EXPERIMENTAL AND THEORETICAL INVESTIGATION OF THE EFFECTIVENESS OF ADAPTIVE FOCUSING OF RADIATION IN A NONLINEAR MEDIUM

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Laboratory experiments and numerical methods were used to investigate the effectiveness of compensation of thermal self-action. The arrangement of the laboratory apparatus and its mathematical model are presented. An algorithm that increases the stability of adaptive correction in the presence of noise in the electron-optic feedback circuit is proposed. The experimental and theoretical results are compared.

An important applied problem in modern adaptive optics is how to make systems which compensate the thermal self-action of laser radiation propagating in natural media more efficient. The most important characteristics of the process of adaptive focusing of laser beams in a nonlinear medium were established primarily theoretically, based on basic mathematical models and extensive use of numerical methods.[1–3] In the last few years it has become possible, in connection with the development of experimental technique, to make a direct comparison of theoretical results and laboratory modeling of adaptive systems. This makes it possible to check the adequacy of the mathematical models employed and to evaluate the reliability of numerical predictions of the maximum capabilities of atmospheric adaptive optics systems.

In this paper a "slow" adaptive system, which is designed for compensation of stationary thermal self-action in a moving regular medium, is investigated theoretically and experimentally. Algorithms that improve the stability of the control of the phase of a laser beam under conditions of natural noise, which naturally appears in a laboratory experiment, are proposed and analyzed. The experimental and theoretical data on control effectiveness are compared.

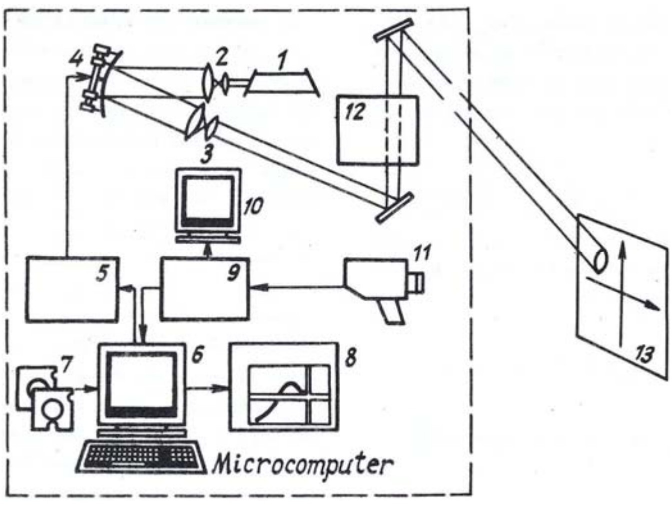

FIG. 1.

1. BLOCK DIAGRAM OF LABORATORY APPARATUS AMD ORGANIZATIQN OF LIGHT-BEAM CONTROL

A laboratory model of an adaptive radiation focusing system[4] is shown in Fig. 1. The laser beam is directed into the phase-forming system, which consists of telescopes 2, 3 and a phase corrector (elastic mirror) 4. After being reflected from the mirror and passing through the telescope 3 the beam enters a cell 12, filled with an alcohol solution of fuchsine. The flow of the medium is simulated by rotating the cell with a constant angular velocity. The image of the beam on the screen 13 is read with a television camera 12 and stored In a computer 6, where the main parameters of the light field are calculated and the corrector control signals are generated.

The source 1 consists of a continuous argon laser with $\lambda = 0.488$ μm. The output beam has a diameter of 1 mm and its power can be regulated from 0.1 to 1 W. The phase corrector 4 consists of a flexible mirror 50 mm in diameter. The center of, the mirror is secured and the edges are free. The mirror is controlled with the help of six external pistons, which are moved by means of stepping motors. When a controlling load is applied to the pistons the mirror is deformed by a system of forces and moments applied along its contour.

In the case of modal control the phase of the beam $\varphi(x, y)$ in the plane of the corrector is formed in the form of a superposition of the basis modes $w_i(x, y)$, reproduced by the mirror:

$$\varphi(x, y) = 2k\, U^T w(x, y), \tag{1}$$

where k is the wave number; $U = \{U_i,\ i = 1, \ldots, N\}$ is the control vector; the index T denotes transposition; and, N is the dimension of the basis. The choice of the modal basis $w = \{w_i,\ i = 1, \ldots, N\}$ depends on by the purpose of the adaptive system. To compensate wind refraction, for which the lowest-order aberrations of the phase predominate, it is natural to choose the first and second order Zernike polynomials ($N = 5$) as the first modes:

$$w_1 = \frac{x}{a_0}, \quad w_2 = \frac{y}{a_0}, \quad w_3 = \frac{2}{a_0^2}\left(x^2 + y^2\right) - 1,$$
$$w_4 = \frac{1}{a_0^2}\left(x^2 - y^2\right), \quad w_5 = \frac{2}{a_0^2}\, xy, \tag{2}$$

where a_0 is the initial radius of the beam. The field $E(x, y, 0)$ at the output of the forming system (in the plane $z = 0$) has the form

$$E(x, y, 0) = E_0(x, y) \exp(i\varphi(x, y)), \tag{3}$$

where E_0 is the amplitude profile of the beam.

When organizing control of the phase of a light beam the adaptive system must be made stable relative to natural noise in the electron-optic feedback circuit. Experience shows that if a scalar quantity is chosen as the goal function of control, then the coordinates of the control U_i are interrelated, and in the presence of noise this degrades the convergence of focusing, especially near an extremum of the goal function. To reduce the crosstalk between the control channels, it is possible to analyze simultaneously all criteria characterizing the distribution of the light intensity $I(x, y, z_0) = \frac{cn_0}{8\pi}|E(x, y, z_0)|^2$ in the observation plane $z = z_0$ (Ref. 5). For the modal basis, which the Zernike polynomials (2) form, it is convenient to take for these criteria the first and second order moments of the intensity $I(x, y, z_0)$:

$$F_1 = x_c; \quad F_2 = y_c; \quad F_{3,4} = \sigma_x^2 \pm \sigma_y^2; \quad F_5 = \sigma_{xy}; \tag{4}$$

where $x_c = M\{x\}$, $y_c = M\{y\}$ are the coordinates of the energy center of gravity

$$\sigma_x^2 = M\{(x - x_c)^2\}; \qquad \sigma_{xy} = M\{(x - x_c)(y - y_c)\};$$

where $M\{f(x, y)\} = \iint f(x, y)\, I(x, y, z_0) dxdy\ /$ $/ \iint I(x, y, z_0) dxdy$.

Then the criterion of focusing of the beam into a given point x_0, y_0 in the plane $z = z_0$ will be that the following components of the vector goal function J must be minimized:

$$J_1 = |x_c - x_0|, \quad J_2 = |y_c - y_0|, \quad J_3 = \sigma_x^2 + \sigma_y^2 - a_D^2,$$
$$J_4 = |\sigma_x^2 - \sigma_x^2|, \quad J_5 = |\sigma_{xy}|, \tag{5}$$

where a_0 is the diffraction-limited radius of the beam.

The phase φ_{opt} with which optimal focusing is achieved is determined by an iteration procedure

$$U_{n+1} = U_n - [A_n]\, J_n, \tag{6}$$

where n is the iteration number. The elements of the control matrix $[A_n]$ are calculated by scanning the phase in the space of the modes $w_i(x, y)$. In a weakly nonlinear medium the matrix $[A]$ is nearly diagonal, and this ensures that the crosstalk between the control channels is weak. In this case it is possible to minimize simultaneously all components of the vector criterion J. To reduce the crosstalk between the channels under conditions of strong nonlinearity it is best to perform successive control first of the astigmatisms U_4 and U_5, followed by control of focusing U_3, and finally control of the wavefront tilts U_1 and U_2.

The algorithm developed was investigated both in a numerical model and on a laboratory setup.[4]

2. MATHEMATICAL MODEL OF THE LABORATORY SETUP

We shall confine the mathematical model to a description of the basic elements of the system for adaptive focusing of radiation, namely, the optical channel and the phase corrector. We shall divide the optical channel into the following sections: 1) from the laser to the mirror (length z_1), 2) from the mirror to the cell (z_2), 3) inside the cell (z_3), and 4) from the cell to the observation plane (z_4). Since the telescopes 2 and 3 are matched and do not introduce amplitude distortions, their effect in the mathematical model is merely to change the length scales on the corresponding sections of the path. On the sections 1, 2, and 4 the beam propagates under conditions of free diffraction, while section 3 simulates a nonlinear atmosphere. Beam propagation in a moving absorbing medium is described by a system of equations for the complex amplitude E and the temperature perturbations T. We shall write this system in a dimensionless form with the standard normalization[1]

$$2i\frac{\partial E}{\partial z} = \Delta_{\perp}E + R_{v}TE - i\theta E; \tag{7}$$

$$\frac{\partial T}{\partial x} = EE^{*}, \tag{8}$$

where $R_v = \dfrac{2k^2|a_0^3|\alpha I_0(\partial n/\partial T)}{n_0\rho C_p V}$ is the nonlinear refraction parameter and $\theta = \alpha n_0 k a_0^2$ is the optical thickness at the diffraction length. On sections of free diffraction the terms describing nonlinear refraction and absorption are eliminated from Eq. (7). Taking into account the change in the radius of the beam by the telescopes the distance z_1 is negligibly small; $z_2 = 0.07$; $z_3 = 0.1$; and, $z_4 = 0.3$ (in units of the corresponding diffraction lengths).

In the mathematical model the phase corrector consists of a model of an octagonal elastic mirror,[6] having free edges and hinged at the center. The deformations of the reflecting surface of the mirror under moments and forces applied along the contour of the mirror are determined by the method of finite elements.

3. EFFECTIVENESS OF PHASE CORRECTION OF WIND-INDUCED REFRACTION

To compare the results of numerical and laboratory modeling of the adaptive focusing system it is convenient to employ relative criteria of quality of control which are defined by the general formula

$$\eta_j = [|j_{opt} - j_0|/|j_0|]\cdot 100\%, \tag{9}$$

where j_{opt} is the value of some parameter characterizing the field distribution obtained by adaptive focusing and j_0 is the value of the same parameter in the absence of control.

The relative changes of the energy half-widths of the beam σ_x and σ_y and the displacement of the energy center x_c which are recorded in one of the typical experiments are presented in Table I. The beam contours seen on the screen before and after control are presented in Fig. 2. Measurements of the parameters of the radiation and nonlinear medium give the following estimates for the dimensionless characteristics of beam propagation in this experiment: $|R_V| \simeq 130$ and $\theta \simeq 5$.

One can see from the data presented that compensation of self-action reduced by a factor of 1.5–2 the energy half-widths of the beam and the wind-induced displacement of the center of the beam was reduced virtually to zero.

TABLE I. Effectiveness of adaptive correction in the laboratory experiment ($|R_V| \simeq 130$, $\theta \simeq 5$).

η_{σ_x} (%)	η_{σ_y} (%)	η_{x_c} (%)
24	55	85

In the numerical experiment, it is primarily of interest to determine more accurately the equivalent parameter of nonlinearity of the medium, since the characteristics of the radiation and the medium are not determined very accurately under laboratory conditions. To this end, we calculated the adaptive focusing of a beam in a medium with nonlinearity parameters in the range $110 \le |R_V| \le 150$. To make it easier to interpret the data it was assumed that the nonlinearity parameter is constant along the path (approximation of a weakly absorbing medium $\theta \simeq 0$).

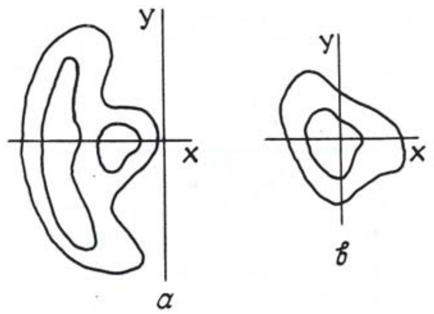

FIG. 2. Laboratory experiment. Change in the visible contour of the light beam in the process of adaptive control: a) before correction, b) after correction.

TABLE II. Numerical investigation of adaptive correction. Modeling of an absorbing medium neglecting radiation attenuation ($\theta \simeq 0$).

	η_{σ_x} (%)	η_{σ_y} (%)	η_{x_c} (%)
$|R_v| = 110$	21	41	91
$|R_v| = 120$	19	41	89
$|R_v| = 130$	16	36	85
$|R_v| = 150$	16	30	80

The computational results are presented in Table II. By comparing them with the data of Table I one can see that the experimental values of the change in the energy half-widths are somewhat lower and the change in the displacement of beam center is somewhat higher than the experimental values. The computational accuracy can be improved by taking into account the fact that as the beam propagates in an absorbing medium the intensity of the light and therefore the equivalent parameter of nonlinearity decrease exponentially as a function of the distance.

The computed values, characterizing the efficiency of correction for different values of the optical thickness θ, are presented in Table III. One can see that introducing into the computational model attenuation of radiation on the nonlinear path makes it possible to improve the accuracy of the numerical prediction by fitting the parameter θ. A qualitative illustration of the possibilities of a numerical experiment is also shown in Fig. 3, where the visible contours of the beam in the observation plane before and after correction are shown (compare with Fig. 2).

TABLE III. Numerical investigation of adaptive correction. Modeling of the absorbing medium, taking into account radiation attenuation ($|R_V|$ = 130).

	η_{σ_x} (%)	η_{σ_y} (%)	η_{x_c} (%)
θ = 10	26	50	92
θ = 5	22	45	88
θ = 2.5	18	40	86

The numerical experiments on control in a "reduced" space of basis modes w_1, w_2, and w_3 using a three-dimensional criterion with the components J_1, J_2, and J_3 showed that in this case the windward displacement of the beam can be compensated, but the beam profile after correction is still spread out transverse to the flow. This suggests that the five-dimensional control basis introduced in this paper and the associated five-dimensional vector quality criterion are not excessive and are close to optimal in the case of compensation of wind-induced refraction in a regular medium.

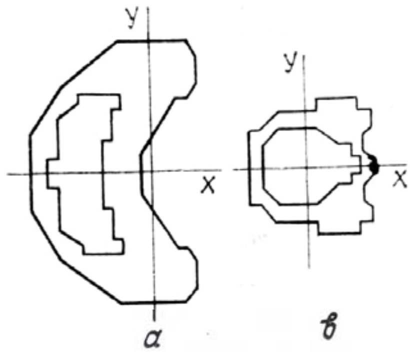

FIG. 3 Numerical experiment. Change in the visible contour of the light beam in the process of adaptive control $|R_V| \simeq 130$ *and* $\theta \simeq 10$)*: a) before correction; b) after correction.*

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