INFLUENCE OF THE SIMPLEX SEARCH ALGORITHM SPEED OF RESPONSE ON THE EFFICIENCY OF ADAPTIVE COMPENSATION FOR DISTORTIONS OF A LIGHT BEAM

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The paper is devoted to development of simplex method as applied to the problems of atmospheric adaptive optics. Special attention is paid to looking for possibilities of increasing the speed of response of the control algorithm by shortening the time interval between successive corrections of the wave front of a beam. The conditions under which a fast algorithm provides for more efficient compensation for nonstationary thermal blooming in randomly inhomogeneous medium are determined.

Recently a search of new control algorithms being stable under both medium and beam parameters fluctuations, providing higher speed of response and the like is of great importance for the theory of adaptive systems. As the numerical calculations[1–4] show, one of the most promising methods extending the possibilities of focusing systems in nonlinear media is a simplex search of the optimal phase. In some aspects it surpasses the gradient procedure[5,6] of "hill climbing" which is usually applied to the cross–aperture sensing systems. A detailed analysis of simplex method potentialities as well as an account of the physical conditions of beam propagation through the self–induced nonstationary field of thermal perturbations along the path allow, apparently, for syntheting the optimal algorithm of control over the wave front meeting all modern requirements to the systems of atmospheric optics.

This paper is devoted to the study of the simplex method efficiency as applied to the problem on compensation for thermal blooming of a light beam in a randomly inhomogeneous medium. We study the dependence of compensation quality on the high speed of operation of the executive elements in the system of phase generation. The numerical simulation is based on the mathematical model of beam propagation through the atmosphere that can simultaneously account for both wind velocity pulsations and large–scale fluctuations in the refractive index.[4] The mean times of frozen wind velocity pulsations as well as of each realization of the random field of the refractive index fluctuations are supposed to be identical in addition, their change occurring at the same instant of time. The quality of control is studied[5] as a function of the parameter $D_s(2\,a_0)$, characterizing the atmospheric turbulence along the path and implying the structural function of phase fluctuations in a spherical wave at the beam diameter (a_0 is the initial radius of the beam).

According to the modal control principle let us assume that the wave front in the form of basis modes superposition is formed at the corrector

$$U(x, y, t) = \theta_x(t)\, x + \theta_y(t)\, y + S_x \frac{x^2}{2} + S_y \frac{y^2}{2} . \qquad (1)$$

In the cross–aperture sensing system under consideration we make use of the alternate variation of controllable coordinates (θ, S) whereas the high speed of operation of the adaptive system is determined by the time τ_c between the two successive wave front corrections. The transient processes along the path occurring due to amplitude and phase field variations at the inlet to a medium lead to considerable intensity changes at the object to be focused on, that influences the stability of adaptive correction. The process of temperature stabilization along the propagation path is characterized by the convective time $\tau_v = a_0/\langle v\rangle$ defined by the beam radius a_0 and average wind velocity along the path $\langle v\rangle$.

In the previous investigations[2–4] the model of the adaptive system characterized by the finite speed of response (namely $\tau_c \geq 0.1\tau_v$) was considered. At the same time it is quite easy to construct a model of the perfect adaptive system with the infinity high speed of response by reducing τ_c indefinitely. The search for an optimal phase and the transient processes in the beam––medium system that occur simultaneously under real conditions, in this case they can be approximately time–separated. Therefore for every instantaneous state of the medium, the phase optimization is carried out in the "frozen" thermal field, and the subsequent relaxation of the medium matches the determined phase with the nonlinear thermal lens for the time τ_r. On the basis of the numerical experiments[2] the quite satisfactory calculation accuracy is determined to be achieved at $\tau_r = 0.1\tau_v$.

Since the choice of the search strategy and determination of the optimal simplex size according to Ref. 2 are the most decisive factors for the effective compensation for thermal blooming in real time it is of interest to analyse the dependence of these factors on the high speed of control algorithm. For further estimating of the phase correction efficiency let us use as before the criterion of focusing

$$J_f(t) = \frac{1}{P_0} \int \exp\left(-\, \mathbf{r}^2 / s_t^2\right) I(\mathbf{r}, z_0, t)\, d^2\mathbf{r} , \qquad (2)$$

characterizing the light beam localization over the fixed aperture with the radius s_t (P_0 is the total beam power, I is the intensity of a light beam, $\mathbf{r} = \{x, y\}$ is the vector radius in the z = const plane, and z_0 is the path length).

Let us use as the basis algorithm, the simplex search with the free reflection of vertices[7] allowing the phase optimization to be successfully performed for each instantaneous state of the medium and the temporal drift of the focusing criterion to be tracked in the process of the subsequent relaxation of the thermal lens. The basic rule of

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this algorithm is to reflect the worst vertex of simplex without any additional conditions.

By choosing the optimal simplex size, it is necessary to take into consideration that its quantity is bound to the limiting speed of response of the adaptive system. Some simple estimates show that for optimal phase determination it is necessary at least $n_{\rm opt} = 2(k + 1)$ iteration steps, where k is the modal basis dimensionality ($k = 4$ in the above–considered case). In fact, the first $k + 1$ measurements of the goal function are required for determination of the initial simplex configuration and about the same number is needed for the reliable "frozen hill climbing". The numerical experiment results confirming these conclusions are demonstrated in Fig. 1, where two dynamic dependences of the focusing criterion for the control over basis (1) are shown for two values of optimizing steps $n_{\rm opt} = 5$ (curve *1*) and 10 (curve *2*) per the time of frozen pulsations. By taking into account that the processes of beam phase optimization and relaxation of a medium cannot be time–separated under real conditions, the following inequality can be used for practical estimation of the required high speed of control system operation:

$$\tau_c \leq \tau_v / 20(k + 1) . \qquad (3)$$

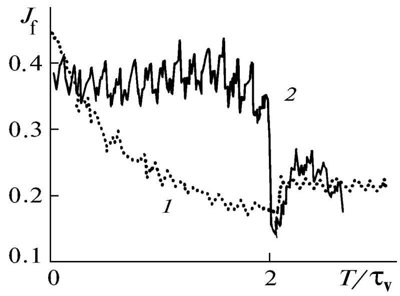

FIG. 1. Focusing criterion J_f versus the control time T for dynamic compensation for thermal blooming based on the simplex method. $n_{\rm opt} = 5$ (curve 1) and 10 (curve 2). Path length $z_0 = 0.5$, nonlinearity parameter $\langle R\rangle = -20$, variance of the wind velocity fluctuations $\sigma_v = 0.3$, and the parameter $D_s(2a_0) \approx 1$.

As is shown in Fig. 1, the high–speed algorithm with the free reflection of vertices provides the efficient correction of nonstationary thermal blooming only for the finite time after which the control "break–down" occurs that cannot be removed by any simplex size fitting. This phenomenon can be explained by a comparison of two algorithms with high ($\tau_c = 0{,}01\tau_v$) and mean ($\tau_c = 0.1\tau_v$) speeds of response illustrated in Fig. 2.

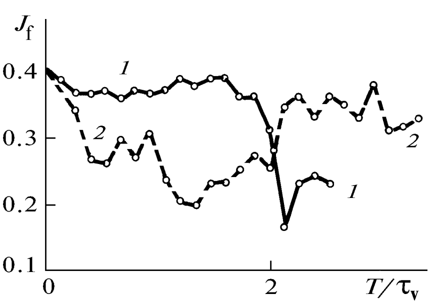

FIG. 2. Focusing criterion J_f versus the control time T for two cases: $\tau_c = 0.01\tau_v$ (curve 1) and $\tau_c = 0.1\tau_v$ (curve 2).

In particular, the second algorithm is stable for larger control time, though for the short time ($\le 2\tau_v$) it loses unfavourably to the first algorithm in efficiency. The reason is that the controllable coordinates S_x, S_y (Fig. 3) change more smoothly than in the former case. It is because of the excessively high rate of increase in the wave front curvature occurring due to the phase optimization over the frozen states of the medium that leads to early (at $t \approx 2\tau_v$) overfocusing of the beam. This effect can be partially weakened by decreasing the simplex size, however in this case the control efficiency at the initial stage of the medium heating (at $t \approx \tau_v$) decreases because the small size simplex has no opportunity to follow the wind drift of the goal function.

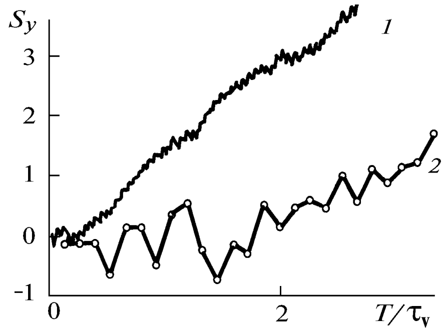

FIG. 3. Dynamics of the controllable coordinate S_y in the course of optimization of the criterion J_f based on the simplex method. Path length $z_0 = 0.5$, nonlinearity parameter $\langle R\rangle = -20$, variance $\sigma_v = 0.3$, and $D_s(2a_0) \approx 1$.

The second and preferable method of the beam overfocusing removal implies the restrictions on the values of controllable coordinates. The size of the restrictions can be estimated, for example, by analysing of the dynamics of $S_x(t)$ and $S_y(t)$ variables during compensation for thermal blooming by the system mean speed of response ($\tau_c = 0.1\tau_v$). As a result, the control process can be represented as simplex moving in a "tunnel" with rigid walls. When reaching these walls the increase of the correspondent coordinates stops, but their decrease is still possible. Figures 4 and 5 display, respectively, the typical temporal dependences of the focusing criterion and variables S_x and S_y for the beam controlled by the high–speed algorithm ($\tau_c = 0.01\tau_v$) with restrictions of the values of the wave front curvature (marked by a dashed line).

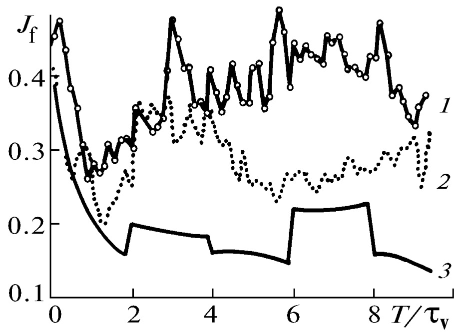

FIG. 4. Typical temporal dependences of focusing criterion. The control with the restrictions imposed on the controllable coordinates at $\tau_c = 0.01\tau_v$ (curve 1), at $\tau_c = 0.1\tau_v$ (curve 2), and without control for collimated beam (curve 3).

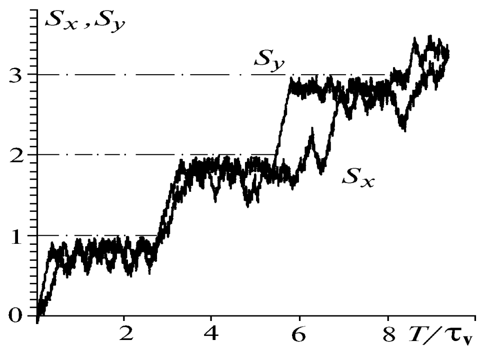

FIG. 5. Characteristic temporal dependences of the controllable coordinates S_x, S_y for the high–speed algorithm ($\tau_c = 0.01\tau_v$) with the restrictions. Path length $z_0 = 0.5$, nonlinearity parameter $\langle R \rangle = -20$, variance of the wind velocity fluctuations $\sigma_v = 0.3$, and $D_s(2a_0) \approx 2$.

For comparison, Fig. 4 shows the analogous dependences in the case of mean–speed control algorithm ($\tau_c = 0.1\tau_v$) and without any control (for collimated beam). It is easy to see that the restrictions imposed on the simplex motion over space of controllable coordinates enable one to get the stable compensation for the beam thermal blooming over sufficiently long time interval ($T \approx 10\tau_v$). The averaging over 10 realizations shows that the application of the proposed algorithm enhances the total light energy incident on the receiving aperture for the control time by a factor of 1.3 in comparison with compensation performed by a system with mean–speed response and of 1.7 – without any control.

In conclusion, it should be noted that the speed of the control algorithm and the simplex optimal size estimated in this paper provide the reliable compensation for the nonstationary light beam distortions. These estimates can be $k = 4$ the optimal simplex size $L_{opt} \approx 0.2$ corresponds to the mirror curvature change at one iteration step $\Delta S \approx 0.05(ka_0^2)^{-1}$, that in its turn is equivalent to the mirror $R = 10a_0$ in radius shift by a factor of about 0.1λ. To determine the mirror frequency characteristics, it is necessary to take into account that stepped changes in controllable coordinates cause the impact on the mirror producing prolonged oscillations[8] that significantly impede the search for an optimal phase. It is obvious that the transient processes in the mirror appear to be insignificant if the instantaneous changes in the controllable coordinates are replaced by more smooth ones, for example, changes linearly varying with time and characterized by τ_R determined by mechanical system Q–factor. In particular, for bimorph piezoelectric mirrors with $Q \approx 10$ we need only to take τ_R being equal to a period of the basic tone of free oscillations. Finally the estimate of the mirror resonance frequency is $f \geq 100/\tau_v$.

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