

# Monte Carlo simulations of polarized CCD lidar returns

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Received August 29, 2007

An upgrade of the software developed at the LMU of Munich and at the ICMMG of Novosibirsk allows for the efficient simulation of CCD lidar returns from highly structured clouds of nonspherical scatterers. The multiple scattering contributions to the lidar return contain additional valuable information about the scattering particles. To make use of it, multi-channel lidars with various fields of view and polarization and CCD lidars have been constructed. The progress in opto-electronics made possible the construction of lidars with charged coupled devices (CCD) which allow for taking time resolved two-dimensional pictures of the diffusion of the lidar beam in a cloud, promising additional gain of information from such CCD lidar returns. Shortly, we discuss the single scattering of polarized light by various types of particles and simulate lidar returns and CCD lidar returns from various types of broken clouds and plumes, compare the two kinds of lidar returns and show the superior capability of CCD lidar returns for the distinction of various types of aerosol clouds and plumes (chimney roses). Here we show simulations of monostatic CCD lidar returns for the distinction of types of clouds and bistatic CCD lidar returns to demonstrate the effect of pulse stretching in different clouds. Our theory is based on the stochastic model of corpuscular multiple scattering of polarized light which is a partially deterministic Markovian jump process and which is equivalent to an appropriate version of the radiative transfer equation for polarized light. For the Monte Carlo simulation we make use of the comfortable and highly effective method of iterative fictitious collisions.

## Introduction

The progress in opto-electronics now allows for the construction of monostatic and bistatic lidars with gated charged coupled devices. With such lidars it is possible to take time resolved two-dimensional pictures of the diffusion of the lidar beam as it is seen from the receiver. This may be done with wide angle or narrow angle receivers with one or several fields of view, with and without polarization. From the (analysis of the) spreading of the diffusion of the beam it is possible to draw conclusions on the extinction coefficient of a cloud, on the scattering particles in a cloud, and on density and turbulence properties within the atmosphere. First experiments with such CCD lidars in the USA, Canada, Europe, and Russia show promising results; see R.F. Cahalan et al.,<sup>1</sup> A.B. Davis et al.,<sup>2</sup> and N. Roy et al.<sup>8</sup> We want to support the construction and the analysis of measurements of such CCD lidars by developing tools for the simulation of monostatic and bistatic CCD lidar returns. These tools are upgrades of the program PBS, which was designed for the simulation of polarized lidar return at the Institute of Mathematics of the Ludwig-Maximilian University in Munich with the assistance of the Institute of Computational Mathematics and Mathematical Geophysics of the SB RAS in Novosibirsk. Our program PBS and its upgrades are based on the stochastic model of the corpuscular multiple scattering process of polarized light. This stochastic process can be shown to be equivalent to radiative transfer equations which coincide

with the classical Chandrasekhar radiative transfer equation in the nonrealistic case of scattering particles with a directional scattering distribution which is rotational invariant to the incident beam (e.g., given by a phase function). The advantage of our stochastic approach is that we can simulate nonstationary solutions of such radiative transfer equations simply by conditioning: e.g., with respect to the order of scattering or with respect to the location of the last scattering point before reception. Furthermore, we can speed up the simulations by variance reduction procedures such as the exponential transform, and the local estimates. Moreover, we can apply the method of iterated fictitious collisions which allows for simulation of returns from broken clouds and has some variance reduction effects in addition. After some remarks on the stochastic model and the directional scattering distributions of different scattering particles we shall show some simulations of returns of a monostatic and a bistatic CCD lidar. The bistatic CCD lidar returns will demonstrate the different pulse stretching of clouds with spherical and non-spherical scatterers.

## 1. The corpuscular multiple scattering process of polarized light

We describe the transport of light through the atmosphere as a stochastic process of corpuscular multiple scattering. Multiple scattering is considered as a sequence of single scatterings of polarized photons at particles of the atmosphere. Each single scattering

of a photon is decomposed into a random collision, a random selection of the type of scattering particle, a random directional scattering, and a deterministic transformation of the Stokes vector and its reference vector.

The Markov kernel (transition probability) describing the random collisions may depend on the direction and the polarization of the photon. The Markov kernel describing the random selection of the scattering particle depends on the local densities of the different types of particles. Finally, the random directional scattering of the photon depends on its polarization and a complete set of Mueller matrices for the incident and all outgoing directions of scattering of the particle. If the new scattering direction is chosen according to this random distribution, the new reference vector and the new Stokes vector are chosen deterministically according to the rules of physics.

The associated Markov kernels for the stepwise transitions yield a Markov kernel for a time discrete Markov process with a high dimensional state space. To this time discrete Markov process a time continuous process is associated which turns out to be a cadlag partially deterministic Markovian jump process (PDMP); see M.H. Davis<sup>3</sup> and Ethier–Kurtz.<sup>4</sup> The infinitesimal generator of this process may be obtained from the Markov kernels of the time discrete process. The associated backward and forward Kolmogorov differential equations may be considered as radiative transfer equations. In the case of unpolarized light with directional scattering distributions which are rotational invariant with respect to the incident beam (which is not true for laser light), the forward Kolmogorov differential equation turns out to be the well-known Chandrasekhar radiative transfer equation; see Lapeyre et al.<sup>5</sup>

The description of the transport of polarized light by a PDMP and the application of the method of iterated fictitious collisions make it possible to design Monte Carlo algorithms which allow for the simulation of the diffusion of pulsed polarized laser beams with respect to density, orientation and mixture of particles structured clouds, broken clouds and exhaustion plumes, such as lidar and CCD lidar

returns with transversal and longitudinal diffusion. In Sections 3 and 4 we shall show some simulations of this type.

## 2. Single scattering of polarized light

To characterize single scattering of a polarized photon at a particle in the atmosphere, Mueller matrices may be used. A Mueller matrix transforms the four-dimensional Stokes vector of an incident photon which is given with respect to a three-dimensional reference vector into the Stokes vector of the outgoing photon. The new reference vector may be chosen to lie in the plane of scattering and to be perpendicular to the new direction of the photon. The plane of scattering is determined by its incident and the outgoing direction. In general, such a Mueller matrix is dependent on these two directions, too. But for certain classes of spherical and for randomly oriented in space non-spherical particles the process of single scattering of a polarized photon may be characterized by a set of Mueller matrices for a single incident direction and all outgoing directions and the set of rotations around the incident direction. The outgoing direction often is decomposed into the off-axis (zenithal,  $\theta$ ) direction and the azimuthal ( $\phi$ ) direction. Under additional geometric and physical assumptions each  $4 \times 4$ -Mueller matrix has only four and six elements unequal to zero, respectively. In most cases the emitted laser beam is linearly polarized. If a photon collides with a particle, the direction of being scattered is determined by the directional scattering distribution which may be obtained from the full set of Muller matrices and the normalized Stokes vector of the incident photon. But even for spherical particles the directional scattering distribution produced by a linearly polarized photon is not rotational invariant with respect to the incident direction; see Fig. 1. Therefore phase functions are not sufficient to describe the directional scattering behavior of polarized photons. The lack of this rotational invariance is especially important in the backward directions ( $\theta = 70\text{--}180^\circ$ ).

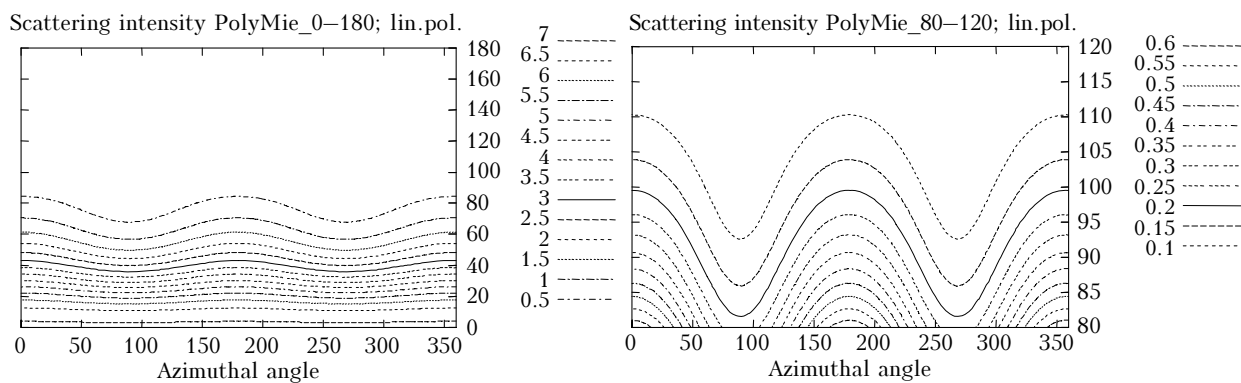
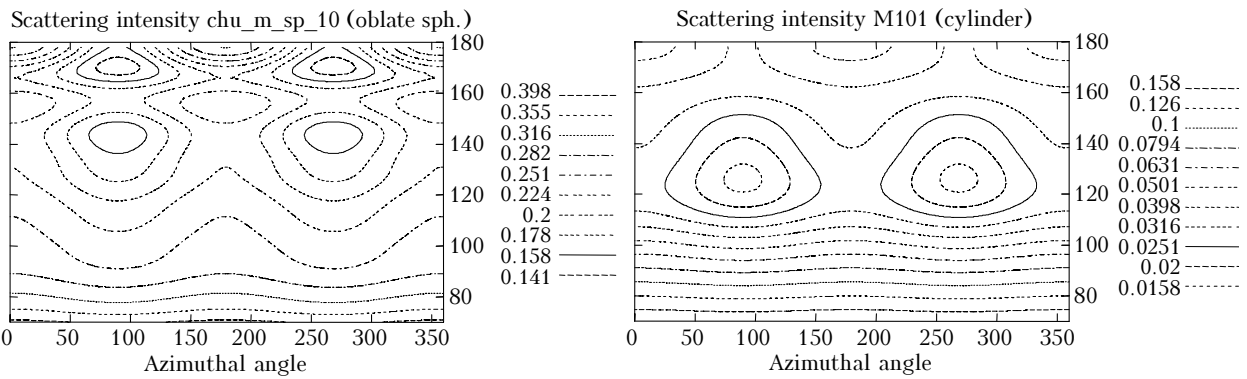


Fig. 1. PolyMie directional scattering distribution of an ensemble of spherical water droplets,  $\theta = 0\text{--}180^\circ$  (left),  $\theta = 80\text{--}120^\circ$  (right).



**Fig. 2.** Directional scattering distribution of an ensemble of randomly oriented oblate spheroids (left) and randomly oriented oblate cylinders (right),  $\theta = 80\text{--}180^\circ$ .

We consider three directional scattering distributions. One is a poly-Mie directional scattering distribution of an ensemble of spherical water droplets (refractive index:  $Re = 1.33$ ,  $Im = 0.1$ ;  $\lambda = 1064 \mu\text{m}$ ;  $\Gamma$  size distribution; “C1 cloud”); see Fig. 1. The other two are directional scattering distributions for randomly oriented oblate cylinders (refractive index:  $Re = 1.311$ ,  $Im = 0.255D-08$ ;  $\lambda = 532 \text{ nm}$ ;  $D/L = 2$ ; M101; “ice cloud”) and randomly oriented oblate spheroids (refractive index:  $Re = 1.53$ ,  $Im = 0.800D-08$ ;  $\lambda = 532 \text{ nm}$ ;  $A/B = 1.7$ ;  $J_{17}$ ; “aerosol cloud”), respectively; see Fig. 2. All these scattering distributions are quite different from each other.

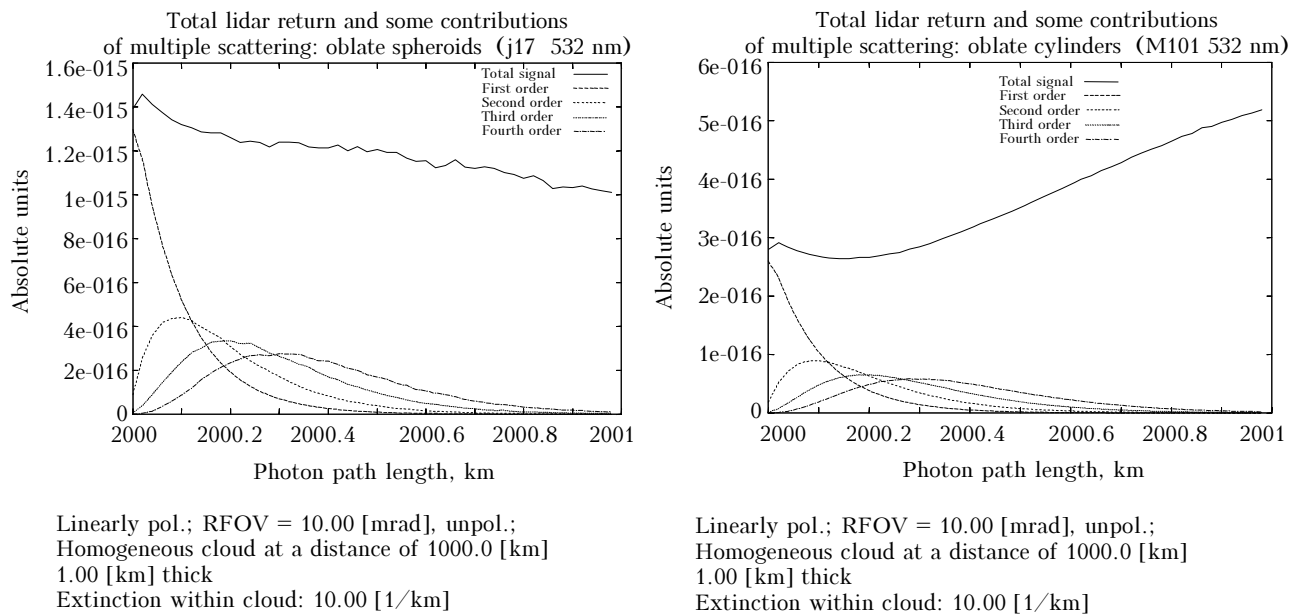
### 3. Monostatic CCD lidar returns

Let us now have a look with a monostatic CCD lidar at a 1 km thick cloud with an extinction

coefficient of  $2 \text{ km}^{-1}$  at a distance of 1000 km, no extinction before and behind the cloud. The linearly parallel-polarized emitter has a 0.2 mrad field of view and a 3 m pulse length. We consider a cloud of oblate spheroids (model of an aerosol) and a cloud of oblate cylinders (model of an ice cloud). First we show the classical lidar return: total return, contributions of first, second, third and fourth order of scattering; see Fig. 3.

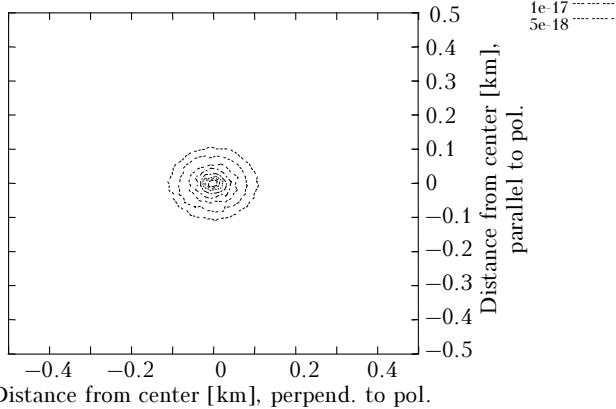
Then we show the CCD lidar returns from different depths of penetration of the cloud. First the total return from the cloud of spheroids for 100–150 and 400–450 m depth of penetration; see Fig. 4.

Then we compare the cross-polarized return from the cloud of spheroids with the one from cylinders, for 300–350 m depth of penetration; see Fig. 5. For further simulations of CCD lidar returns see U.G. Oppel and M. Wengenmayer<sup>6</sup> and U.G. Oppel et al.<sup>7</sup>

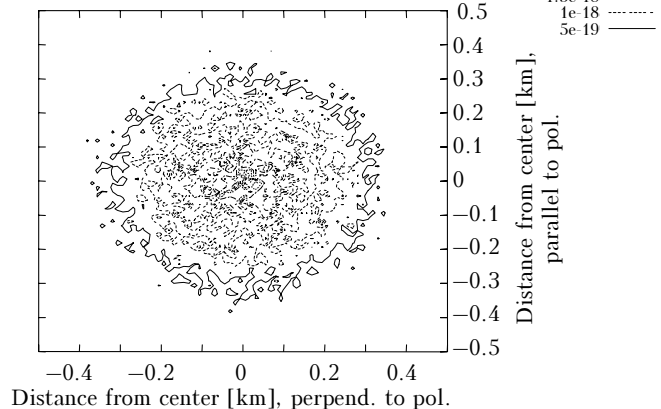


**Fig. 3.** The classical lidar return from a cloud of randomly oriented oblate spheroids (left) and randomly oriented oblate cylinders (right): total return, contributions of first, second, third and fourth order of scattering.

Diffusion of beam seen from receiver, total return  
 linearly pol.; RSFOV = 10 [mrad], unpol.;  
 homogeneous cloud at a distance  
 of 1000.0 [km], 1.00 [km] thick;  
 extinction within cloud: 1.00e+001 [1/km]; j17;  
 range (~ time interval): 1000.10–1000.15 km

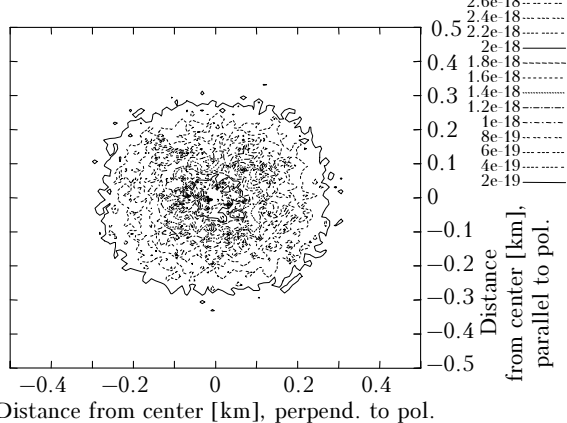


Diffusion of beam seen from receiver, total return  
 linearly pol.; RSFOV = 10 [mrad], unpol.;  
 homogeneous cloud at a distance  
 of 1000.0 [km], 1.00 [km] thick;  
 extinction within cloud: 1.00e+001 [1/km]; j17;  
 range (~ time interval): 1000.40–1000.45 km

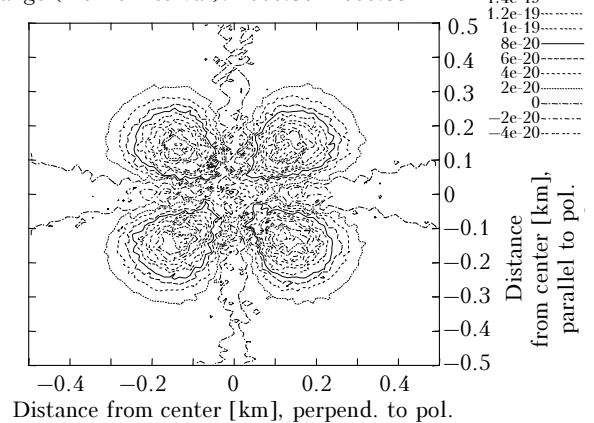


**Fig. 4.** The total planar CCD lidar return from a cloud of randomly oriented oblate spheroids from a penetration depth of 100–150 m (left) and 400–450 m (right).

Diffusion of beam seen from receiver, total return  
 linearly pol.; RSFOV = 10 [mrad], cross-pol.;  
 homogeneous cloud at a distance  
 of 1000.0 [km], 1.00 [km] thick;  
 extinction within cloud: 1.00e+001 [1/km]; j17;  
 range (~ time interval): 1000.30–1000.35 km



Diffusion of beam seen from receiver, total return  
 linearly pol.; RSFOV = 10 [mrad], cross-pol.;  
 homogeneous cloud at a distance  
 of 1000.0 [km], 1.00 [km] thick;  
 extinction within cloud: 1.00e+001 [1/km]; m101;  
 range (~ time interval): 1000.30–1000.35 km



**Fig. 5.** The total cross-polarized planar CCD lidar return from a cloud of randomly oriented oblate spheroids (left) and of randomly oriented cylinders (right) from a penetration depth of 300–350 m.

### 4. Bistatic CCD lidar returns: pulse stretching

Let us now have a look at the phenomenon of pulse stretching. It is well known that multiple scattering will produce a prolongation and broadening of a pulsed laser beam. However, it is very hard, if not impossible, to calculate this diffusion effect of the laser beam exactly. Obviously, this diffusion is caused by the collisions of the photons of the beam with scattering particles of the atmosphere and by the resulting backward, forward and sideward scattering. The collisions depend on the extinction coefficient. The direction of scattering is determined by the directional

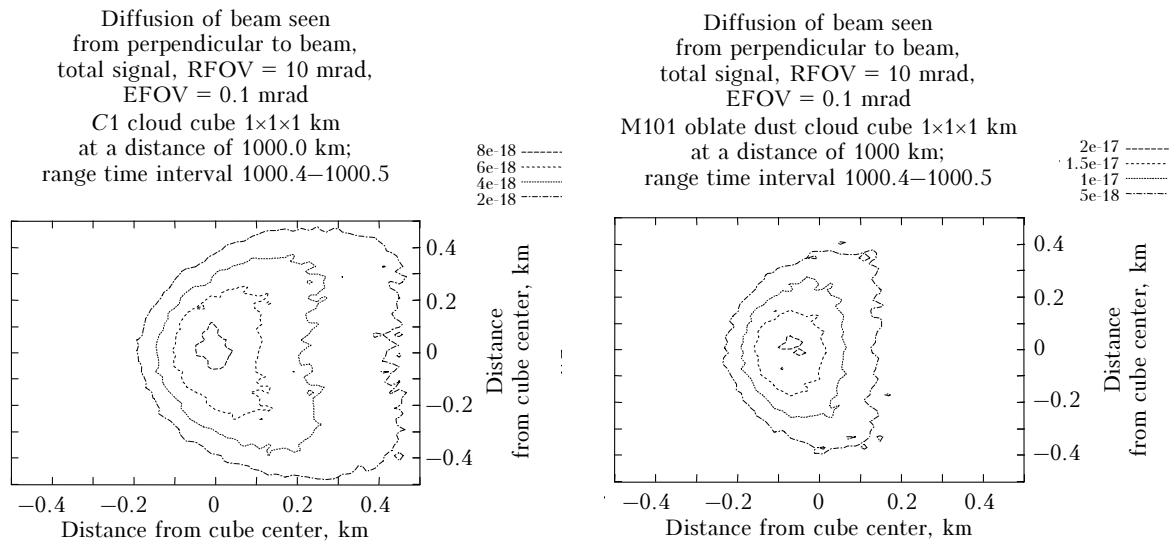
scattering distribution of the particles which depend on the polarization of the multiply scattered photons, too. On the basis of our corpuscular stochastic process of multiple scattering with polarization and with the programs based on this theory it is possible to simulate this diffusion phenomenon of pulse stretching in detail and to make it visible. For this purpose we use the setup of a bistatic CCD lidar.

For example, let us consider a cloud of the form of a cube with 1 km side length at a distance of 1000 km from the emitter with a 0.1 mrad field of view pointing perpendicularly on one side of the cube. The pulse length is 3 m. The receiver field of view axis is perpendicular to the emitter field of view axis pointing at the center of a side of the cube, its

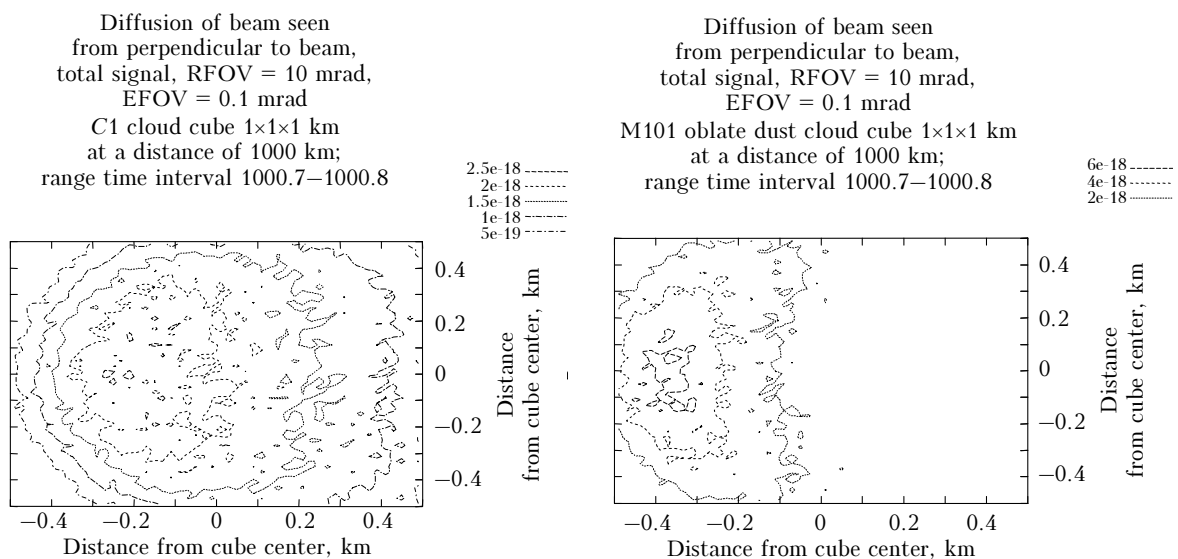
aperture is 10 mrad. The receiver is at a distance of 1 km from the center of the cube. In one case the cloud is a C1 cloud of water droplets whose directional scattering distributions are determined by the set of PolyMie Mueller matrices. In the second case the cloud is an ice cloud whose directional scattering distributions are determined by the set of randomly oriented oblate cylinders (M101). The pulse stretching is increasing with the depth of penetration, the one of the water cloud is much higher than the pulse stretching of the ice cloud, finally more than double as much; see Figs. 6 and 7.

### Conclusions

Monostatic and bistatic returns of CCD lidars may be simulated using programs which have been developed at the Institute of Mathematics of the Ludwig-Maximilian University of Munich on the basis of the stochastic processes of corpuscular multiple scattering with polarization for the description of transport of light through the atmosphere. These simulations can be done in a reasonable time, e.g., half an hour for the pulse stretching problem.



**Fig. 6.** Pulse stretching of a laser beam in a cloud of water droplets (left) and an ice cloud (right) at a penetration depth of 400–500 m.



**Fig. 7.** Pulse stretching of a laser beam in a cloud of water droplets (left) and an ice cloud (right) at a penetration depth of 700–800 m.

Our simulations show that CCD diffusion returns are sensitive against many parameters of scattering particles like shape, extinction, and size. The information obtained from inspection with the naked eye or from statistical and graphical analysis of CCD diffusion pictures are good for classification of clouds and for obtaining a priori knowledge about the possible types of scattering particles within the cloud and, hence, for assisting in solving parameter retrieval problems. Therefore our programs can assist in designing a new generation of CCD lidars. Obviously, the diffusion pictures obtained from these CCD lidars contain lots of information.

### Acknowledgements

We are grateful to Lisheng Xu from the Chengdu University of Information Technology from whom we received the full sets of Mueller matrices of randomly oriented prolate and oblate spheroids and cylinders.

This research is supported by the International Association for the Promotion of Cooperation with Scientists through the New Independent States of the Former Soviet Union (INTAS) and the Russian Foundation for Basic Research (RFBR) under projects INTAS No. 05-1000008-8024 and RFBR No. 06-05-64484a.

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