EFFECTS OF THE ATMOSPHERIC TURBULENCE OUTER SCALE ON THE INSTANTANEOUS AND LONG-EXPOSURE RADII OF A LASER BEAM.

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The behavior of laser beam radius is analyzed numerically by Monte-Carlo method for the case of a horizontal path in the atmosphere under conditions of weak, strong, and moderate turbulence. Phase screens are generated by new sub-harmonic method of Johansson and Gabel to imitate atmospheric turbulence. The comparison is made of the experimental data with the results obtained by statistical trials and analytical approach.

Spatial inhomogeneities of air refractive index in turbulent atmosphere cause an increase in angular width of a laser beam and may result in its displacement as a whole. These phenomena may essentially affect the operation of optical systems and therefore their investigation is of practical importance.

The relation of a beam radius, at long averaging, to its instantaneous value is expressed by the following formula[1]:

$$a_l^2 = a_S^2 + \sigma_c^2, \qquad (1)$$

where σ_c^2 – is the variance of the laser beam random displacements. The most strict analytical estimations of these values with the account for influence of the outer scale of atmospheric turbulence were obtained using phase approximation of the Huygens-Kirchhoff method.[2] From these estimations it follows, that the outer scale affects the displacement variance in the region of weak turbulence broadening

$$D_s(2a_0) \ll \Omega^{4/3} \, (1 - (1 + \beta)^{-1/6}), \qquad (2)$$

where $D_s(2a_0) = 1.1 \, C_n^2 \, k^2 \, L(2a_0)^{5/3}$ is the structure function of the phase of a spherical wave on the diameter of the transmitter aperture a_0, C_n^2 is the structure constant of the air refractive index, $\beta = (0.54 \, L_0)^2/(2\pi^2 \, a_0^2)$; L is the length of the propagation path, $\Omega = k \, a_0^2/L$ is the Fresnel number of the transmitter aperture, k is the wave number. In the region of strong broadening

$$D_s(2a_0) \gg \Omega^{5/3} \, (1 + \beta)^{5/6}. \qquad (3)$$

In the intermediate region of the parameters

$$\Omega^{5/3} \ll D_s(2a_0) \ll \Omega^{5/3} \, (1 + \beta)^{5/6} \qquad (4)$$

the estimations predict independence of the variance of random displacement of the beam on the outer scale of turbulence.

On the contrary, the calculated long-exposure radius of the beam, a_l with the account for the outer scale coincides with those obtained without the account of the outer scale L_0 accurate to 4%.

The data of estimations have an asymptotic character. Different approximations have been used for such estimations. Impossibility of obtaining exact analytical solutions for the stochastic light field in randomly inhomogeneous media stimulates the development of numerical methods for solving that type of problems in atmospheric optics.

The method of statistical trials (Monte-Carlo method) has enjoyed the widest application.[3] This method based on the model of phase screens (MPS) allows one to simulate practically any conditions of radiation propagation and to effectively investigate various statistical properties of light field in a unified approach. Strong fluctuations of plane wave intensity[4] and statistics of small-scale fluctuations of a narrow beam[5] are investigated with the help of Monte-Carlo method.

In this paper, the variance of the beam displacements and of its radius is investigated by this method at short-term and long-term averaging in turbulent atmosphere. The results obtained with statistical trial, when analyzing propagation of collimated and focused laser beams, are compared with those of analytical estimations.

To simulate large-scale inhomogeneities of atmospheric turbulence, we have used the phase screens constructed by a modified technique of sub-harmonic.[6] When obtaining the low-frequency part of the phase field spectrum $\tilde{\varphi}_{\mathrm{lf}}$, near the zeroth harmonic, we divide this region into sub-harmonics. The resulting field is defined as a sum of the high-frequency part $\tilde{\varphi}_{\mathrm{hf}}$, obtained by an ordinary spectral technique, and the low-frequency one $\tilde{\varphi}_{\mathrm{lf}}$ created by the above sub-harmonics approach

$$\tilde{\varphi}(n, m) = \tilde{\varphi}_{\mathrm{hf}}(n, m) + \tilde{\varphi}_{\mathrm{lf}}(n, m). \qquad (5)$$

At every division of the two-dimensional region near zero frequency 32 sub-harmonics are added. Such

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a division matches the iteration step in formation of the phase screen.

In a computer experiment a random position of the center of gravity of an optical beam in a plane transverse to the direction of propagation is defined by the following expression:

$$\tilde{\rho}_c^{(i)}(z) = \frac{1}{P_0} \int d^2\rho \, \rho \, \tilde{I}^{(i)}(z, \rho), \quad (6)$$

and the long-exposure radius of the beam by the expression

$$a_l^2(z) = \frac{1}{P_0} \left< \int d^2\rho \, \rho^2 \, \tilde{I}^{(i)}(z, \rho) \right>_M, \quad (7)$$

where $P_0 = \int d^2\rho \, I(0, \rho)$ is the total power of the beam; $\tilde{I}^{(i)}(z, \rho)$ is the random distribution of the intensity in the cross section of the beam at some realization; $<\!-\!>_M$ denotes the operation of averaging over M realizations. The beam spread σ_c^2 was defined as the $\{\tilde{\rho}_c^{(i)}(z),\ i = 1 \dots M\}$ result of statistical treatment of an ensemble of realizations.

Shown in Fig. 1 are the results of numerical experiments for a collimated beam of radiation at 0.5 μm wavelength with a Gaussian profile having the radius $a_0 = 2$ cm propagated along a horizontal near-ground path of the length $L = 2$ km. We used Karman spatial spectrum with the outer scale L_0 equal to 50 m, 5 m, and 50 cm, the structure constant used was taken to be $C_n^2 = 5\cdot10^{-15}$ cm$^{-2/3}$. There were 20 screens along the path. The statistical ensemble involved 100 realizations. The phase screens were generated by a modified method of sub-harmonics on a 512 by 512 grid with the number of iterations $N_j = 4$. For the conditions of beam propagation considered the results of experiment embrace the regions of weak as well as strong broadening. At $L_0 = 50$ cm, the region of weak broadening is at $z < 500$ m, intermediate region is at $700 < z < 1000$ m, and the region of strong broadening is at $z > 1200$ m, at $L_0 = 5$ and 50 m the weak broadening is at $z < 500$ m and the intermediate region is at $600 < z < 2900$ m.

Some experiments show (see Fig. 1), that the variance of the displacements of the beam center of gravity essentially depends on the outer scale of turbulence not only in the regions of strong and weak turbulent broadening, but in the intermediate regions as well. Regardless of the outer scale effects the variations of σ_c^2, the long-exposure radius of the beam do not practically depend on L_0, that coincides with theoretical estimates. Thus, when calculating long-exposure radius of the beam a_l, one can neglect the influence of L_0. However, the analytical formula[1]

$$a_l^2 = a_0^2 \left[\left(1 - \frac{L}{F}\right)^2 + \Omega^{-2} \left(1 + \frac{4}{3} \frac{a_0^2}{\rho_{pl}^2}\right) \right], \quad (8)$$

$$\rho_{pl} = (1.4\, C_n^2\, k^2\, L)^{-3/5}$$

shows a more rapid a_l increase in a with increasing distance, than it follows from the statistical approach.

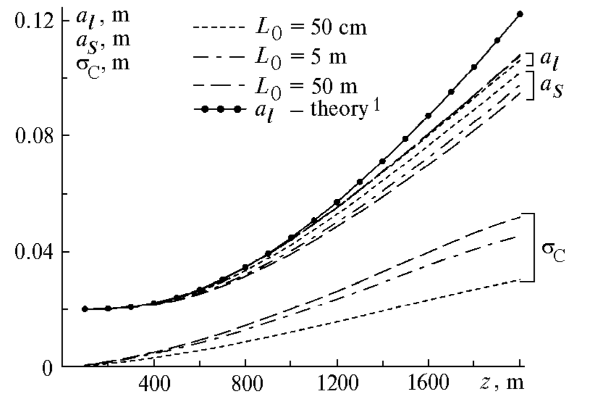

FIG. 1. Long-exposure radius a_l, short-exposure radius a_S, root-mean-square deviation of the displacement of the center of gravity σ_c of a collimated beam with the initial radius $a_0 = 2$ cm, λ = 0.5 μm, propagating along a near-ground path with the parameters of turbulence $C_n^2 = 5\cdot10^{-15}$ cm$^{-2/3}$ and different values of the outer scale L_0.

Relative contribution of the random displacements to the long-exposure broadening is characterized by the parameter $\alpha = 2\,\sigma_c^2/a_S^2$. The dependence of a value on the turbulence parameter $D_s(2a_0)$ (Fig. 2) well agrees with the theoretically predicted one, and this shows that turbulent broadening prevails over the random displacements of the beam within a wide range of D_s values. It is seen, that with the increasing outer scale L_0, the contribution of displacements becomes comparable with the turbulent broadening.

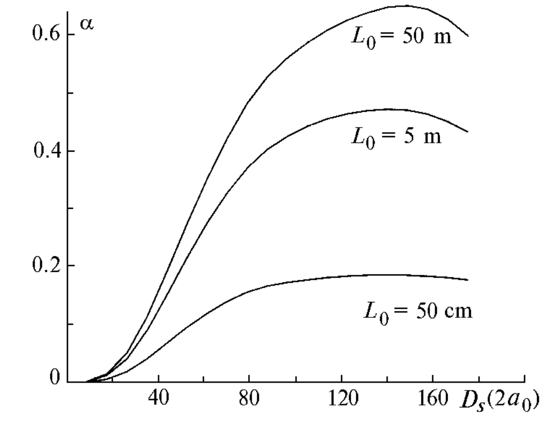

FIG. 2. Dependence of the magnitude of the parameter $\alpha = 2\,\sigma_c^2/a_S^2$ of a collimated beam with the initial radius $a_0 = 2$ cm, λ = 0.5 μm, propagating along a near-ground path with parameter of turbulence $C_n^2 = 5\cdot10^{-15}$ cm$^{-2/3}$ on parameter of turbulence $D_s(2a_0)$ at different values of the outer scale L_0.

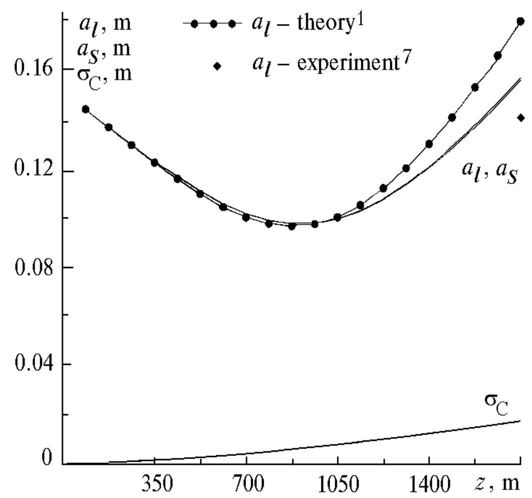

FIG. 3. Long-exposure radius a_l, short-exposure radius a_S, root-mean-square deviation of the displacement of the center of gravity of a focused beam of the initial radius a_0 = 15.1 cm, *wavelength* λ = 0.63 μm, *propagating along the path at* H = 2 m *at the distance* $L = F$ = 1750 m *at* C_n^2 *value equal to* $1.5\cdot10^{-14}$ cm$^{-2/3}$.

The results of computer simulations of a field experiment[7] with a focused beam are presented in Fig. 3 (a_0 = 15.1 cm, λ = 0.63 μm, focal length $F = L$ = 1750 m, $C_n^2 = 1.5\cdot10^{-14}$ cm$^{-2/3}$, L_0 = 0.8 m). The computer experiment is performed on a 1024 by 1024 grid to provide for high spatial resolution (diffraction beam radius at focus is a_d = 1.16 mm). The phase screens were generated by a modified method of sub-harmonics with the number of iterations N_j = 2. At that number of iterations the accuracy of simulating the wave phase fluctuation variance on a screen, determined by inhomogeneities of the spatial scale comparable with L_0, is not worse than 5%. There were 20 screens along the path and the statistical ensemble involved 50 realizations. It is seen that the rms displacement of the center of gravity S of a focused beam is much less than the instantaneous radius of the beam propagated along the entire path. Theoretical estimations of a_l coincide with the data of numerical experiment at the distance z up to the turbulent beam waist and give enhanced values behind it. The value a_l obtained in full-scale experiment is close to the data obtained by statistical trial approach.

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