LIDAR EQUATION FOR A WEAKLY ANISOTROPIC MEDIUM

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Lidar equation has been derived for the case when the anisotropy of a scattering medium is assumed to be weak to introduce any essential distortions into the wave front of a sounding radiation. It is shown in the paper that this equation may successfully be used when interpreting data of lidar sensing of crystal clouds in the atmosphere.

The crystal clouds that are quite frequently observed in the atmosphere are, from the standpoint of atmospheric optics, an optically anisotropic medium. That means that the extinction of light propagated through such a medium, as well as the polarization of direct and scattered radiation, should depend on the direction of propagation and on the polarization state of incident radiation.

The lidar equation that is most commonly used to describe lidar returns does not allow for this circumstance since it is written for the intensity of light thus being actually a solution, in the single scattering approximation, of a scalar-form radiation transfer equation for the backscatter. In so doing it is assumed that the scattering properties of the medium may be described by such scalar values as extinction and scattering coefficients and the scattering phase function. The other one approach enables one to relate the Stokes parameters of the backward scattered radiation to those of a sounding beam. To do this one should replace the backscattering coefficient entering the scalar lidar equation by a backscattering phase matrix. At the same time it is still assumed that the extinction of radiation along the path to the scattering volume and back may be described using scalar representation for the extinction coefficient of the medium. As a result this approach ignores possible dependence of the extinction on the direction of a sounding beam propagation, with respect to the axes characteristic of an optically anisotropic medium. Moreover, this approach does not allow for possible variations in the state of polarization of the direct and scattered radiation when propagated through the portion of a sounding path within an anisotropic medium.

This approach seems, *a priori*, to be quite sufficient for the case of sounding crystal clouds sense, because of their low optical thickness, one may assume that the anisotropy of their scattering properties should only weakly affect the polarization of the direct (non-scattered) radiation and its attenuation.

However, from our estimates[1] it follows that significant corrections for the extinction anisotropy may be needed to the extinction along the sounding path, if described using this approach.

In any case it seems to be relevant if some expressions for estimating the applicability limits of this approach are obtained to introduce corrections when necessary. Regardless of the case of sounding crystal clouds the derivation of lidar equation discussed below may be helpful for understanding the problems that may arise when sounding other optically anisotropic media.

Propagation of a polarized radiation through an anisotropic medium is described by a system of four coupled integro-differential equations for the Stokes parameters.[2] In a compact vector-matrix form this system is as follows:

$$\left(\frac{\partial}{c\partial t} + \boldsymbol{\omega}\nabla + \boldsymbol{\varepsilon}(\mathbf{r}, \boldsymbol{\omega})\right) \mathbf{S}(t, \mathbf{r}, \boldsymbol{\omega}) =$$

$$= \int\limits_{4\pi} \mathrm{d}\boldsymbol{\omega}' \, \mathbf{M}(\mathbf{r}, \boldsymbol{\omega}, \boldsymbol{\omega}') \, \mathbf{S}(t, \mathbf{r}, \boldsymbol{\omega}') + \mathbf{C}(t, \mathbf{r}, \boldsymbol{\omega}), \qquad (1)$$

where $\mathbf{S}(t, \mathbf{r}, \boldsymbol{\omega})$ is the Stokes vector of radiation at a point set by the radius vector $\mathbf{r}$; $\boldsymbol{\omega}$ is the unit vector along the direction of light propagation that coincides with the direction of wave vector $\mathbf{k}$. The operator in brackets involves the partial derivative with respect to time divided by the speed of light in the medium, the operator $\boldsymbol{\omega}\nabla$ that means taking a directional derivative along the direction $\boldsymbol{\omega}$, and the extinction matrix $\boldsymbol{\varepsilon}$ of the medium for the radiation that is being propagated along the direction $\boldsymbol{\omega}$. The integral in the right-hand side of the equation defines the radiation that is being propagated along the direction $\boldsymbol{\omega}$ and which has originated at the point $\mathbf{r}$ due to re-distribution of radiation incident on a unit scattering volume around this point from various directions $\boldsymbol{\omega}'$. The elements of the scattering phase matrix $\mathbf{M}$ have dimensionality of $[\mathrm{m}^{-1}\mathrm{sr}^{-1}]$. The vector $\mathbf{C}$ defines the field from light sources.

The lidar equation could be derived as a solution to system (1) that would explicitly relate the flux of

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the Stokes vector incident onto the lidar receiving antenna to the parameters $\boldsymbol{\varepsilon}$ and $\mathbf{M}$ of the medium provided that the Stokes vector of sounding radiation is known. However, this system of equations can be solved only numerically. For this reason we make, in what follows, some assumptions that enable a simplification of the system. Certain explanations of the assumptions made are also given below, in the course of consideration.

Let us first consider, as a heuristic item of the discussion, the radiation transfer equation in a scalar form that has the view similar to that of equation (1)

$$\left(\frac{\partial}{c\partial t} + \boldsymbol{\omega}\nabla + \varepsilon(\mathbf{r})\right) J(t, \mathbf{r}, \boldsymbol{\omega}) = \frac{\sigma(\mathbf{r})}{4\pi} \times$$

$$\times \int\limits_{4\pi} \mathrm{d}\boldsymbol{\omega}'\, \gamma(\mathbf{r}, \boldsymbol{\omega}, \boldsymbol{\omega}')\, J(t, \mathbf{r}, \boldsymbol{\omega}') + q(t, \mathbf{r}, \boldsymbol{\omega}), \qquad (2)$$

where $J(t, \mathbf{r}, \boldsymbol{\omega})$ is the intensity of radiation; ε is the extinction coefficient; σ is the scattering coefficient; γ is the scattering phase function.

This equation may also be written in the operator form[3]

$$\mathbf{L}\, J = \mathbf{B}\, J + q. \qquad (3)$$

The meaning of the differential, $\mathbf{L}$, and integral, $\mathbf{B}$, operators may be understood from the comparison of this equation with the equation (2).

Solution of the equation (3) may be presented in the form a series of the following view:

$$J = \sum_{n=0}^{\infty} (\mathbf{L}^{-1}\,\mathbf{B})^n\, \mathbf{L}^{-1}\, q = \sum_{n=0}^{\infty} J_n, \qquad (4)$$

where n means the order of scattering.

As a consequence, for the direct, non-scattered radiation we have the following expression:

$$J_0 = \mathbf{L}^{-1}\, q\,, \qquad (5)$$

while for the first-order scattering the expression is as follows:

$$J_1 = \mathbf{L}^{-1}\,\mathbf{B}\,\mathbf{L}^{-1}\, q. \qquad (6)$$

The inverse operator $\mathbf{L}^{-1}$ is related to the $\mathbf{L}$ operator as $\mathbf{L}\,\mathbf{L}^{-1} = \mathbf{I}$, where $\mathbf{I}$ is the identity operator that, in turn, is an integral operator with the Green's function of the operator $\mathbf{L}$ as the kernel. The Green's function, in this case, has the following view:

$$\mathbf{G}(t, t', \mathbf{r}, \mathbf{r}') = h(t - t')\, \mathrm{e}^{-\varepsilon c(t-t')} \times$$

$$\times\, \delta\, [(\mathbf{r} - \mathbf{r}') - c(t - t')\, \omega], \qquad (7)$$

where $h(t)$ is the unit step function and $\delta(\xi)$ is the delta function.

Solution of the equation (5) is as follows:

$$J_0 = c \int\limits_{-\infty}^{\infty} \mathrm{d}t' \int\limits_{-\infty}^{\infty}\!\int\!\!\int \mathrm{d}\mathbf{r}'\, \mathbf{G}(t, t', \mathbf{r}, \mathbf{r}')\, q(t', \mathbf{r}', \boldsymbol{\omega}). \qquad (8)$$

Let us assume that $\varepsilon = \mathrm{const}$ while the source function being set as follows:

$$q(t', \mathbf{r}', \boldsymbol{\omega}) = P_0(\Delta\omega_0)^{-1}\, \delta(t)\, \delta(\mathbf{r})\, [h(\theta - \Delta\theta/2) -$$

$$-\, h(\theta + \Delta\theta/2)]\, [h(\varphi) - h(\varphi + \pi)], \qquad (9)$$

where θ and φ are the polar and azimuth angles of the direction $\boldsymbol{\omega}$.

Formula (9) describes the action of an instant point source of radiation uniformly emitting into the conical solid angle $\Delta\omega_0 = \pi(\Delta\theta)^2/4$ the radiation of P_0 power.

Under the assumptions made the expression (8) yields an obvious result that intensity of the backscattered radiation

$$J_0(\mathbf{r}) = P_0(\Delta\omega_0)^{-1}\, \mathbf{r}^{-2} \exp\{-\, \varepsilon r\} \qquad (10)$$

falls off inversely proportional to the squared range and that the extinction of radiation obeys the Bouguer law.

When considering the case of an anisotropic medium we shall accept the following suppositions. Let us omit the integral term in equation (1), thus considering only direct, non-scattered radiation.

The vector $\mathbf{C}(t, \mathbf{r}, \omega)$ is set similarly to expression (9) assuming that at $t = 0$ and at the point $\mathbf{r} = 0$ we have

$$\mathbf{C}_0 = P_0(\Delta\omega_0)^{-1}\, \mathbf{s}_0, \qquad (11)$$

where $\mathbf{s}_0$ is the normalized Stokes vector of radiation coming from the source.

Assume also that there exists an operator $\mathbf{L}^{-1}$ inverse to the operator $\mathbf{L}$, from the left-hand side of equation (1), such that when applied to vector (11) it yields a solution that has same structure as solution (10)

$$\mathbf{S}_0(\mathbf{r}) = \mathbf{L}^{-1}\, \mathbf{C} = \mathbf{r}^{-2}\, \mathbf{Y}\, P_0(\Delta\omega_0)^{-1}\, \mathbf{s}_0, \qquad (12)$$

where $\mathbf{Y}$ is the unknown, for the present, operator that characterizes the transformation of the Stokes parameters of the radiation coming from the source. Actually, this formula results from the assumption of weak anisotropy of the medium. It simply means that the train of radiation from the source is being washed out due to purely geometric causes. No possible distortions of the wave front due to, for instance, the birefringence is considered. As will be clear from the discussion below the condition of weak anisotropy reduces to a requirement that the off-diagonal elements of the extinction matrix be small, as compared to the diagonal ones

$\varepsilon_{ii} \gg \varepsilon_{ij}$.

Then we demand that radiation propagates along the z axis within a small solid angle $\Delta\omega_0$ so that one may consider that

$$\boldsymbol{\varepsilon}(\mathbf{r}, \boldsymbol{\omega}) = \boldsymbol{\varepsilon}(z, \mathbf{e}_z), \tag{13}$$

where $\mathbf{e}_z$ is the unit vector.

When seeking the view of the operator $\mathbf{Y}$ note that without the integral term in equation (1) and after the termination of the source action, i.e., when $\mathbf{C}(t, \mathbf{r}, \boldsymbol{\omega}) = 0$, the equation (1) takes the form a system of homogeneous linear equations. By making use of the condition (13) we may write the system for a one-dimension case having in mind that by making a substitution $z = ct$ one may exclude the explicit dependence on time

$$\frac{\mathrm{d}}{\mathrm{d}z}\, \mathbf{S}(z) = -\, \boldsymbol{\varepsilon}(z, \mathbf{e}_z)\, \mathbf{S}(z). \tag{14}$$

Solution to this system is unambiguously determined by the boundary condition (11).

As known from the theory of systems of linear differential equations a solution to system (14) can be written in the following form[4]:

$$\mathbf{S}(z) = \mathbf{Y}(z, z_0, \mathbf{e}_z)\, \mathbf{S}(z_0), \tag{15}$$

where $\mathbf{Y}(z, z_0, \mathbf{e}_z)$ is the fundamental matrix of the system that satisfies the boundary condition

$$\mathbf{Y}(z, z_0, \mathbf{e}_z) = \mathbf{I}, \tag{16}$$

where $\mathbf{I}$ is the unit matrix.

The matrix $\mathbf{Y}(z, z_0, \mathbf{e}_z)$ may be expressed through the matrix $(-\boldsymbol{\varepsilon})$ with the following iteration series:

$$\mathbf{Y}(z, z_0) = \mathbf{I} - \int\limits_{z_0}^{z} \boldsymbol{\varepsilon}(z_1)\, \mathrm{d}z_1 + \int\limits_{z_0}^{z} \boldsymbol{\varepsilon}(z_1)\, \mathrm{d}z_1 \int\limits_{z_0}^{z_1} \boldsymbol{\varepsilon}(z_2)\, \mathrm{d}z_2 - $$
$$- \int\limits_{z_0}^{z} \boldsymbol{\varepsilon}(z_1)\, \mathrm{d}z_1 \int\limits_{z_0}^{z_1} \boldsymbol{\varepsilon}(z_2)\, \mathrm{d}z_2 \int\limits_{z_0}^{z_2} \boldsymbol{\varepsilon}(z_3)\, \mathrm{d}z_3 + \dots\, . \tag{17}$$

For the sake of simplification we omitted $\mathbf{e}_z$ in the arguments of the matrices.

If ε is independent of z the series (17) coincides with the definition of the exponential function with a matrix as an argument

$$\exp\,(-\,\mathbf{A}) = \sum_{n=0}^{\infty} (-\,\mathbf{A})^n / n!\ . \tag{18}$$

So, from Eq. (17) it follows that at $\boldsymbol{\varepsilon}(z, \mathbf{e}_z) = \mathrm{const}$

$$\mathbf{Y}(z, z_0, \mathbf{e}_z) = \mathrm{e.\,!}\ \{-\,(z - z_0)\, \boldsymbol{\varepsilon}\}. \tag{19}$$

From the known property of the fundamental matrix

$$\mathbf{Y}(z,z_0) = \mathbf{Y}(z,z_{n-1})\, \mathbf{Y}(z_{n-1},z_{n-2}) \dots \mathbf{Y}(z_1,z_0) \tag{20}$$

it follows that at $\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}(z, \mathbf{e}_z)$

$$\mathbf{Y}(z, z_0, \mathbf{e}_z) = \mathrm{e.\,!}\ \left\{- \int\limits_{z_0}^{z} \boldsymbol{\varepsilon}(z', \mathbf{e}_z)\, \mathrm{d}z' \right\}. \tag{21}$$

Substituting the operator (21) into the expression (15) and having in mind the boundary condition (11) one finds from a comparison with the expression (12) that the operator $\mathbf{L}^{-1}$ has, under the assumptions made, the following view:

$$\mathbf{L}^{-1} = r^{-2}\ \mathrm{e.\,!}\ \left\{- \int\limits_{z_0}^{z} \boldsymbol{\varepsilon}(z', \mathbf{e}_z)\, \mathrm{d}z' \right\}. \tag{22}$$

According to expression (12) one obtains for the forward propagated radiation that

$$\mathbf{S}_0(z, \mathbf{e}_z) = P_0[\Delta\omega_0\, r^2]^{-1}\ \mathrm{e.\,!}\ \left\{- \int\limits_{z_0}^{z} \boldsymbol{\varepsilon}(z', \mathbf{e}_z)\, \mathrm{d}z' \right\} \mathbf{s}_0. \tag{23}$$

Then, following the formal expression (6), one should apply the scattering operator to the Stokes vector $\mathbf{S}_0(z)$ in order to calculate the Stokes vector of scattered radiation and after that to apply again the operator (22) to $\mathbf{S}_0(z)$, while, however, changing the direction for $-\,\mathbf{e}_z$.

To seek the scattering operator, it is worth making use of a known method of assuming that at every moment t in time, or at the distance $z = ct$, there appears a source of light whose Stokes vector is

$$\mathbf{S}_0(z, -\,\mathbf{e}_z) = \mathbf{M}_\pi(z)\, \Delta V\, \mathbf{S}_0(z, \mathbf{e}_z), \tag{24}$$

where $\mathbf{M}_\pi(z) = \mathbf{M}_\pi(z, \mathbf{e}_z, -\,\mathbf{e}_z)$ is the backscattering phase matrix and

$$\Delta V = r^2\, \Delta\omega_0\, c\Delta t / 2 \tag{25}$$

is the portion of scattering volume that contributes to the backscattered flux at the moment t. At the moment $2t$ this flux reaches the lidar receiving antenna. The quantity Δt is the duration of a rectangular pulse that is equivalent in the radiation energy to a realistic laser pulse.

Transformation of scattered radiation, on its way from the illuminated scattering volume toward the lidar receiver, is defined by the operator

$$\mathbf{L}^{-1}(-\,\mathbf{e}_z) = r^{-2}\, \mathbf{Y}(z, z_0, -\,\mathbf{e}_z). \tag{26}$$

Having in mind that $\mathbf{r} = z$ and $z_0 = 0$, let use unite the equations (22)–(25) following the scheme (6) then multiply this result by the area of the lidar receiving antenna and obtain the equation sought

$$P(z)\ \mathbf{s}(z) = \frac{1}{2} c\, W_0\, A z^{-2}\, \mathbf{Y}(z, -\mathbf{e}_z)\ \mathbf{M}_\pi(z)\ \mathbf{Y}(z,\ \mathbf{e}_z)\ \mathbf{s}_0, \quad (27)$$

where $\mathbf{s}(z)$ and $\mathbf{s}_0$ are the dimensionless normalized Stokes vectors of the scattered and sounding radiation, respectively; c is the speed of light; $W_0 = P_0 \Delta t$ is the energy of sounding pulse; A is the area of the lidar receiving antenna; z is distance to the scattering volume. The elements of the backscattering phase matrix have the dimensionality of $[\mathrm{m}^{-1}\mathrm{sr}^{-1}]$.

This equation relates, in the single scattering approximation, the flux of the Stokes vector of radiation emitted by a transmitter to that of scattered radiation collected by the receiving antenna of a lidar.

Let us now consider properties of $\mathbf{Y}$ operators that have the view (21). When writing the equation (27) we have assumed that the anisotropic medium is immediately adjacent to the lidar that is $z_0 = 0$. If, however, there is a portion of the sounding path, $[0,\ z_0]$, where the extinction may be described by a scalar extinction coefficient $\alpha(z)$, then one can write, based on the properties of the fundamental matrix that

$$\mathbf{Y}(z, \mathbf{e}_z) = \mathbf{Y}(z,\ z_0,\ \mathbf{e}_z)\ \mathbf{Y}(z,\ 0,\ \mathbf{e}_z) \quad (28)$$

and

$$\mathbf{Y}(z, -\mathbf{e}_z) = \mathrm{e.!} \left\{ -\int_{z_0}^{z} \boldsymbol{\varepsilon}(z')\, \mathrm{d}z' \right\} \mathrm{e.!} \left\{ -\int_{0}^{z_0} \alpha(z')\, \mathrm{d}z' \right\}. \quad (29)$$

Having in mind the expression (20) one can divide the interval $[z,\ z_0]$ into n subintervals Δz_i such that within each subinterval one may assume that $\boldsymbol{\varepsilon} = \mathrm{const}$ and $\varepsilon_{ij}\ \Delta z \ll 1$. In that case the operator $\mathbf{Y}(z,\ z_0,\ \mathbf{e}_z)$ may be presented as the product

$$\mathbf{Y}(z, z_0, \mathbf{e}_z) = (\mathbf{I} - \Delta z_n\, \varepsilon_n) \ldots (\mathbf{I} - \Delta z_i\, \varepsilon_i) \ldots (\mathbf{I} - \Delta z_1\, \varepsilon_1), \quad (30)$$

where $\mathbf{I}$ denotes the unit matrix. The matrix ε_1 refers to the subinterval $\Delta z_1 = (z_1 - z_0)$, and so on.

Since in the general case we have that

$$\varepsilon_i\, \varepsilon_j - \varepsilon_j\, \varepsilon_i \neq 0,$$

we see that the co-factors in expression (30) are noncommutative.

Since the direct and backward scattered radiation travel along the same path the scattering particles are in the reciprocal positions relative to them. According to the reciprocity theorem the amplitude scattering matrix should be transposed and the signs at its off-diagonal elements inverted.[5]

As to the extinction matrix in this case the situation is as follows. If the extinction matrix for the direct radiation is written in the form of a block matrix

$$\boldsymbol{\varepsilon}(z,\ \mathbf{e}_z) = \begin{pmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{pmatrix},$$

then the corresponding matrix for the backward going radiation is

$$\boldsymbol{\varepsilon}^*(z) = \boldsymbol{\varepsilon}(z,\ -\mathbf{e}_z) = \begin{pmatrix} \mathbf{A} & -\mathbf{B} \\ -\mathbf{C} & \mathbf{D} \end{pmatrix}. \quad (31)$$

Taking into account the expression (31) and assuming that the interval is subdivided as in formula (30) the operator $\mathbf{Y}(z, z_0, -\mathbf{e}_z)$ may be written as follows:

$$\mathbf{Y}(z,\ z_0,\ -\mathbf{e}_z) = (\mathbf{I} - \Delta z_1\, \boldsymbol{\varepsilon}_1^*) \ldots (\mathbf{I} - \Delta z_i\, \boldsymbol{\varepsilon}_i^*) \ldots (\mathbf{I} - \Delta z_n\, \boldsymbol{\varepsilon}_n^*), \quad (32)$$

that is the sequence of the co-factors inverts.

In the general case solving equation (27) faces many difficulties. The matter is that as in the case with the scalar lidar equation it needs for supplementing of a definition by some matrix relationship like

$$\boldsymbol{\varepsilon} = \boldsymbol{\Gamma}\ \mathbf{M}_\pi,$$

that would set an *a priori* ratio between the extinction matrix and the backscattering phase matrix of the medium under study. Having supplemented the definition and substituting the operators $\mathbf{Y}(z,\ z_0,\ -\mathbf{e}_z)$ and $\mathbf{Y}(z_0,\ z,\ -\mathbf{e}_z)$, in their corresponding forms (30) and (32), into the equation (27) one may iteratively solve it.

Fortunately, in the case of sensing crystal clouds the situation seems to be much simpler. We have performed certain preliminary theoretical study of the extinction matrices of some simple models of ensembles of ice crystal particles.[1] In so doing we have calculated extinction matrices for the ensembles of cylinder particles having different modal radii and types of orientation. The calculations made show that normally the magnitudes of the off-diagonal elements of the extinction matrix, in these cases, do not exceed one per cent of the diagonal ones. It is also important to note that the ratio is larger for small particles whose size compares to the wavelength of incident radiation, while decreasing at increasing size of the scattering particle. This is obviously indicative of the fact that in the case of large particles the main contribution to the extinction of radiation comes from diffraction on the particle edges, while the interference of the non-scattered wave with the wave passed through the particle makes an essentially low contribution. It seems reasonable to consider that the extinction due to the diffraction will hardly become relatively lower when taking into account hexagonal shapes of the particles and the birefringence effect. For this reason it seems to be very close to reality the following representation:

$$\boldsymbol{\varepsilon}(z,\ \mathbf{e}_z) = \alpha(z,\ \mathbf{e}_z)\ \mathbf{I}. \quad (33)$$

If this condition holds, the operators $\mathbf{Y}(z, z_0, -\mathbf{e}_z)$ and $\mathbf{M}_\pi$ become commutative and more over

$$\mathbf{Y}(z, z_0, -\mathbf{e}_z) = \mathbf{Y}(z, z_0, \mathbf{e}_z).$$

Then, finally the equation (27) takes the form

$$P(z)\,\mathbf{s}(z) = \frac{1}{2}\,c\,W_0\,A z^{-2}\,\mathbf{M}_\pi(z)\,\mathbf{s}_0 \times$$

$$\times\ \text{e.!}\left\{-2\int_0^{z_0} \alpha(z', \theta, \varphi)\,\mathrm{d}z'\right\}. \tag{34}$$

In the relation to describing the radiation extinction along the sounding path this equation differs from the equation in the scalar form only by the fact that the extinction coefficient entering it may appear to be dependent on the polar, θ, and azimuth, φ, angles that determine the direction of the sounding path with respect to the axes that characterize the medium anisotropy.

Based on the considerations presented above it seems that merely substituting a one-dimension matrix instead of the backscattering coefficient is quite an effective way of passing to a vector form of the lidar equation, at least in the case of sensing crystal clouds. Moreover, if the direction of sounding is fixed no difference in the way the extinction of radiation by the medium along the path is being taken into account occurs, as compared to the scalar form of the equation. Certain peculiarities may appear when comparing the sounding results obtained along the paths oriented at different azimuth and zenith angles. But, if the diffraction component dominates in the total extinction then the dependence of extinction on the direction of sounding may easily be interpreted. This circumstance opens some new possibilities of studying the microphysical properties of the ensembles of crystal particles and their orientation as well.

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