ON THE POSSIBILITY OF CONTROLLING THE PHASE OF RADIATION OF AN OPTICALLY COUPLED LASER ARRAY

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The structure and the threshold amplification of collective modes of the diffraction–coupled laser array are theoretically investigated. The dependence of threshold gain on the order of diffraction coupling (i.e., on the number of coupled lasers) is analyzed within the framework of the perturbation theory. Functions are found describing the response of the phase of the output radiation on the misalignment of optical lengths of individual lasers in the laser array.

Promising technique for obtaining the powerful laser radiation of high quality, which is of interest for various problems in laser technology, is the use of module multibeam laser systems. Frequency and phase synchronization of modules via optical coupling of different lasers makes it possible to sum over the coherent fields of individual lasers, so that much higher power density can be obtained in the far zone of diffraction ($I \sim N^2$, where N is the number of lasers in array). The efficiency of various means for such coupling and the problems of stability of coherent regime of generation by the laser array were theoretically analyzed in Refs. 1–3. Optical coupling can be realized in the simplest way for large laser arrays accounted for the diffractional exchange of radiation between the active elements placed inside the common cavity (Fig. 1). For example, phased generation by large number of waveguide CO_2 lasers (about 60), placed in a two–dimensional array with cross section forming a periodic triangular grid, was obtained in Ref. 4. Synchronization of both the two– and one–dimensional arrays of gas and semiconductor lasers was obtained in Refs. 3, 5, and 6 on the basis of self-reproduction of periodic fields (the Talbot effect).

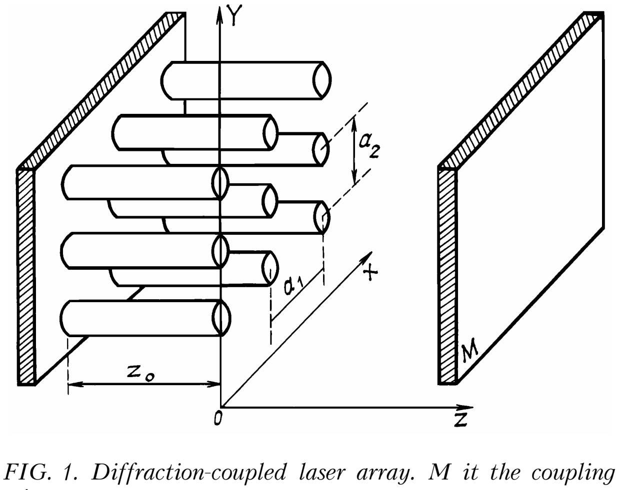

FIG. 1. Diffraction-coupled laser array. M it the coupling minor.

Both theoretical[7] and experimental[8,9] studies pointed out the strong influence of the spread in the parameters of individual lasers (for example, their optical lengths) on the efficiency of phase synchronization. Therefore, it becomes quite important to compensate for such distortions as well as to actively control the phase profile of output radiation of the laser array.

In the present paper we theoretically investigate the effect of misalignment of the optical lengths of the diffraction coupled lasers on the structure and threshold amplification of the proper field distribution (of its collective modes). Response functions are found for the phase of the output radiation.

To analyze this problem, we employ the model of periodic array of the diffraction coupled lasers (Fig. 1). The individual active elements are optically coupled due to diffraction spreading of radiation reflected from the coupling mirror M.

To retrieve the collective modes in the array let us consider the transformation of the field after the radiation has passed around the cavity. The field in the plane $z = 0$ has the form:

$$E(x,\ y,\ z = 0) = \sum_n \sum_m e_{nm} f(x - na_1; y - ma_2), \qquad (1)$$

where $f(x,\ y)$ is the distribution of the given transverse mode of an individual waveguide and e_{nm} is the complex field amplitude in the waveguide $(n,\ m)$. Transformation of the field after the radiation passed round the cavity may be represented in the operator form:

$$E(x,\ y,\ 2(z + z_0)) = \hat{T}\,\hat{P}\,\hat{G}\,\hat{E}\,(x,\ y,\ 0), \qquad (2)$$

where $\hat{G}$ is the linear diffraction operator describing the propagation of radiation to the coupling mirror M and back to the plane of the exit aperture of waveguides, $\hat{P}$ is the operator of projection upon the given transverse mode of the waveguide, and $\hat{T}$ is the operator of radiation propagation along the waveguides.

Equating Eqs. (1) and (2) (in accordance with the conduction of complete field reproduction after the radiation has passed around the cavity) we obtain an eigenvalue problem:

$$\gamma^e_{nm} = \sum_k \sum_l M^{nm}_{kl} e_{kl}, \qquad (3)$$

$$M^{nm}_{kl} = \iiiint G(x - \xi; y - \eta; 2z) f(\xi - ka_1; \eta - la_2) \times$$
$$\times f(x - na_1; y - ma_a) \mathrm{d}\xi \mathrm{d}\eta \mathrm{d}x \mathrm{d}y = M \begin{vmatrix} m - l \\ n - k \end{vmatrix}, \qquad (4)$$

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where M_{kl}^{nm} is the complex coefficient of diffractional coupling of the waveguides (k, l) and (n, m), γ is the eigenvalue whose modulus and phase prescribe the Q–factor and frequency of the corresponding collective mode. Under condition that $f(x, y) = f_1(x) f_2(y)$, problem (3) is factorized for a square array and is reduced to the problem of collective modes in a linear array:

$$\gamma^{(k)} \mathbf{E}^{(k)} = \hat{M} \mathbf{E}^{(k)}, \quad (5)$$

where $\hat{M}$ is the matrix of coefficients of diffraction coupling and $\mathbf{E}^{(k)}$ is the complex envelope of the kth collective mode.

Solution (5), which corresponds to the complete reproduction of periodic fields at a distance $z_T = \frac{2a_1^2}{\lambda}$ is well known for the infinite laser array. In that case two modes appear to separate out in accordance with their Q–factor ($|\gamma| = 1$): the cophased ($e_n = \text{const}$) and the antiphased ($e_n = (-1)^n$ const) ones (see Ref. 1).

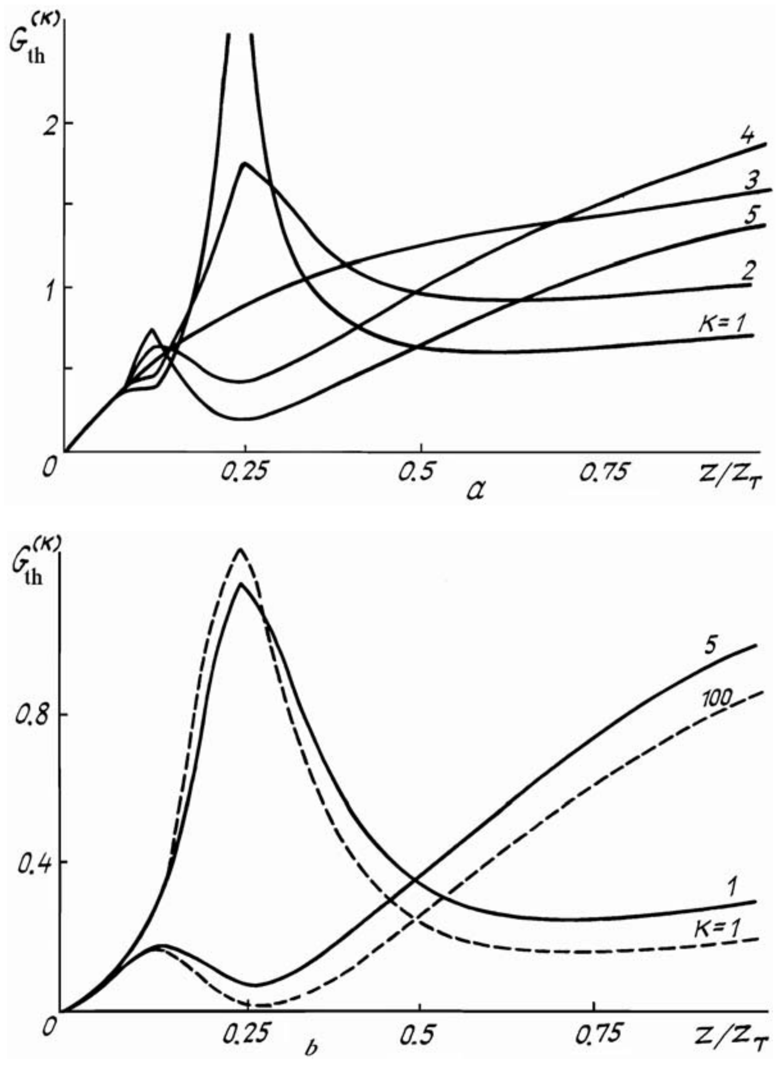

FIG. 2. Threshold gains for collective modes $G_{th}^{(k)}$ vs. distance z to the coupling mirror (κ is the number of a mode): a) $a/\sigma = 6$ and $N = 5$; b) $a/\sigma = 4$ and $N = 5$ (solid curve) and $N = 100$ (dashed curve).

The general solution of problem (5) has not yet been found for a large but finite array of lasers. However, taking into account that $|M_1| \gg |M_2| \gg |M_3|$ ..., we may restrict ourselves to considering only the optical coupling of the "closest neighbours". In this approximation the matrix $\hat{M}$ of diffraction coupling is tridiagonal and the solution of problem (5) acquires the form

$$\gamma^{(k)} = M_0 + 2M_1 \cos\frac{\pi k}{N+1}, e_n^{(k)} = \sqrt{\frac{2}{N+1}} \sin\frac{\pi k n}{N+1},$$

$$k = 1,\ldots,N; n = 1,\ldots N,$$

where k is the number of the collective mode. Note that $k = 1$ corresponds to the cophased mode and $k = N$ – to the antiphased mode.

For $n \sim 1$ and $n \sim$ N the field amplitudes in the waveguides decrease due to uncompensated diffraction losses of radiation at the edges of the array. The dependence of threshold gains $G_{th}^{(k)} = -\ln|\gamma^{(k)}|$ on the distance to the coupling mirror z is shown in Fig. 2 for

$$f(x,y) = \sqrt{\frac{2}{\pi}\frac{1}{\sigma}} \exp\left\{-\frac{x^2+y^2}{\sigma^2}\right\}.$$

The threshold gain of the antiphased mode ($k = N$) reaches minimum at $z = z_T/4$, since the self-reproduction results in such a distribution of the corresponding field after its reflection from the mirror, which coincides with the initial distribution in the plane of the waveguides output aperture at a distance of $z_T/2$. As for the cophased mode ($k = 1$), the distribution of the reflected field in the plane of the output aperture is shifted at half the period of the laser array relative to the initial distribution at $z = z_T/4$, and the threshold gain appears to be maximum then.

At $z = z_T/2$ the threshold gains of cophased and antiphased modes coincide with each other since the field distributions in the plane of the waveguides output aperture are reproduced after reflection for both modes.

An increase of threshold gains with a/a is explained by wider divergence of radiation which results in large diffractional losses. With increase of the number of lasers in array, edge losses diminished, which results in lower threshold gains for all collective modes.

It follows from the solution of problem (5) for eigenmodes and eigenvalues, available for the infinite array that threshold gains for both the cophased and the antiphased modes reached minima at $z = z_T/2$. As can be seen from Fig. 2, the model of coupling of the "closest neighbours" does not describe that effect. Increasing the number N of lasers in array does not introduce any qualitative changes.

It is natural to assume that this effect results from the neglect of diffractional coupling of remote lasers in the array. Indeed, the absolute values of matrix elements M_2, M_3,... increase with the distance z to the coupling mirror and the tridiagonal approximation for the matrix of diffraction coupling fails. Corrections for eigenvalues of collective modes may then be then retrieved using the perturbation method.

We obtain in the first order of the perturbation theory

$$\gamma_S^{(k)} = M_0 + 2\sum_{l=1}^{S} M_l \left\{ \cos\frac{\pi k l}{N+1} - \frac{1}{N+1} \times \right.$$
$$\left. \times \left[l \cos\frac{\pi k l}{N+1} - \sin\frac{\pi k l}{N+1} \cot\frac{\pi k}{N+1} \right] \right\}, \quad (6)$$

where $\gamma_S^{(k)}$ is the eigenvalue of the kth collective mode and S is the order of diffraction coupling ($S = N - 1$ corresponds to an account of the optical coupling of all the lasers in the array). The eigenvalue yielded by the perturbation method for $S = 1$ coincides with the corresponding eigenvalue of the tridiagonal matrix $\hat{M}$ (6).

Figure 3 shows the dependence of threshold gains for the cophased and antiphased modes for different orders of diffraction coupling S. It can be seen that with increase of S the minima of the collective modes in threshold gains are formed in the vicinity of $z = z_T/2$. The positions of these minima at the z axis tend toward the theoretical value $z = z_T/2$ found for the infinite laser array.

It should be noted, however, that according to the available experimental data on the behavior of threshold gains,[5,6] the model of coupling of the "closest neighbours" is more adequate to situations found in practice. Indeed, the spread in the parameters of individual lasers results, under condition that $|M_1| \gg |M_2| \gg |M_3| \ldots$, in direct coupling of the "closest neighbours" alone.

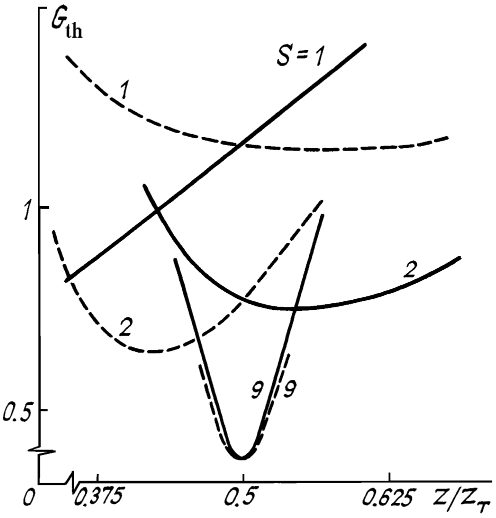

FIG. 3. Threshold gain for the cophased (dashed curves) and antiphased (solid curves) mode vs. the distance to coupling minor z for different orders of diffraction coupling S. N = 10 and a/σ = 12.

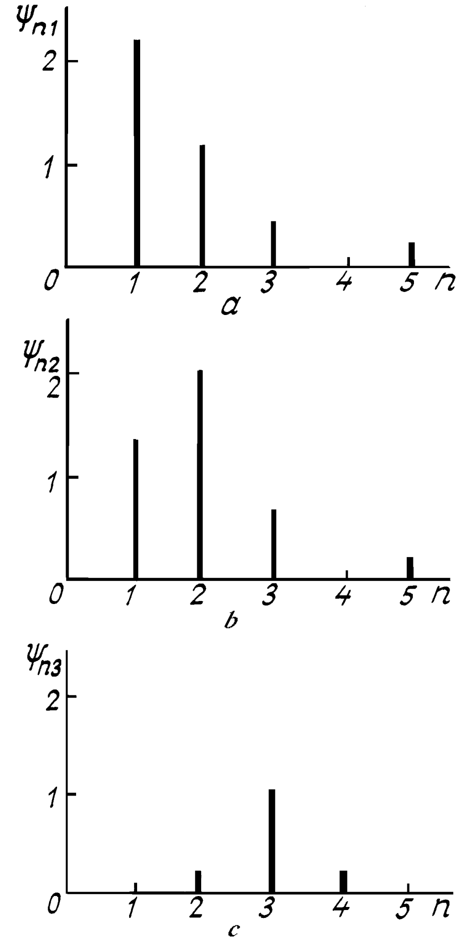

FIG. 4. Response function of phase Ψ_{nl} of the output radiation by the laser array vs. misalignment of the optical length of the lth channel of generation: a) l = 1, b) l = 2, and c) l = 3. N = 5, a/σ = 4, and z = 0.7 z_T.

If the optical lengths of different generation channels are misaligned by $\xi_n = z_n - z_0$, $n = 1, ..., N$, the problem of eigenmodes and eigenvalues (5) can be written in the form:

$$\gamma^{(k)} \mathbf{E}^{(k)'} = \hat{A} \hat{M} \mathbf{E}^{(k)'};$$

$$A_{nm} = \delta_{nm} \exp\{i\varphi_n\}; \ \varphi_n = 2k_0 \xi_n , \quad (8)$$

where k_0 is the wave number and k is the serial number of the collective mode. If we restrict ourselves to the case of small misalignments $|\varphi_n| \ll 1$, we may obtain, in the first order of the perturbation theory, the response of phase of the radiation of the cophased collective mode:

$$\Phi = 2k_0 \hat{\Psi} \xi, \quad (9)$$

where Φ is the vector of phase of the output radiation and ξ is the vector of misalignment between the optical lengths of lasers. The elements of matrix $\hat{\Psi}$ which are equal to

$$\Psi_{nl} = \sum_{k=2}^{N} \mathrm{Re}\left[\frac{\gamma^{(1)}}{\gamma^{(1)} - \gamma^{(k)}}\right] e_l^{(k)} e_n^{(k)}, \quad (10)$$

determine the response of the phase of radiation in the nth generation channel to unit misalignment in the optical length of the fth channel (see Fig. 4).

The presence of coupling of the lasers in the array causes a distributed function of phase response. The change in length of a single channel results in a change in the phase profile of the entire array. Note that such a distributed response determines the characteristic correlation length of the phase profile of output radiation in the case of random misalignment in the optical lengths $\langle \Phi_n \Phi_k \rangle = 4k_0^2 \sum_l \Psi_{nl} \Psi_{kl}$ (see Ref. 7). The amplitude of the phase response decreases with displacement of the channel, in which the misalignment is introduced, toward the centre of the array. The effect is explained by the fact that the edge lasers have fewer neighbours in the array with which they may effectively couple, than the central ones, so that they undergo greater changes in response to unit external action.

The given-above results demonstrate that both the phase profile and the mode composition of the output radiation of the array of optically coupled lasers may be controlled by varying the misalignment in their optical lengths. Further study of the efficiency of such control results in the need to account for the effect of the saturation of gains in the laser active media and for the dynamics of formation of fields so as to determine the limits of coherent regime of generation.

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