GRADIENT METHOD OF MINIMIZATION OF ANGULAR DIVERGENCE OF A LIGHT BEAM

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An algorithm is proposed for optimization of the parameters of propagation of a light beam through a nonlinear medium against the criterion of angular divergence in the far diffraction zone. A system of equations is derived for two auxiliary functions which can be used for construction of an iterative procedure of search for the optimal phase by the method of conditional gradient. To demonstrate the efficiency of the algorithm, the problem of compensation for the stationary wind refraction is solved.

Propagation of light beams through the real atmosphere is accompanied by diverse effects associated with nonlinear refraction, fluctuations of the optical parameters, and so on. These effects cause amplitude phase distortions of the beam propagated along the path and, as a consequence, its additional angular divergence, which is, generally speaking, not additive to its diffractional divergence. Studying the possibilities of minimization of the beam divergence in the far diffraction zone by means of control of the beam phase front at the transmitting aperture is of interest for certain problems in atmospheric optics.

The most promising approach seems to be the determination of the initial phase profile of the beam in the process of numerical solution of the problem of optimization. With the use of the gradient methods, which are well developed at present, to maximize the radiant power density at the object of a given radius.[1,2]

The problem of optimal control consists in finding such parameters of the system, which ensure the necessary conditions of interaction of the beam with the medium. In the formulation of the problem of control of the beam parameters, at first it is necessary to formulate the quality criterion (the goal function), which specifies the aim of such a control.

The angular divergence in the far diffraction zone can be minimized by maximizing the relative fraction of the light power concentrated within a given solid angle at a sufficiently distant target. The last condition can be conveniently formulated as that of maximizing the spectral criterion

$$J_X = \int \tilde{\rho}(\mathbf{k}) |\tilde{A}(\mathbf{k})|^2_{z=z_0} \, d^2\mathbf{k}, \tag{1}$$

where $\mathbf{k} = \{k_x, k_y\}$, k_x and k_y are the projections of the wave vector onto the plane perpendicular to the direction of beam propagation, $\tilde{A}(\mathbf{k})\big|_{z=z_0}$ is the spectrum of complex amplitude of the electric field of light wave at the target ($z = z_0$), and $\rho(\mathbf{k})$ is the function specifying the solid angle within which the radiant energy is concentrated. Propagation of quasistationary radiation through the moving weakly absorbing medium is described by a system of equations which is given below in dimensionless form

$$2i\,\frac{\partial A}{\partial z} = \frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} + R_V T A\,, \tag{2}$$

$$\frac{\partial T}{\partial x} = A A^*\,. \tag{3}$$

An initial condition is prescribed upon entering the nonlinear medium ($z = 0$)

$$A(x, y, 0) = A_0(x, y) \exp\,[iU(x, y)]\,. \tag{4}$$

We will find the increment ΔJ_Ω to functional (1) due to the variation of the initial phase profile ΔU of the emitted wave

$$\Delta J_\Omega = \int \tilde{\rho}(\mathbf{k})\, \Delta\, |\tilde{A}(\mathbf{k})|^2_{z=z_0}\, d^2\mathbf{k} =$$

$$= 2\mathrm{Re} \int \tilde{\rho}(\mathbf{k})\, [\tilde{A}^*(\mathbf{k})\, \Delta\tilde{A}(\mathbf{k})]|^2_{z=z_0}\, d^2\mathbf{k}\,. \tag{5}$$

Going over from the spectral to the initial variables, we transform ΔJ_Ω into the form

$$\Delta J_\Omega = (1/\sqrt{2\pi})^3\, 2\mathrm{Re} \int (\psi \Delta A)|_{z=0}\, d^2\mathbf{r} + J_1\,, \tag{6}$$

where

$$J_1 = (1/\sqrt{2\pi})^3\, 2\mathrm{Re} \int_0^{z_0} dz \int \left[\Delta A\, \frac{\partial\psi}{\partial z} + \psi\, \frac{\partial A}{\partial z} \right] d^2\mathbf{r}\,,$$

where $\mathbf{r} = \{x, y\}$ and $\Psi(\mathbf{r}, z)$ is some auxiliary function. We choose the function Ψ in such a way that the term J_1 in Eq. (5) will vanish. It has been demonstrated in Ref. 3 that $J_1 = 0$ when the function $\Psi(\mathbf{r}, z)$ and the second auxiliary function $G(\mathbf{r}, z)$ both satisfy the conjugated system of equations

$$-2i\,\frac{\partial\psi}{\partial z} = \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + R_V T\psi + 2iR_V A^* G\,, \tag{7}$$

$$-\frac{\partial G}{\partial x} = \mathrm{Im}(A\psi)\,. \tag{8}$$

When spectral criterion (1) is maximized, the boundary conditions for functions Ψ and G take the form

$$\psi(\mathbf{r}, z_0) = \int \rho(\mathbf{r}' - \mathbf{r})\, A^*(\mathbf{r}')|_{z=z_0}\, d^2\mathbf{r}'\,,\quad G(\mathbf{r}, z_0) = 0\,. \tag{9}$$

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Thus the increment in the functional ΔJ_X is expressed as follows:

$$\Delta J_\Omega = (1/\sqrt{2\pi})^3\, 2\mathrm{Re} \int (\psi \Delta A)\big|_{z=0}\, \mathrm{d}^2\mathbf{r}\ . \qquad (10)$$

in terms of variation of the field at the radiating aperture $\Delta A(\mathbf{r}, 0)$.

Equation (10) is used to find the gradient of the functional J_Ω. As was demonstrated in Ref. 3, when the phase front of the beam is controlled and its amplitude profile is fixed, the $(n + 1)$th order approximation of phase is related to its nth order approximation by the iterative procedure

$$U_{n+1}(\mathbf{r}) = U_n(\mathbf{r}) - \alpha\, [U_n(\mathbf{r}) + \arg\psi_n(\mathbf{r}, 0)]\ ,$$

where $0 < \alpha < 1$ is the size of the gradient step. Direct calculation of boundary condition (9) would entail lengthy computations. At the same time, it is easy to demonstrate that this condition can be reduced to the form

$$\psi(\mathbf{r}) = 1/\sqrt{2\pi} \int \rho(\mathbf{k}) A^*(\mathbf{k}) \exp(i\mathbf{k}\mathbf{r})\, \mathrm{d}^2\mathbf{k}\ , \qquad (11)$$

which makes it possible to implement the FFT algorithm.

Below we present some results of numerical simulation against the spectral criterion. Calculations were performed for beams with the Gaussian amplitude profile propagating along the path of length $z_0 = 0.5$. The nonlinearity parameter R_v was chosen to be equal to -15, and the relation

$$\rho(\mathbf{k}) = \exp(-k^2/s^2) \qquad (12)$$

was used for the function $\rho(\mathbf{k})$ which specifies the spectral region of energy concentration (parameter s here varied in the course of numerical experiment).

Calculations demonstrated that the efficiency of the algorithm of optimization strongly depends on the value of the parameter s. The fraction of radiant energy concentrated within a given solid angle and the peak intensity of radiation at the target varies in different ways in the course of the iterative process. Varying s, the criterion J_Ω can be increased by 10–15% and the peak intensity at the target – – by 20–30%. The initial approximation of the beam phase and the size of the gradient step α affects insignificantly the convergence of the iterative process.

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