E}(z) - E(z)|| = o(\Delta z^2) ,$$

where the exact solution

$$E(z) = \exp\left\{ \int_0^z (L_D(\zeta) + L_R(\zeta))\, d\zeta \right\} E(0)$$

and the approximate one

$$\tilde{E}(z) = \exp\left\{ L_D(\Delta z/2) + \sum_{j=2}^{N} (L_D^j(\Delta z) + L_R^j(\Delta z)) + \right.$$

$$\left. + L_R(\Delta z) + L_D(\Delta z/2) \right\} E(0)$$

are written in the operator form.

Let us introduce a control operator L_C to the algorithm of the approximate solution

$$L(z) = L_D(\Delta z/2) + \sum_{j=2}^{N} (L_R^j(\Delta z) + L_D^j(\Delta z) + L_C^j(\Delta z)) +$$

$$+ L_R(\Delta z) + L_D(\Delta z/2) .$$

The representation of the operator $L_C^j(\Delta z)$ can be naturally related to the algorithm of the step operator $L_D^j(\Delta z)$, which makes the main contribution to calculations.

For solving the diffraction problem we use the algorithm of fast Fourier transform (FFT), which makes it possible to calculate the step operator in the spectral space

$$F_k(\Delta z) = F_k(0) \exp(-i k^2 \Delta z/2) ,$$

where

$$F_k = \sum_m E_m \exp(-ikm) , \quad k, m = 0, 1, 2, \ldots, n-1 .$$

Therefore, a control of the computational stability conservation should be performed in a spectral space. This makes it possible to use directly the spectral criterion of stability or to combine it with any of the energy criteria.[6]

Since the norm of the step operator is equal to unity the increase in the high frequency harmonics is possible only due to the energy transfer from the low frequency portion of the spectrum. If the control operator is represented in the form of the filtering function

$$H(k) = F_k(\Delta z) \exp(-k^2/k_0^2) ,$$

the algorithm stability will be provided owing to conservation of the spectral radius k_0.

However, owing to such a procedure this calculational scheme becomes nonconservative with respect to energy.

More preferable procedure is that of estimating the increase of derivative of the field spectrum

$$G_1 = |F_k(\Delta z) - F_k(0)| \le C_1 |F|$$

together with the energy criterion at the step Δz:

$$G_2 = ||F_k(\Delta z) - F_k(0)|| \le C_2 .$$

In the case of violation of the stability condition the execution of the problem is either terminated or, according to the methods of interval arithmetic, there occurs return to the previous step and correction of the length of the step Δz and of other parameters of the problem.

2. INSTABILITY WITHIN LIMITED BEAMS

Formally Eqs. (1) and (2) are of one and the same type, i.e., they are of evolution type. However, the physical meanings of the evolution variables in these equations are different: in parabolic wave equation (1) the evolution variable is the longitudinal coordinate z while in transfer equation (2) it is time t. Therefore, in the case of appearance of unstable processes one could expect their further development both in the space and time.

Let us consider a stationary regime of thermal blooming, which can be described by the system of equations

$$\begin{cases} \dfrac{\partial E}{\partial z} = \dfrac{i}{2}\left(\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2} + T\right) E, & (4) \\ \left(V_x \dfrac{\partial}{\partial x} + V_y \dfrac{\partial}{\partial y} \right) T = EE^* . & (5) \end{cases}$$

From the classical theory of nonlinear waves it is well known that a plane monochromatic wave conserves its stability only conditionally when propagating through a focusing medium with cubic nonlinearity, i.e., until its power exceeds certain critical value.[7] Thermal nonlinearity of the atmosphere is defocusing and a plane wave stability in such a medium is absolute. However, for limited wave beams such cases of propagation are feasible in which not only a decay instability of the beam can occur but also the soliton–like solutions can appear.

Let us set the boundary conditions for Eq. (4) in the form of a super–Gaussian beam

$$E(x, y) = E_0 \exp\left[-(x^2 + y^2)^m \right] ,$$

where $m = 1, 2, \ldots, n$.

We can vary the smoothness of the function $E(x, y)$ at the input to the nonlinear medium by varying the parameter m and by increasing the amplitude E_0 to make the medium nonlinearity stronger.

Equation (4) describes the evolution of the field E along the longitudinal coordinate z and Eq. (5) does the evolution of the field T in the plane (x, y) perpendicular to the direction of the vector of the transfer velocity with the components V_x and V_y. Taking into account the asymmetry of the problem in the case of a strong nonlinearity of the medium one could expect an increase of nonlinear effects in the direction of the transfer vector. Figure 2 shows the results of calculations for a one–dimensional beam for the parameters $E_0 = 33$ and $m = 1$. At the initial portion of the path the beam is deflected toward the medium flow (Fig. 2*a*), however, it loses the axial symmetry because of the dependence of the beam displacement on the intensity. The appearance of the sharp front of temperature gradient would lead to the whole beam instability and to a decay of the beam into fragments. Further deflection of the beam and the energy redistribution occur in the process of the beam propagation (Fig. 2*b*).

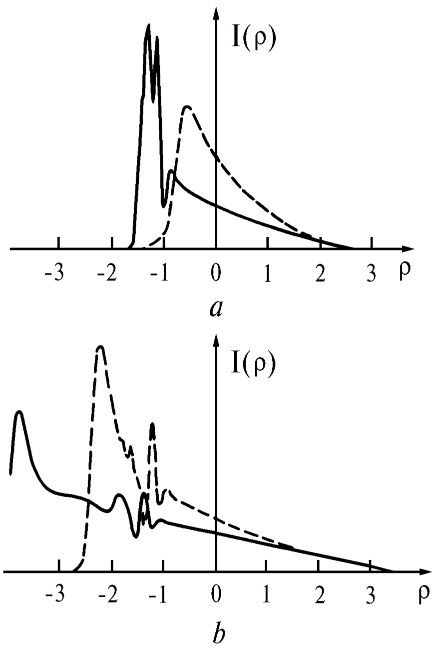

FIG. 2. Evolution of the intensity profile of a Gaussian beam along the path: a) $z = 0.04$ (dashed curve) and 0.08 (solid curve) and b) $z = 0.1$ (dashed curve) and 0.12 (solid curve).

The dimensionless parameter $R = E_0 z$ characterizes the instability degree of thermal blooming of a limited beam in a nonlinear medium (when $R > 1$ the process becomes unstable). In this example $z = 0.1$, therefore $R = 3.3$ and instability is strongly pronounced.

A super–Gaussian beam with $m = 8$ demonstrates an interesting behavior. At the initial stage of propagation (Fig. 3*a*) the Fresnel diffraction leads to appearance of ripple from which the stable soliton–like formation is then formed. During thermal blooming this soliton moves toward the medium flow conserving its configuration.

Instability of a limited beam is observed at the beam power exceeding the optimum power of a transmitter at given parameters of numerical simulations. Introduction of a feedback into the beam–medium system leads to the beam instability at the transmitter power lower than the optimum one.

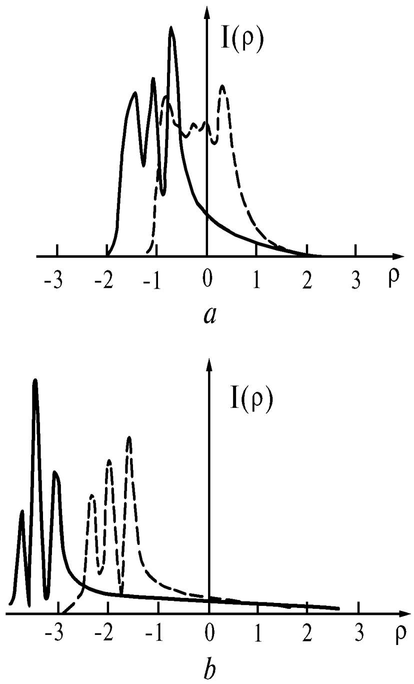

FIG. 3. Evolution of the intensity profile of a super–Gaussian beam along the path: a) $z = 0.04$ (dashed curve) and 0.08 (solid curve) and b) $z = 0.1$ (dashed curve) and 0.12 (solid curve).

3. INSTABILITY OF A BEAM UNDER AN ADAPTIVE CONTROL

Starting with the earliest studies of numerical simulation of adaptive systems for thermal blooming correction an unstable behavior of a beam has been noted in the case of the phase distribution control by a phase conjugation method. Such local characteristics of a beam as the maximum intensity and its position (coordinates in its cross section) were subjected to strong oscillations in the process of solving the nonstationary problem.[9–12]

Such an instability was called the phase conjugation instability (PCI). The reference–wave phase, for which a portion of a powerful beam reflected from an object can serve, is used to introduce predistortions into the emitted wave front. The reference wave closes an optical feedback loop because it passes the same optical path. This condition is necessary to satisfy the principle of reciprocity that guarantees compensation for the distortions. This same condition leads at the same time to an instability, because the feedback amplifies any negligible perturbations appearing at the path between the emitter and the object.

The mathematical model of an adaptive system operating in the nonstationary regime in the case of dimensionless variables has the form

$$\begin{cases} \dfrac{\partial E}{\partial z} = \dfrac{i}{2}\left(\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2} + T\right) E \,; & (6) \\ \dfrac{\partial T}{\partial t} + \left(V_x \dfrac{\partial}{\partial x} + V_y \dfrac{\partial}{\partial y} \right) T = EE^* \,; & (7) \\ \dfrac{\partial E^R}{\partial z} = \dfrac{i}{2}\left(\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2} + T\right) E^R & (8) \end{cases}$$

with the initial and boundary conditions

$$E(x, y, 0, t) = E(0)\, f(x, y, t)\ ;$$

$$T(x, y, z, 0) = T_0(x, y, z)\ ;$$

$$\lim_{x,\, y \to \infty} E = 0\ ; \qquad (9)$$

$$f(x, y) = \exp\left[\, -(x^2 + y^2)\,(1/2 + i\delta)\right]$$

which are valid at the boundaries of the computational grid.

Boundary conditions for the reference wave were prescribed in terms of an independent Gaussian beam

$$E^R(x, y, L, t) = \exp\left[\, -(x^2 + y^2)/2\, a^2\right] \qquad (10)$$

or the wave reflected from a specular reflector

$$E^R(x, y, L, t) = E(x, y, L, t)\, \exp\left[\, -(x^2 + y^2)/2\, a^2\right]\ . \qquad (11)$$

We have developed a software package for work stations (AT–386/387 and AT–486 types) compatible with IBM PC to study numerically models (6)–(8). A procedure of the computational stability control is used in the algorithm of solving the equations. The graphic interface makes it possible to store the main output data such as profiles of the intensity and the phase, power spectra of the emitted and reference beams that are calculated at each point of the time with a subsequent reproduction of the process dynamics.

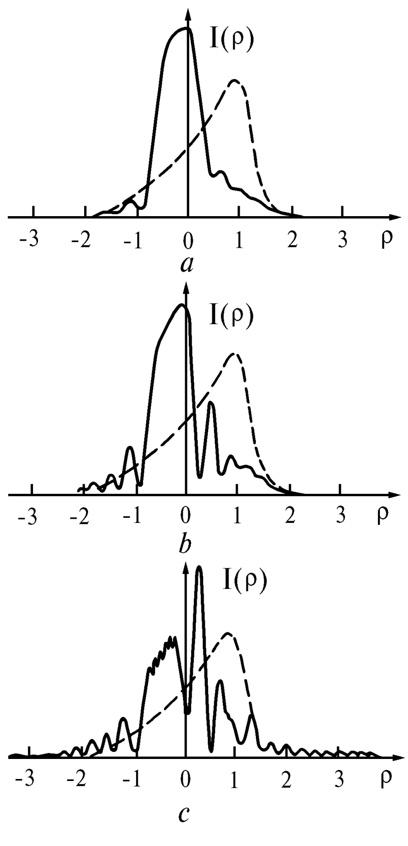

FIG. 4. Evolution of the intensity profile of a Gaussian beam vs time: stationary solution (dashed curve) and $t = 0.56$ *(a),* $t = 0.72$ *(b), and* $t = 0.81$ *(c) (solid curve).*

Figures 4 and 5 show the intensity profiles and power spectra of the main beam at three time points calculated as the ratio of the medium transfer time to the length unity in the transverse plane that is equal to the beam radius. With the development of the instability the small–scale inhomogeneities are amplified in the beam up to its decay into fragments (Figs. 4*a* and 4*b*). The spectral analysis reveals an increase of the high frequency harmonics with time (Figs. 5*a* and 5*b*).

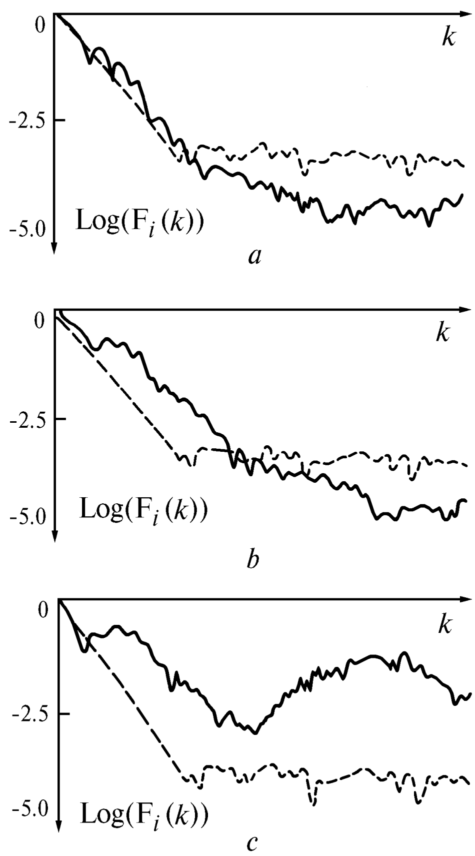

FIG. 5. Evolution of the spatial spectrum of the field vs time: the stationary solution (dashed curve) and $t = 0.56$ *(a),* $t = 0.72$ *(b), and* $t = 0.81$ *(c) (solid curves).*

The phase conjugation instability begins to develop when the beam power is lower than the optimum one, that could be of certain interest for applications. Recent theoretical studies mostly performed by numerical methods have revealed a number of physical mechanisms affecting the development of the PCI (Ref. 1). Virtually all the factors causing small–scale fluctuations of the amplitude and phase in the beam result in amplifying of the PCI, these are apodization by a sharp aperture, turbulent inhomogeneities of the refractive index of the atmosphere, and irregularities of the wave front of the laser source itself.

Attenuation of the PCI is possible due to processes that introduce strong change in the medium during the adaptive mirror response and thus affecting the conservation of the principle of reciprocity (or, in other words, the quality of the amplifier) in the beam–medium system. Among such factors there are spatial variations of the medium parameters (along the path and within the cross section of a beam) and their temporal variations caused by regular and fluctuation variations of the wind velocity in the atmosphere, turbulent mixing of inhomogeneities, fluctuations in the laser source, scanning by the beam when following the object motion, etc.

Thus, the same factors (in particular, the atmospheric turbulence) can both attenuate and amplify the PCI. Therefore, for a reliable prediction of the atmospheric distortions of the powerful laser beams and designing the adaptive optical systems it is necessary to introduce the maximum possible number of physical parameters into the imitation calculational model. These parameters can be either directly measurable or preset in terms of average statistical values (for the atmosphere latitudinal, seasonal, diurnal, altitude, and other models of thermodynamic parameters can be used).

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