NEW APPROACH TO SOLUTION OF THE INVERSE PROBLEM OF THERMAL SOUNDING OF THE ATMOSPHERE FROM SATELLITES

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We propose in this paper a new approach to solution of the inverse problem as applied to thermal sounding of the atmosphere. The approach is based on application of the Green's function to the regularized, by Tikhonov method, integro-differential equation. It is tested by solving a closed numerical model problem and then comparing the result with that obtained by classical Tikhonov method. It is noted that the approach provides a higher stability of the regularized solution. The data obtained with the HIRS/2 device (NOAA satellite) have been processed using this approach.

STATEMENT OF THE INVERSE PROBLEM

The equation of radiation transfer is the basic instrument to relate the measured intensity of the outgoing radiation (IOR) to the temperature distribution in the Earth's atmosphere. The expression for IOR at the upper boundary of the atmosphere (assume that there is no extinction and transformation of radiation along the path from the upper boundary of the atmosphere to a satellite) can be presented in the following form[1–14]:

$$I_\nu(\mu) = \delta_\nu\, B_\nu(T(h_0))P(h_0, H, \mu) + \int\limits_{h_0}^{H} B_\nu(T(z))\, \frac{\mathrm{d}P_\nu(z, H, \mu)}{\mathrm{d}z}\, \mathrm{d}z + (1 - \delta_\nu)I^{\downarrow}_\nu, \tag{1}$$

where ν is the center of the spectral channel; δ_ν is the surface emissivity (blackbody emissivity $\delta_\nu = 1$); $\mu = 1/\cos\varphi$, φ is the angle measured from the nadir direction; P is the transmittance of the atmospheric layer $(z - H)$ in the direction of φ; B_ν is the Plank function at the given channel frequency and temperature T; h_0 is the altitude of the lower atmospheric level considered (ground level $h_0 = 0$ or the cloud level); H is the altitude of the upper boundary of the atmosphere (in our problem it is taken to be $H = 100$ km); $I^{\downarrow}_\nu$ is the intensity of radiation reflected from the Earth's surface.

The Plank function at the frequency ν and temperature T is determined by the equation

$$B_\nu(T) = 1.1910659\cdot10^{-5}\, \nu^3/[\exp(1.438833\nu/T) - 1]\,, \tag{2}$$

where ν is the wave number, in cm^{-1}.

The transmission of the atmospheric layer between the levels z and H is described by the expression

$$P(z,H,\mu) = \exp\left(-\int\limits_{z}^{H} \mathrm{d}z' \sum_{j=1}^{M} K_j(\nu,T(z'))\, \rho_j(z')\, Q(z',\mu)\right). \tag{3}$$

Here K_j is the absorption coefficient that is determined by absorption by atmospheric gases with concentrations ρ_j and by the continuos absorption by the H_2O; Q is the path function accounting for the Earth's surface curvature at angular measurements,

$$Q(z', \mu) = \frac{\mu(R + z')}{\sqrt{(R + z')^2\mu^2 - (R + H_s)^2(\mu^2 - 1)}}\,,$$

$H_s \le z' \le 0$,

H_s is the flight altitude of a satellite.

The atmospheric absorption is caused by light absorption by such gases as H_2O, CO_2, O_3, and others. Absorption spectra of these gases are shown in Figs. 1 and 2.

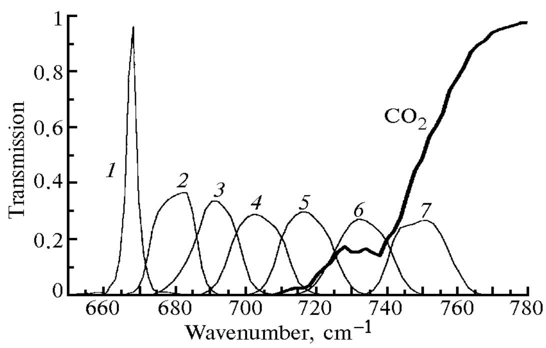

FIG. 1. Spectral channels of the HIRS/2 radiometers and the CO_2 *transmission spectrum for the midlatitude summer meteorological model.*

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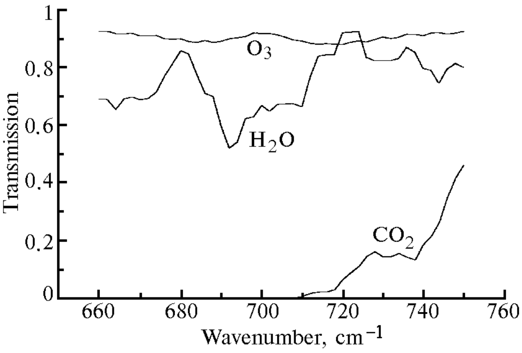

FIG. 2. Atmospheric transmission spectrum for the midlatitude summer model of the atmosphere in the region of spectral channels 1–7 of the HIRS/2 radiometer.

Each spectral channel of the HIRS device has its own spectral instrumental function $A(\nu)$, therefore, in the general case the measured parameter may be written in the following form:

$$J_\nu(\mu) = \int_0^\infty A(\nu - \nu') \, I_\nu(\mu) \, d\nu \, . \quad (4)$$

To determine the temperature profile of the atmosphere, most suitable channels of HIRS/2 are channels *1–7* (14.95–13.34 μm) coinciding with the 15-μm absorption band of CO_2. Figures 3 and 4 demonstrate the spectral dependence of the IOR, in mW/(m^2·sr·cm^{-1}), for the channels *1–7* and the altitude dependence of the kernel of the integral equation (1). When solving the problem of thermal sounding of the atmosphere, the carbon dioxide is normally considered as a homogeneously mixed gas with known concentration.

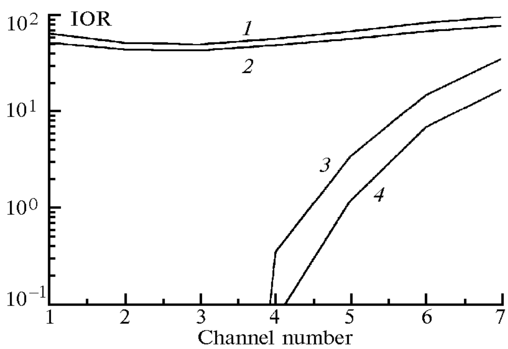

FIG. 3. Spectrum of the outgoing radiation for the system "underlying surface + atmosphere" for the meteorological models of midlatitude summer (1) and winter (2) as compared to the Earth's outgoing radiation for summer (3) and winter (4).

In Eq. (1) it is assumed that $\delta_\nu = 1$, i.e. the underlying surface is considered as a blackbody. The third term vanishes in this approximation. The approximation $\delta_\nu = 1$ is true to a high accuracy for water surface. For land it is violated, i.e. $\delta_\nu < 1$. In addition, the spectral dependence of the emissivity δ_ν is different for different underlying surfaces. All the above-said significantly complicates solution of the inverse problem of thermal sounding (TSP) over land. In fact, separate problem on determining the temperature of the underlying surface and its emissivity δ_ν is to be solved. In this paper, we do not consider this problem. For this reason, we assume, hereinafter, $\delta_\nu = 1$.

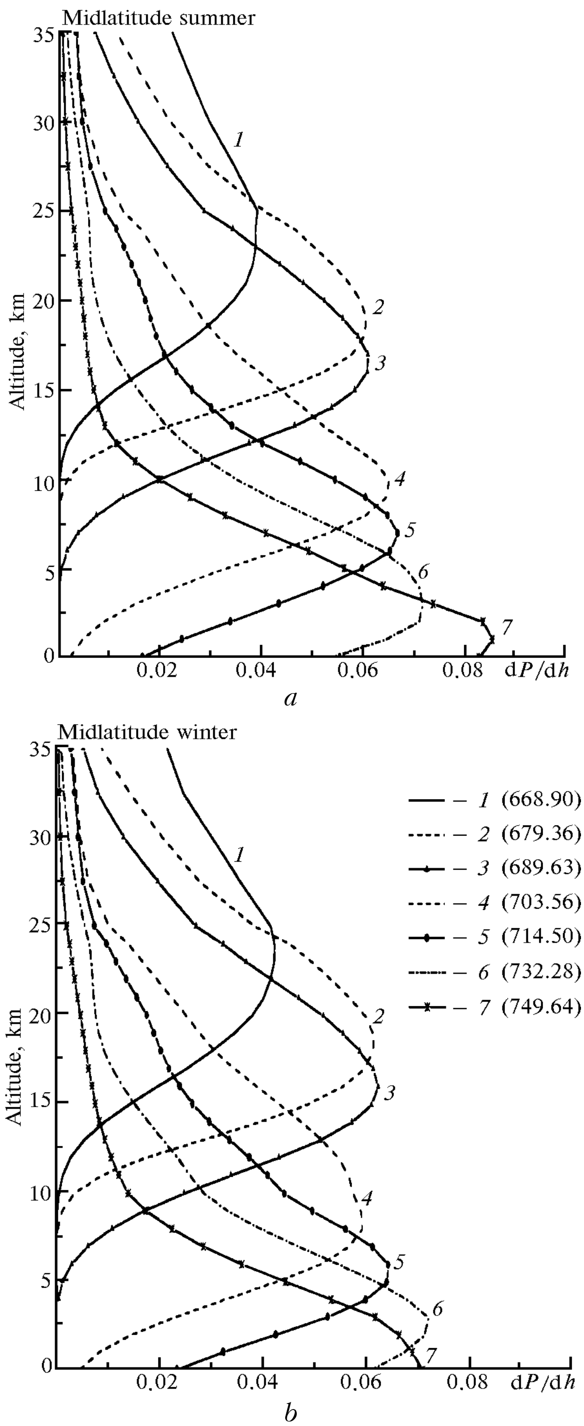

FIG. 4. Weighting functions of the HIRS/2 radiometer for the spectral channels 1–7 and two meteorological models.

Thus, the initial equation for reconstruction of the temperature profile under clear sky conditions in the atmosphere is

$$I_\nu(\mu) = B_\nu(T(h_0))P(h_0, H, \mu) + + \int_{h_0}^{H} B_\nu(T(z)) \frac{dP_\nu(z, H, \mu)}{dz} dz . \quad (5)$$

Equation (5) can be reduced to the integral Fredholm equation of the first kind

$$\int_a^b K(\nu, s)\, y(s)\, ds = f(\nu) , \quad (6)$$

where

$$f(\nu) = I_\nu(\mu) - B_\nu(T(h_0))P(h_0, H, \mu) ;$$

$$h_0 = a = 0 ;\ b = H ;$$

$$K(\nu, s) = \frac{d}{ds} P(s, H, \mu) ; \quad y(s) = B_\nu(T(s)) .$$

Let us describe the problems arising, when solving the TSP.

1. The account of the out-of-integral terms in Eq. (1) responsible for the emission of the underlying surface (US) (Figure 3 compares the outgoing radiation of the system “US + atmosphere“ (curves *1* and *2*) with the US outgoing radiation (curves *3* and *4*)), including thermal radiation reflected from the Earth's surface.

2. Detection of clouds within the device's field of view, determination of the cloud type, and taking them into account. This is necessary, because with clouds the first and third terms in Eq. (1) are determined by the radiation coming from clouds, rather than the Earth's surface. In this case, the parameters to be known are the altitude of a cloud, its type, emissivity, etc.

3. Inadequacy of the optical model of the atmosphere to the real situation during measurements. The kernel of the integral equation is calculated for some model situation in the atmosphere. We can conditionally make some refinements of the kernel to adjust it to, for example, a season and, thus, to allow for the statistical information about variations of the corresponding parameters in different seasons. Of course, it is impossible to take into account any real situation in relation to that wide list of the parameters determining the optical model. The only assumption, which is supported by many evidences,[3] is here the spatiotemporal stability of the CO_2 content.

Note, that the problem of thermal sounding is the ill-posed problem. The conditionality number for the matrix of the system of linear algebraic equations, to which the integral equation is reduced, is equal to $\approx 10^{10}$. That large value indicates that the problem is ill-posed, i.e., it is very sensitive to infinitesimal errors in the right-hand side of the equation and its parts.

Since Eq. (6) is significantly ill-posed, it is usually solved with the use of Tikhonov regularization method. Therefore, let us consider briefly the Tikhonov method of solution and the new approach based on the Green’s functions we propose in this paper.

TIKHONOV REGULARIZATION METHOD

Let us consider the Tikhonov regularization method as applied to solution of the Fredholm integral equation of the first kind[15]:

$$Ay = \int_a^b K(x, s)\, y(s)\, ds = f(x) ,\ c \le x \le d \quad (7)$$

(the kernel of Eq. (7), $K(x, s)$, is assumed real and continuous function on the rectangle $\{a \le s \le b, c \le x \le d\}$).

The Euler equation for the extremum problem

$$\Phi_\alpha[y_\alpha, f] = \inf_{y \in Y} [y, f] ,$$

where

$$\Phi_\alpha[y_{\alpha_1}, f] = \rho(Ay, f) + \alpha \int_a^b \left[y^2(s) + q(s) \left(\frac{dy(s)}{ds}\right)^2 \right] ds ,$$

has the following form:

$$\alpha[y_\alpha(t) - q y''_\alpha(t)] + \int_a^b R(t, s)\, y_\alpha(t)\, ds = F(t) ;$$

$$a \le t \le b , \quad (8)$$

where

$$R(t,s) = R(s,t) = \int_c^d K(x, t)\, K(x, s)\, dx ;$$

$$F(t) = \int_c^d K(x, t)\, f(x)\, dx .$$

Thus, one should solve Eq. (8) instead of the ill-posed equation of the first kind (7).

The main problem in the Tikhonov regularization method is how to choose the proper regularization parameter which gives the optimal solution.

The simplest way to determine this parameter is the trial-and-error method (or the exhaustive search), when the discrepancies of the following form

$$\delta = \left(\int_a^b \{Ay_\alpha - f\}^2 \, dx \right)^{1/2} = \rho_f(Ay, f) \qquad (9)$$

are first calculated for the monotonically decreasing sequence of the regularization parameters $\alpha_1 > \alpha_2 > \ldots > \alpha_n$ (for example, $\alpha_1 = 10^0$, $\alpha_2 = 10^{-1}$, $\alpha_3 = 10^{-2}, \ldots$). The parameter, which makes the discrepancy (9) minimal, is chosen as the best result.

There are many other criteria for making choice of the regularization parameter with the corresponding algorithms described in Refs. 16 and 17.

METHOD OF THE GREEN'S FUNCTIONS

Some problems may arise in numerical realization of the integro-differential equation (8) because of the algebraization of the second derivative needed. There exist several schemes based on finite differences technique, but the use of those may lead to instability of a solution. To overcome this problem, let us use the Green's method and solve the equation. As a result, we obtain the following equation for solution of the problem of thermal sounding instead of the Eq. (8):

$$\int_0^1 K_1(t, t') \, y(t') \, dt' + \alpha y(t) = f_1(t, \alpha, C_1, C_2) , \qquad (10)$$

where

$$K_1(t, t') = \int_0^1 \tilde{K}(t'', t) \, G(t'', t') \, dt'' ; \qquad (11)$$

$$f_1(t,\alpha,C_1,C_2) = \int_0^1 \tilde{f}(t'') G(t'', t) \, dt'' + \alpha y_0(t,C_1,C_2) , \qquad (12)$$

$$\tilde{K}(t, z) = (d - c) \, (b - a) \times$$

$$\times \int_0^1 K(x(t'), s(t)) \, K(x(t'), s(z)) \, dt';$$

$$\tilde{f}(t) = (d - c) \int_0^1 K(x(t'), s(t)) \, f(x(t')) \, dt';$$

$$x(t) = c + (d - c)t ; \quad s(z) = a + (b - a)z ;$$

$G(x, y)$ is the Green function:

$$G(x, z) = \begin{cases} \dfrac{1}{2(1 - e^2)} \, (e^z - e^{2-z}) \, (e^x - e^{-x}), & 0 \le x \le z, \\ \dfrac{1}{2(1 - e^2)} \, (e^z - e^{-z}) \, (e^x - e^{2-x}), & z \le x \le 1 . \end{cases} \qquad (13)$$

The function y_0 is the solution of the homogeneous equation

$$y_0(t, \alpha, C_1, C_2) = C_1 \, e^t + C_2 \, e^{-t} . \qquad (14)$$

The constants C_1 and C_2 can be found from the boundary conditions:

a) $y\big|_{t = 0} = A_0$; $\quad y\big|_{t = 1} = A_1$ or

b) $y'\big|_{t = 0} = B_0$; $\quad y'\big|_{t = 1} = B_1$.

Equation (10) is the Fredholm integral equation of the second kind. It can be easily solved numerically.[17] The approximate boundary conditions can be chosen from a meteorological model corresponding to the season.

NUMERICAL EXPERIMENT

There exit two schemes to reconstruct the atmospheric temperature. The one that is based on expansion of the Plank function into a series over temperature relative to some reference profile. The solution in this scheme is the deviation of temperature from the reference profile. In the second scheme, the unknown parameter is the Plank function itself. Then the obtained Plank function is used for calculating temperature by Eq. (2). Our research, as well as the results from literature[1,3,4,6–8] indicate that the second scheme is preferable.

We solved the model problem in the following way. First, we calculated the "measured" IOR $I_\nu(\mu)$ by Eq. (5), and then it was distorted by a random variable $\varepsilon(\nu)$ for imitating the measurement noise

$$\int_a^b K(\nu,s) \, y(s) \, ds - f(\nu) = \varepsilon(\nu) .$$

Then the inverse problem was solved with the use of the new approach for several levels of errors characteristic of real experiments (0.5 – 3%).

Figure 5 presents the temperature profiles retrieved from the solution of the model problem. Figure 5*a* presents the reconstructed profile of temperature T_r compared with the model profile T_0 (midlatitude summer) at the error of 0.5%, while Fig. 5*b* shows the same profiles at the 3-% error.

Analysis of Fig. 5 shows how unstable is the model problem to experimental errors. The errors of even 0.5% level lead to distortions in the reconstructed temperature profile (Δ_T = 2 K), while the errors of 3% result in a more significant distortion of the temperature profile (the maximum error Δ_T reaches 13 K). It is known from literature, that the level of errors in IOR measurements does not exceed 1%. The results of modeling show that, if the physical uncertainties described by the terms out of the integral are taken into account most efficiently, then we might expect to achieve a suitable accuracy in temperature profile reconstruction with this level of measurement errors.

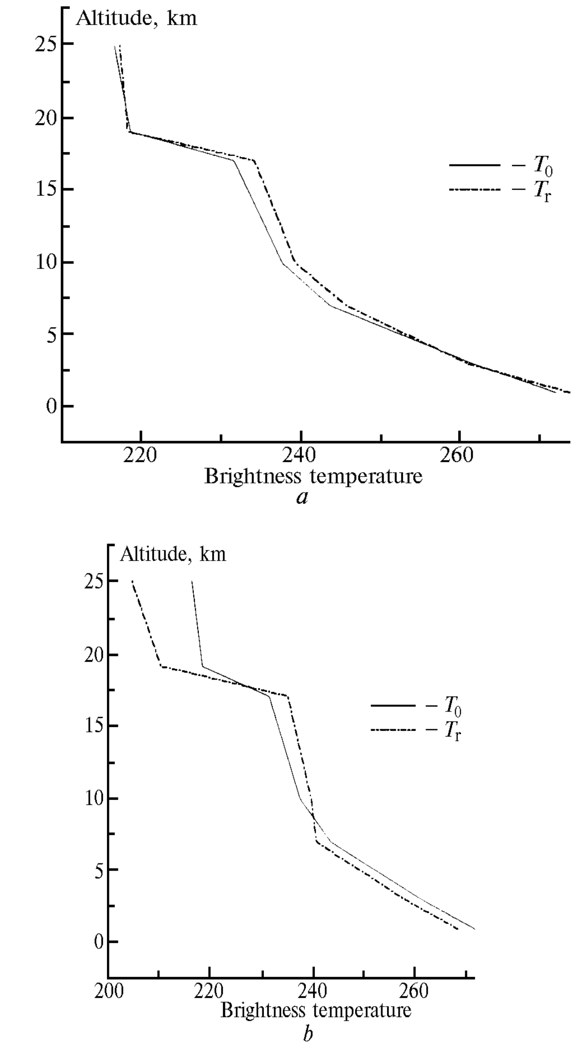

FIG. 5. Temperature profiles retrieved from solution of the model problem: reconstructed temperature profile T_r *as compared with the model profile* T_0 *(midlatitude summer) with errors at the level of* 0.5% *(a); the same but with errors at the level of* 3% *(b).*

Let us consider now the errors resulting from uncertain determination of the land emissivity.

Figure 6 gives the example of reconstruction of temperature profiles at different levels of errors in determination of the US temperature. As seen, underestimation of the US temperature does not lead to significant errors, while overestimation may grossly change the reconstructed profile relative to the exact one. However, this result does not guarantee that the same tendency (different errors depending on whether the US temperature is underestimated or overestimated) will be observed in practice. Therefore, we should consider the maximum obtained value as an obtained error. As seen, the error of reconstruction of the temperature profile is below 1° (at the altitude of 7.5 km). And only at the ground level it reaches 2°.

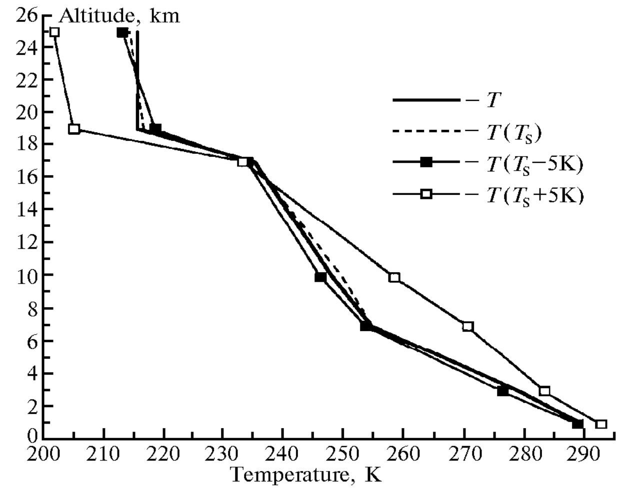

FIG. 6. Temperature profiles reconstructed at different values of errors in estimation of the underlying surface temperature (the average value is 295 K*).*

Figure 7 illustrates the influence of errors in the kernel of the integral equation (they are caused by systematic errors in the spectroscopic parameters) to the accuracy of the temperature profile reconstruction.

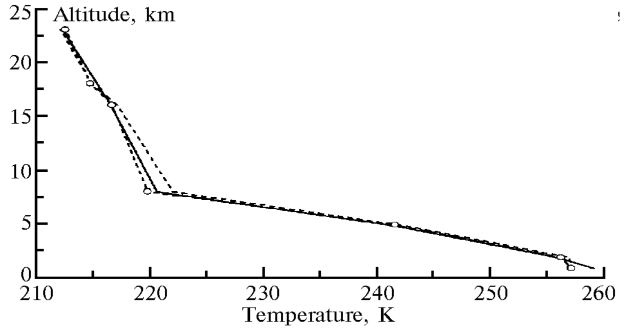

FIG. 7. An example of influence of spectroscopic errors upon reconstruction of the temperature profile in the numerical experiments: the exact profile (solid curve); the reconstructed profile with 5-% errors in the kernel and the exact right-hand side (dashed line with circles); the reconstructed profile with the exact kernel and 0.5-% error in the right-hand side (dashed curve).

PROCESSING OF THE EXPERIMENTAL DATA

Now consider solution of the inverse problem of thermal sounding based on data recorded on June 1, 1998, at the Institute of Atmospheric Optics with the SKANEKS station. Figure 8 presents the OR fluxes measured in the nadir direction in the seven channels within the 15-μm absorption band of CO_2 and for several trajectories as the satellite flew from north to south (measurements in the vicinity of the city of Tomsk).

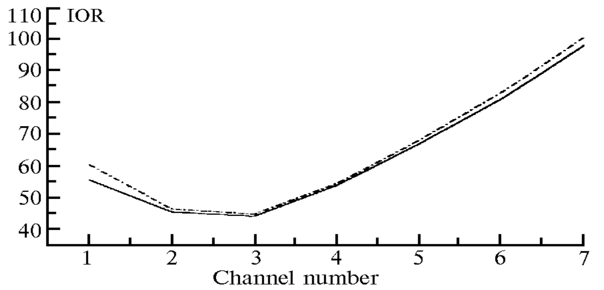

FIG. 8. Measured OR fluxes for several flight trajectories from north-to-south.

The land emissivity has been modeled and then removed from calculations. As seen from Fig. 9, the reconstructed values of the temperature do not contradict, at least to the common sense, and they agree with the model concepts about the thermal state of the atmosphere. To make more specific conclusions, the comparison with data of radio sounding of the temperature is needed.

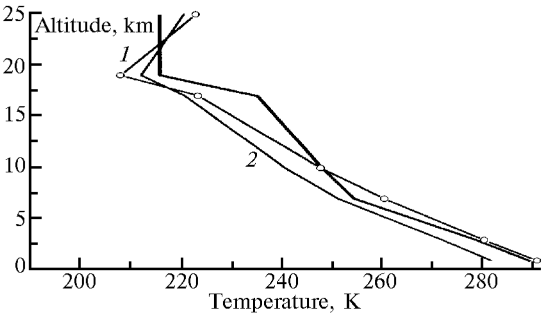

FIG. 9. Temperature profiles reconstructed for several flight trajectories (curves 1 and 2) in the vicinity of Tomsk as compared to the model temperature profile (midlatitude summer).

CONCLUSIONS

1. To solve numerically the integro-differential equation regularized by Tikhonov method, the equation should first be transformed using the Green's function. This procedure allows a significant increase in the stability of the regularized solution.

2. The accuracy of reconstruction of the temperature profile depends on the error in the right-hand side of the equation. If the measurement error in the IOR is below 0.5%, then the accuracy of reconstruction of the temperature profile is 2°, what is comparable with the results obtained earlier by other authors.

3. The errors in spectroscopic data influence the accuracy of reconstruction of the temperature profile only if the systematic shift of the kernel of the integral equation takes place. The error in temperature estimation reaches 2° at 5-% shift of the initial spectral data.

4. The term of the Eq. (1) which is responsible for the contribution coming from the underlying surface, has a strong effect on the accuracy of temperature estimation. Variations of the US temperature within 5° result in the errors of estimation of the atmospheric temperature up to 10° at some altitudes.

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