COMPUTER SIMULATION OF IMAGING OF EXTENDED OBJECT THROUGH A TURBULENT ATMOSPHERE. PART 1. THE METHOD.

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Received May 13, 1997

The method for direct numerical simulation of random images of an extended object viewed through the atmosphere is developed based on a short-exposure optical transfer function concept and the phase screen approach. The model of point spread function is formulated that allows for the random movement as well as for the diffraction and small-scale turbulent blurring. The results obtained by testing with the Monte-Carlo method have demonstrated high statistical accuracy of the model proposed.

The classical theory of image transfer through the randomly-inhomogeneous media has rather long history being developed in numerous publications most of which are based on the averaged amplitude and phase fluctuations of light waves in the atmosphere (see, for example Refs. 1 to 4). Thus, the analytical assessments, generalized in Ref. 1, allow one to relatively easily predict the so called long-exposure image degradation, provided that the spatial spectrum of the refractive index fluctuations is known.

Recently a number of papers was published in which the point spread functions are simulated using instantaneous fields of $\tilde{n}$ along the path.[5–9] These papers are basically devoted to the problems of observation of astronomical objects that, on the one hand, are located outside the Earth's atmosphere, and, on the other hand, represent, practically, the point objects. At the same time, the problem on predicting the distortions in the images of extended objects in the Earth's atmosphere attracts an enhanced interest of the applications in optical sounding, navigation, and the development of information systems.

The problem of a reliable, in the statistical sense, imitation of short-exposure images of incoherently illuminated objects is an urgent one in a number of the information and wave optics theories and modern applications such as the development and testing of image processing algorithms, pattern recognition, and simulation of the adaptive systems for image correction. In this paper the procedure of direct computer solution of this problem is described based on the idea of short-exposure optical transfer function and the phase screen method.[10]

1. THE MODEL OF IMAGE TRANSFER

Consider the following model of the image formation and its transfer through a clear turbulent atmosphere. Assume that an incoherent radiation from an object in the plane $\{\mathbf{r}'\}$ passes through the phase screen $\tilde{\varphi}$ and comes to the entrance aperture of an imaging optics in the plane $\{\mathbf{r}''\}$ (Fig. 1). For simplicity, the optical system is presented by a lens of the diameter d and focal length f. Having in mind that normally $f \ll z$, the image is formed in the plane $\{\mathbf{r}\}$ that practically coincides with the focal plane of the lens.

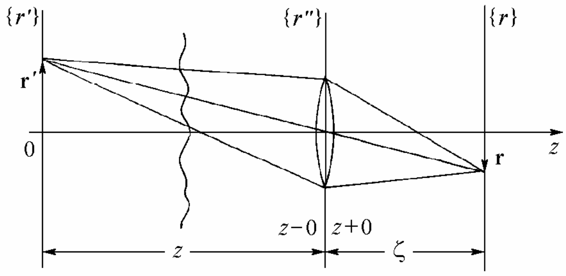

FIG. 1. Model of the image transfer through the turbulent atmosphere: $\mathbf{r}'$ *is the point source coordinate in the object plane* $\{\mathbf{r}'\}$; $\mathbf{r}''$ *is the coordinate in the lens plane;* $\mathbf{r}$ *is the coordinate in the image plane;* z *is the distance between the object plane* $\{\mathbf{r}'\}$ *and a lens;* ζ *is the distance between the image plane* $\{\mathbf{r}\}$ *and a lens;* $\zeta \approx f$ *at* $f \ll z$, *where* f *is the focal length of a lens.*

The phase of radiation coming from different points of an incoherently illuminated object varies randomly. The intensity $I(\mathbf{r}, t)$ of an instantaneous image of the object is a superposition of images of the object points with the intensity distribution $I(\mathbf{r}')$ averaged over time that exceeds the coherence time τ_c while being, at the same time, less than the time τ_{atm} characteristic of fluctuations in the frozen atmosphere

$$I(\mathbf{r}, t) = \int_{-\infty}^{\infty} \tilde{S}(\mathbf{r}, \mathbf{r}', t)\, I(\mathbf{r}')\, \mathrm{d}^2\mathbf{r}'. \qquad (1)$$

0235-6880/98/04 349-04 $02.00

Here $\tilde{S}(\mathbf{r}, \mathbf{r}', t)$ is the point spread function (PSF) of the imaging system, that is, the intensity distribution over the image of an individual point source as recorded instantaneously, compared to τ_{atm}. The PSF $\tilde{S}(\mathbf{r}, \mathbf{r}', t)$ is a random function of $\mathbf{r}$ and $\mathbf{r}'$ coordinates, in addition, it varies randomly in time due to the dynamic fluctuations of phase in the turbulent atmosphere.

To obtain the PSF of the system under consideration, let us use the expression describing, in the image plane $\{\mathbf{r}\}$, the complex amplitude $E(\mathbf{r}, \mathbf{r}', t)$ of the wave propagated from a point source in the object plane $\{\mathbf{r}'\}$. In paraxial approximation this expression can be written as[1]

$$E(\mathbf{r}, \mathbf{r}', t) = -\frac{AM}{\lambda^2 \zeta^2} \exp\left\{i \frac{k\mathbf{r}^2}{2\zeta}\left(1 + \frac{1}{M}\right)\right\} \times$$

$$\times \int_{-\infty}^{\infty} P(\mathbf{r}'') \exp\left\{-\frac{ik\mathbf{r}''}{\zeta}(\mathbf{r} + M\mathbf{r}')\right\} \times$$

$$\times \exp\{i\,\tilde{\varphi}(\mathbf{r}', \mathbf{r}'', t)\}\, \mathrm{d}^2\mathbf{r}'', \qquad (2)$$

where A is the amplitude of the wave coming from a point source; λ is the mean wavelength at the incoherent radiation; $\tilde{\varphi}(\mathbf{r}', \mathbf{r}'', t)$ is a random phase shift in the atmosphere whose value depends on the coordinates $\mathbf{r}'$ and $\mathbf{r}''$ of the object and lens points, respectively; $P(\mathbf{r}'')$ is the pupil function; $l = \zeta/z$ is the magnification of the system, the values ζ, z, and f being related to each other by the formula of a thin lens. Assuming that $z \gg f$, ζ value in (2) can be substituted by f. According to (2)

$$\tilde{S}(\mathbf{r}, \mathbf{r}', t) = \frac{M^2}{\lambda^4 f^4} \int_{-\infty}^{\infty} P(\mathbf{r}''_1)\, P(\mathbf{r}''_2) \times$$

$$\times \exp\left\{-\frac{ik\,(\mathbf{r} + M\mathbf{r}')}{f}(\mathbf{r}''_2 - \mathbf{r}''_1)\right\} \times$$

$$\times \exp\{i\,(\tilde{\varphi}(\mathbf{r}', \mathbf{r}''_1, t) - \tilde{\varphi}(\mathbf{r}', \mathbf{r}''_2, t))\}\, \mathrm{d}^2\mathbf{r}''_1\, \mathrm{d}^2\mathbf{r}''_2. \qquad (3)$$

Let us represent the random value of the phase difference $\tilde{\varphi}(\mathbf{r}', \mathbf{r}''_1, t) - \tilde{\varphi}(\mathbf{r}', \mathbf{r}''_2, t)$ of a wave passed from the $\mathbf{r}'$ point of the source through different $\mathbf{r}''_1$ and $\mathbf{r}''_2$ points of the lens aperture in the form

$$\tilde{\varphi}(\mathbf{r}', \mathbf{r}''_1, t) - \tilde{\varphi}(\mathbf{r}', \mathbf{r}''_2, t) =$$

$$= \tilde{\theta}(\mathbf{r}', t)\,(\mathbf{r}''_1 - \mathbf{r}''_2) + \tilde{\varphi}_S(\mathbf{r}', \mathbf{r}''_1, \mathbf{r}''_2, t). \qquad (4)$$

The first term in (4) describes large-scale atmospheric inhomogeneities which cause the random tilt of the wave-front for radiation that reaches $\mathbf{r}''_1$ and $\mathbf{r}''_2$ points from the source located at the $\mathbf{r}'$ point. The second term in (4) is the contribution coming from the small-scale phase fluctuations.

Parameter $\tilde{\theta}(\mathbf{r}', t)$ is the random function of source coordinate $\mathbf{r}'$ and time t. Considering that for the characteristic time τ_L of tilts and of small-scale fluctuations τ_l, the condition $\tau_l \ll \tau_L$ holds, one may introduce the "slow" time η for $\tilde{\theta}$ parameter and the "quick" one for $\tilde{\varphi}_S$ fluctuations. Let us define as a "short" the image exposure whose duration, τ_{exp}, satisfies the condition

$$\tau_c < \tau_l < \tau_{exp} < \tau_L. \qquad (5)$$

Thus, the small-scale fluctuations $\tilde{\varphi}_S$ are averaged over the time during τ_{exp}. As a result the short-time exposure PSF $\tilde{S}_{exp}(\mathbf{r}, \mathbf{r}', \eta)$ is expressed by the integral of instantaneous $\tilde{S}(\mathbf{r}, \mathbf{r}', t, \eta)$ over the fast t

$$\tilde{S}_{exp}(\mathbf{r}, \mathbf{r}', \eta) = \int_0^{\tau_{exp}} \tilde{S}(\mathbf{r}, \mathbf{r}', t, \eta)\, \mathrm{d}t. \qquad (6)$$

The PSF $\tilde{S}_{exp}(\mathbf{r}, \mathbf{r}', \eta)$ is a random function of the slow time η connected with the wave front tilt evolution in the atmosphere. In accordance with (3) $\tilde{S}_{exp}(\mathbf{r}, \mathbf{r}', \eta)$ equals to

$$\tilde{S}_{exp}(\mathbf{r}, \mathbf{r}', \eta) = \frac{M^2}{\lambda^4 f^4} \int_{-\infty}^{\infty} P(\mathbf{r}''_1)\, P(\mathbf{r}''_2) \times$$

$$\times \exp\left\{-\frac{ik\,(\mathbf{r} + M\mathbf{r}')}{f}(\mathbf{r}''_2 - \mathbf{r}''_1)\right\} \times$$

$$\times \exp\{i\tilde{\theta}(\mathbf{r}', \eta)\,(\mathbf{r}''_2 - \mathbf{r}''_1)\} \int_0^{\tau} \mathrm{d}t \times$$

$$\times \exp\{i\,(\tilde{\varphi}_S(\mathbf{r}', \mathbf{r}''_1, t) - \tilde{\varphi}_S(\mathbf{r}', \mathbf{r}''_2, t))\}\, \mathrm{d}^2\mathbf{r}''_1\, \mathrm{d}^2\mathbf{r}''_2. \qquad (7)$$

Taking into account the hypothesis of field $\tilde{\varphi}_S$ ergodicity the averaging over time can be replaced by statistical averaging. Let us introduce the autocorrelation function $\Gamma_{\varphi\, S}(\mathbf{r}''_1 - \mathbf{r}''_2)$ of the transmission coefficient of a phase screen with small-scale inhomogeneities which is independent of the point source coordinate $\mathbf{r}'$ for a statistically uniform field of the atmospheric turbulence.

$$\Gamma_{\varphi\, S}(\mathbf{r}''_1 - \mathbf{r}''_2) = \langle \exp\{i\,(\tilde{\varphi}_S(\mathbf{r}', \mathbf{r}''_1) - \tilde{\varphi}_S(\mathbf{r}', \mathbf{r}''_2))\}\rangle. \qquad (8)$$

As a result short-exposure PSF, $\tilde{S}_{exp}(\mathbf{r}, \mathbf{r}', \eta)$, takes the form

$$\tilde{S}_{exp}(\mathbf{r}, \mathbf{r}', \eta) = \frac{M^2}{\lambda^4 f^4} \int_{-\infty}^{\infty} P(\mathbf{r}''_1)\, P(\mathbf{r}''_2) \times$$

$$\times \exp\left\{-\frac{ik}{f}\,[\mathbf{r} + M\mathbf{r}' + f\,\tilde{\theta}(\mathbf{r}',\eta)]\,[\mathbf{r}_2'' - \mathbf{r}_1'']\right\} \times$$
$$\times\, \Gamma_{\varphi\ s}(\mathbf{r}_2'' - \mathbf{r}_1'')\, d^2\mathbf{r}_1''\, d^2\mathbf{r}_2''. \qquad (9)$$

The averaged contribution of small-scale fluctuations does not produce any effect on the PSF structure and in the absence of large-scale fluctuations ($\tilde{\theta} = 0$) the optical system is isoplanatic like a diffraction-limited one. The PSF $S_{0S}(\mathbf{r}, \mathbf{r}')$ of such a subsystem (that is the system without large-scale fluctuations) is expressed by a convolution of the PSF $S_0(\mathbf{r}, \mathbf{r}')$ of a diffraction-limited system with the PSF $S_S(\mathbf{r}, \mathbf{r}')$ representing the case with small-scale fluctuations

$$S_{0S}(\mathbf{r}, \mathbf{r}') = S_0(\mathbf{r}, \mathbf{r}') \otimes S_S(\mathbf{r}, \mathbf{r}'). \qquad (10)$$

Small-scale fluctuations of the phase lead, on average, to an additional blurring of the image formed with a diffraction-limited system. As this takes place, the intensity maximum in the image of an isolated point coincides with its position determined by the geometric optics laws. The image $I(\mathbf{r})$ is expressed, as in the case with a diffraction-limited system, by the convolution of the PSF with the geometric-optics image.

Because of the isoplanatism of the subsystem with averaged small-scale fluctuations the PSF S_{0S} can be expressed in the form

$$S_{0S}(\mathbf{r}, \mathbf{r}') = S_{0S}(\mathbf{r} - \xi), \qquad (11)$$

where $\xi = -M\mathbf{r}'$. Such a PSF relates via a Fourier transform to the averaged optical transfer function (OTF) $H_{0S}(\mathbf{\Omega})$ of the system

$$S_{0S}(\mathbf{r}) = \mathrm{F}^{-1}\,\{H_{0S}(\mathbf{\Omega})\}, \qquad (12)$$

where $\mathbf{\Omega} = \mathbf{v}f$, $\mathbf{v}$ is the vector of spatial frequencies in the plane perpendicular to the optical axis of the system. In its turn

$$H_{0S}(\mathbf{\Omega}) = H_0(\mathbf{\Omega})\, H_S(\mathbf{\Omega}), \qquad (13)$$

where $H_0(\mathbf{\Omega})$ is the OTF of a diffraction-limited system. The function $H_S(\mathbf{\Omega})$ is the OTF caused by small-scale fluctuations.

The diffraction-limited OTF, H_0, of a lens with the diameter d has the form[11]

$$H_0(\mathbf{\Omega}) = \begin{cases} \dfrac{2}{\pi}\left[\arccos\left(\dfrac{\Omega}{\Omega_0}\right) - \dfrac{\Omega}{\Omega_0}\sqrt{1 - \left(\dfrac{\Omega}{\Omega_0}\right)^2}\,\right], & \Omega \le \Omega_0, \\ 0, & \Omega > \Omega_0, \end{cases} \qquad (14)$$

where $\Omega_0 = d/\lambda$ is the upper boundary of the spatial spectrum. The averaged short-exposure OTF, $H_S(\mathbf{\Omega})$, has been calculated in Ref. 1 by excluding the wave-front tilts contribution into the instantaneous OTF of an incoherent system with the subsequent averaging over an ensemble. Using Kolmogorov spectrum of the refractive index fluctuations the expression for $H_S(\mathbf{\Omega})$ has been derived in the form

$$H_S(\mathbf{\Omega}) = \exp\left\{-3.44\left(\frac{\lambda\Omega}{r_0}\right)^{5/3}\left[1 - \alpha\left(\frac{\Omega}{\Omega_0}\right)^{1/3}\right]\right\}, \qquad (15)$$

where r_0 is Fried's radius describing the distorting influence of the atmospheric turbulence. For a statistically homogeneous near-ground path

$$r_0 = 0.185\,[\lambda^2/(C_n^2\, z)]^{3/5}. \qquad (16)$$

The value of the parameter α is chosen in accordance with the optical system geometry. For the near field it is assumed that $\alpha = 1$, while for the far field $\alpha = 0.5$. At $\alpha = 0$ the expression (15) describes the averaged OTF at the exposure time exceeding the characteristic time τ_L for large-scale inhomogeneities.

The contribution of the large-scale fluctuations into the short-exposure PSF $\tilde{S}_{\exp}(\mathbf{r}, \mathbf{r}', \eta)$ manifests itself in random displacements $\tilde{\xi}$ of the image intensity maximum

$$\tilde{\xi}(\mathbf{r}', \eta) = -f\,\tilde{\theta}(\mathbf{r}', \eta). \qquad (17)$$

As this takes place, the image displacement of each point of the object varies randomly with time η and coordinate $\mathbf{r}'$. The blurring of a point image, however, is independent of its displacement. The image blurred as it is defined by diffraction and by the averaged contribution of small-scale fluctuations is randomly shifted and deformed. Thus, the short-exposure PSF $\tilde{S}_{\exp}(\mathbf{r}, \mathbf{r}', \eta)$ can be represented in the following form:

$$\tilde{S}_{\exp}(\mathbf{r}, \mathbf{r}', \eta) = S_{0S}\,(\mathbf{r} - \xi - \tilde{\xi}(\mathbf{r}', \eta)). \qquad (18)$$

The system having such a PSF is not spatially invariant since there is random term $\tilde{\xi}$ in the difference between the image and object coordinates $\mathbf{r}$ and ξ.

The expression for the intensity distribution of a short-exposure image follows from (1) after averaging over the "quick" time:

$$\tilde{I}(\mathbf{r}, \eta) = \int_{-\infty}^{\infty} \tilde{S}_{\exp}(\mathbf{r}, \mathbf{r}', \eta)\, I(\mathbf{r}')\, d^2\mathbf{r}'. \qquad (19)$$

Taking into account (18), the expression for $\tilde{I}(\mathbf{r}, \eta)$ assumes the form

$$\tilde{I}(\mathbf{r}, \eta) = \int_{-\infty}^{\infty} S_{0S}\,(\mathbf{r} - \xi(\mathbf{r}') - \tilde{\xi}(\mathbf{r}', \eta))\, I(\mathbf{r}')\, d^2\mathbf{r}'. \qquad (20)$$

The algorithm constructed is schematically shown in Fig. 2 for a simple example of imaging two points with the brightness J_1 and J_2

$$I(\mathbf{r}') = J_1\,\delta(\mathbf{r}' - \mathbf{r}_1') + J_2\,\delta(\mathbf{r}' - \mathbf{r}_2'). \qquad (21)$$

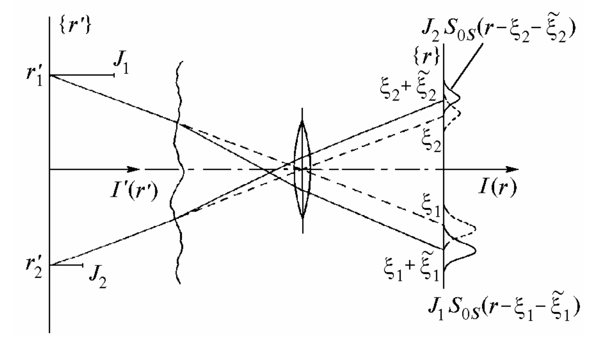

FIG. 2. The scheme of constructing the algorithm of $\tilde{I}(\mathbf{r})$ realization for a short-exposure image in the case with a two-point object; $\mathbf{r}'_1$, $\mathbf{r}'_2$ are the coordinates of points in the object plane; J_1, J_2 are their radiation powers; ξ_1, ξ_2 are the geometrical-optical displacements of points in the image plane.

2. TESTING THE METHOD

Some results of computer simulations of the PSF of an optical system made assuming specific atmospheric conditions are presented in Fig. 3 for a point object located at the coordinates origin ($\mathbf{r}' = 0$).

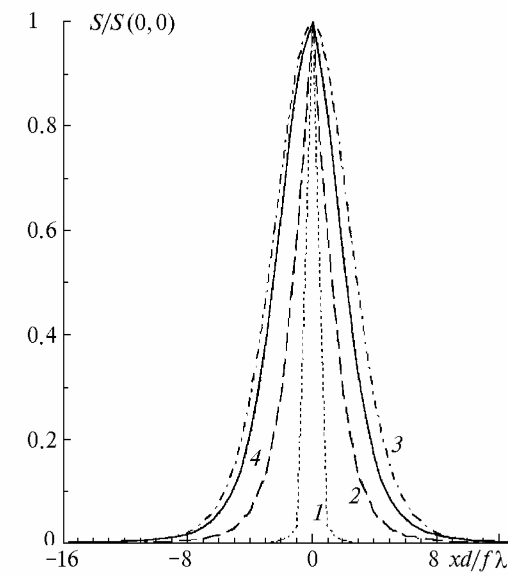

FIG. 3. The normalized PSFs as functions of the normalized transverse coordinate x. Curves are as follows: the diffraction-limited PSF S_0 (1); the short-exposure PSF of a subsystem without large-scale fluctuations S_{0S} (2); the theoretical long-exposure PSF S_L (3); the long-exposure PSF S_S^N, obtained by Monte-Carlo method over 100 realizations (4). Simulation conditions: $d = 10$ cm, $z = 2$ km, $l = 0.5$ μm, $C_n^2 = 10^{-15}$ cm$^{-2/3}$, dimensions of the grid in the image plane is 64×64, the grid's step $\Delta x = f\lambda/(2d)$.

Diffraction-limited $S_0(\mathbf{r})$ and the short-exposure PSF $S_{0S}(\mathbf{r}, 0)$ were obtained using formulas (12)–(15). It is seen that the presence of a small-scale turbulence leads to a considerable broadening of the short-exposure PSF $S_{0S}(\mathbf{r}, 0)$ (curve *2*) as compared with diffraction-limited $S_0(\mathbf{r})$ one (curve *1*).

The averaging of PSF $\tilde{S}_{\text{exp}}(\mathbf{r}, 0, \eta)$ over the slow time η results in a long-exposure PSF

$$S_L(\mathbf{r}, 0) = \lim_{T\to\infty} \frac{1}{T}\int_0^T S_{0S}(\mathbf{r} - \tilde{\xi}(0, \eta))\, d\eta, \qquad (22)$$

which, in accordance with Ref. 1, can be obtained by the Fourier transform of the long-exposure PSF

$$H_{0L}(\Omega) = H_0(\Omega)\exp\left\{-3.44\left(\frac{\lambda\Omega}{r_0}\right)^{5/3}\right\}. \qquad (23)$$

Thus obtained, theoretical PSF $S_L(\mathbf{r}, 0)$ is shown in Fig. 3 by curve *3*.

In a computer experiment the long-exposure PSF $S_S^N(\mathbf{r}, 0)$ was calculated by averaging N random samplings of the short-exposure PSFs, $\tilde{S}_{\text{exp}}(\mathbf{r}, 0, \eta)$. The corresponding results obtained at $N = 100$ are presented in Fig. 3 by curve *4*.

Good agreement between the theoretically predicted long-exposure PSFs and those calculated by Monte-Carlo method is well seen from the above results that confirms the reliability of the model constructed. Somewhat lesser width of the S_S^N as compared with S_L is caused by errors in the reproduction of large-scale inhomogeneities on the phase screen of a limited size (in numerical experiment it was of 2.5 m). Further development of the phase screen generation algorithms is likely to enable one to make this error negligible.

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