

APPLICATION OF AN INTEGRATED FORECASTING PROCEDURE TO THE SPATIAL EXTRAPOLATION OF METEOROLOGICAL FIELDS INTO THE TERRITORY UNCOVERED WITH OBSERVATIONAL DATA

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A refined integrated forecasting procedure is considered based on integration of two alternative prediction schemes (the method for optimal extrapolation and the modified method of clustering arguments underline these schemes) and developed to solve the problems of spatial extrapolation of mesometeorological fields into the territory uncovered with aerological data. The quality of this forecasting procedure is numerically estimated based on the data of many-year observations at the radiosonde network being typical of the mesometeorological site.

1. INTRODUCTION

A problem of spatial extrapolation of the vertical profiles of mesometeorological fields (that is, the fields with characteristic scales varying from several tens to several hundreds of kilometers¹) into the territory uncovered with aerological data occupies a highly important place among modern and urgent problems of applied meteorology. This is caused by the fact that the speedy solution of this problem is necessary to solve a great many problems in national economy and defence, in particular:

- local numerical weather forecast for regions with scarce aerological network;
- evaluation of spatial spreading of technogenic pollutants (including radionuclides) at short distances (up to 100–200 km) from their sources in case of industrial emergency;
- routine evaluation of the physical state of the atmosphere (and primarily, its temperature and wind regimes) to provide prescribed ballistic trajectories of missiles and rockets and to increase the accuracy and efficiency of hitting targets on enemy territories;
- meteorological support of launching and landing of various vehicles including Shuttles, etc. Here it should be noted that to solve the above-enumerated problems, mesometeorological fields should be extrapolated with high horizontal resolution (with a step of 5–50 km, see Ref. 1) and even more higher vertical resolution (with steps of 100–500 m at altitudes up to 3 km and 1–1.5 km between 3–16 km, see Ref. 2). Satisfactory accuracy of estimation of the parameters also should be ensured at an examined point. Thus, for example, the average temperature and the zonal and meridional wind velocity components (these parameters are averaged over the given atmospheric layers) should be estimated with rms errors

of no more than 1.0–1.2° and 1.0–1.2 m/s, respectively, to provide geophysical support to modern rocket and gun systems.³

It is quite clear that in practice the extrapolation of a 3-D structure of mesometeorological fields faces a number of problems.

First, the existing global aerological network gives no way of estimating the 3-D structure of mesometeorological fields due to its extreme inhomogeneity and insufficient covering (even in Europe and North America most extensively covered with observational data the shortest distance between two neighboring stations is 300–400 km (see Ref. 4), with rare exception).

Second, even the method of optimal extrapolation, the most widespread method for numerical prediction schemes described in detail in Ref. 4, obligatorily calls for a large body of initial information and calculation (from the data of many-year observations) of different characteristics of the spatial structure of meteorological parameters and primarily, the autocorrelation functions.

Third, in the existing numerical prediction schemes, including the optimal extrapolation scheme, the parameters of centered fields are used most often as the input information. These parameters are calculated from the global archive of climatic averages (norms) typically assigned in the nodes of the regular geographic grid⁵ with a latitudinal step of 5° and a longitudinal step of 10°. This grid gives no way of extrapolating the mesometeorological fields with the required spatial resolution.

Fourth, there is a special class of applied problems connected, for example, with geophysical support for military technical systems during combat operations, when the international data exchange is impossible. This situation calls for the development of special methods of spatial extrapolation of mesometeorological

fields based only on the data collected by the local aerological network.

From the preceding, considering the lack of publications on the examined problem, in the last few years the specialists from the Institute of Atmospheric Optics, SB RAS (under supervision of Valerii S. Komarov) have developed special methods and algorithms for numerical extrapolation of the 3-D structure of mesometeorological fields under conditions of informational uncertainty, that is, for a limited body of experimental data (the first results of investigations on this problem were presented in Refs. 6–8). These methods and algorithms were based on an integrated approach harnessing the procedure of optimal integration of alternative forecasting methods, namely, the optimal extrapolation method (OEM) and the modified method of clustering arguments (MMCA).

Statistical estimation of the quality of this approach for the prediction of mesoscale temperature and wind velocity fields⁷ have shown that it provides quite satisfactory data for practical needs and noticeably increases (in comparison with the OEM alone) the accuracy of spatial extrapolation. However, because of a small body of experimental information used to obtain these results, they must be further tested with the use of more representative samples of aerological data obtained with sufficient statistics.

In the present paper, we discuss the results of quality testing of spatial extrapolation of mesometeorological fields performed by the complex of alternative methods (OEM and MMCA) for the empirical data obtained with a sufficient statistic, in contrast with Ref. 7.

2. SOME METHODOLOGICAL ASPECTS OF SOLVING THE PROBLEM ON SPATIAL EXTRAPOLATION OF MESOMETEOROLOGICAL FIELDS

Before consideration of some methodical aspects of the solution of the above-formulated problem, we define in general the notion of spatial extrapolation of fields of the meteorological parameters.

Let a function $\xi(x^{(1)}, x^{(2)}, \dots, x^{(n)})$ of several independent variables be specified. Draw an n -D sphere of minimum radius through the points with the known values of $\xi(x^{(1)}, x^{(2)}, \dots, x^{(n)})$. Then, according to Ref. 4, by the notion of extrapolation of the function ξ is meant the procedure of calculation of its values beyond the n -D sphere. From this it follows that the extrapolation problem can be reduced to the calculation of the function value at a given point from the known values of this functions in the other points.

This problem can be formulated more rigorously for a random field with the use of a body of mathematics.

Let the values of a uniform centered field ξ be specified at points $\mathbf{r}_i \in W_x \subset R^m$ (here, \mathbf{r}_i is the radius-

vector of the point determined by its spatial coordinates x, y, z and time t and $i = 1, 2, \dots, n$ is the number of points in a closed set W_x of the finite Euclidean space R^m). Then the procedure of spatial extrapolation of the field into the point $\mathbf{r}_0 \notin W_x \subset R^m$ (that is, calculation of its value $\xi(\mathbf{r}_0)$ beyond the set W_x from the known values $\xi(\mathbf{r}_i)$ at the points $\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_n$ belonging to the set W_x) is reduced to the expression of the form

$$\hat{\xi}(\mathbf{r}_0) = \Xi[\xi(\mathbf{r}_1), \xi(\mathbf{r}_2), \dots, \xi(\mathbf{r}_n)], \tag{1}$$

where the form of the function Ξ is determined by the employed extrapolation procedure and the relative positions of the points $\mathbf{r}_0, \mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_n$.

Because in the problem examined we are dealing with the extrapolation of a uniform and centered mesometeorological field, within its boundaries, according to Ref. 4, it is horizontally uniform and isotropic and hence the equalities

$$\bar{\xi}(\mathbf{r}_i) = \bar{\xi}(\mathbf{r}_k) = \dots = \bar{\xi}; \tag{2}$$

$$\sigma_{\xi}^2(\mathbf{r}_i) = \sigma_{\xi}^2(\mathbf{r}_k) = \sigma_{\xi}^2; \tag{3}$$

$$\mu_{\xi}(\mathbf{r}_i, \mathbf{r}_k) = \mu_{\xi}(\rho) \tag{4}$$

are valid (here, $\bar{\xi}$ is the mean, σ_{ξ}^2 is the variance, and $\mu_{\xi}(\rho)$ is the normalized correlation function, and $\rho = |\mathbf{r}_i - \mathbf{r}_k|$ is the distance between the points \mathbf{r}_i and \mathbf{r}_k). Therefore, in the present paper as earlier⁶ we used an original approach based on the procedure of optimal integration of two alternative methods for statistical extrapolation (OEM and MMCA) as the main method for spatial prediction of the structure of this field.

In so doing, the optimal extrapolation method was used to choose the atmospheric altitude level for which the rms prediction error was minimum and to reconstruct (for the same level) the value of the field ξ in the given point (with the coordinates x_0 and y_0) from its values measured in the surrounding points \mathbf{r}_i (that is, at the aerological stations) located beyond the extrapolation region. At the same time, the modified method of clustering arguments was used for numerical reconstruction of the vertical structure of the field at the point (x_0, y_0) throughout the entire thickness of the examined layer.

Because the detailed description of theoretical principles and OEM and MMCA algorithms have already been given in Refs. 4, 6, and 8, in the present paper we only outline the essence of these methods somewhat refined as applied to the solution of the examined problem.

a) Optimal Extrapolation Method

By the optimal extrapolation method, the field at the point $\mathbf{r}_0 \notin W_x \subset R^m$ is calculated from

measurements at points \mathbf{r}_i (that is, from measurements at the neighboring aerological stations with coordinates x_i, y_i) using the formula⁴

$$\xi(\mathbf{r}_0) = \sum_{i=1}^n a_i \xi(\mathbf{r}_i), \quad (5)$$

where a_i are the weighting coefficients to be calculated, and n is the number of points (stations).

In so doing, for optimal estimation of the field ξ at the point \mathbf{r}_0 , the condition

$$\mathbf{E}[\mathbf{a}] = \mathbf{M}\{[\tilde{\xi}(\mathbf{r}_0) - a_i \xi(\mathbf{r}_i)]^2\} \rightarrow \min, \quad (6)$$

must be fulfilled when we calculate the weighting coefficients a_i , where $\tilde{\xi}(\mathbf{r}_0)$ is the observed value of the field ξ at the point \mathbf{r}_0 , and \mathbf{M} is the operator of mathematical expectation.

The quantity $\min \mathbf{E}[\mathbf{a}]$ is called the optimal extrapolation error.

To calculate the weighting coefficients a_i , a system of linear equations (SLEs) of the form⁴

$$\sum_{j=1}^n a_j \mu_{ij} + \eta^2 a_i = \mu_{0i} \quad (i = 1, 2, \dots, n) \quad (7)$$

is commonly used, where μ_{ij} and μ_{0i} are the coefficients of spatial correlation between the true values of the field ξ at points $\mathbf{r}_i, \mathbf{r}_j$ and $\mathbf{r}_0, \mathbf{r}_i$, respectively; $\eta^2 = \Delta^2 / \sigma_\xi^2$ is the so-called measure of the measurement error (here, Δ^2 is the variance of this error, and σ_ξ^2 is the variance of the meteorological parameter ξ).

Because in the problem to be solved we deal with the mesometeorological fields that, according to Ref. 4, can be classified as uniform and isotropic, the values of the autocorrelation functions μ_{ij} and μ_{0i} depend only on the separation between the points we have taken. In addition, the measures of the measurement error at points \mathbf{r}_0 and \mathbf{r}_i will be identical, because the variance σ_ξ^2 remains unchanged (within the mesoscale experimental site) and the errors of radiosonde observations also remain constant.

In practice different analytic functions are used in the OEM algorithms to estimate the coefficients μ_{ij} and μ_{0i} as functions of the distance

$$\rho_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}. \quad (8)$$

In our previous papers,⁶⁻⁸ the following analytic functions were used to approximate the spatial correlation functions

of the temperature (T):

$$\mu_T(\rho) = \exp(-0.825 \rho^{0.92}) \quad (9)$$

and of the zonal (V_x) and meridional (V_y) wind components:

$$\mu_{V_x}(\rho) = \mu_{V_y}(\rho) = (1 - 0.98\rho) \exp(-0.98\rho). \quad (10)$$

In the present paper we took the other analytic dependences to approximate the empirical spatial correlation functions of the temperature and zonal and meridional wind components at the ground level (this level is commonly used to predict the near-ground values of the fields by the optimal extrapolation method). These analytic dependences were borrowed from Ref. 9 and have the following forms:

for the temperature

$$\mu_T(\rho) = \{\exp(-\alpha\rho)\} \cos(\beta\rho), \quad (11)$$

where $\alpha = 0.436$ and $\beta = 0.863$;

for the zonal and meridional wind components

$$\mu_{V_x}(\rho) = \mu_{V_y}(\rho) = (1 - \alpha\rho) \exp(-\rho)^2, \quad (12)$$

where $\alpha = 1.162$.

The distance ρ in Eqs. (11) and (12) is in thousand kilometers.

b) Modified Method of Clustering Arguments

Now we outline the essence of the modified method of clustering arguments (it was described in detail in Ref. 6).

Let the values of the uniform centered field be defined at points \mathbf{r}_i ($i = 1, 2, \dots, n$) at discrete times $t = 1, 2, \dots, N$.

Then from the known value of this field at the point \mathbf{r}_i nearest to the point \mathbf{r}_0 and from the results of its optimal extrapolation into this point performed at the altitude level for which the prediction error is minimum at time $t = N + 1$, we compose a sample of spatiotemporal observations of the form

$$\{\xi_i(h, t), h = 0, 1, \dots, h_k; t = 1, 2, \dots, N\},$$

$$\{\xi_0(h, t), h = 0, 1, \dots, \bar{h} \leq h_k; t = N + 1\}, \quad (13)$$

where $\xi_i(h, t)$ is an ensemble of values of the field ξ at the point with coordinates x_i, y_i at altitude h recorded during the time interval t over which the discrete measurements are carried out; $\xi_0(h, t)$ is the sought-after values of this field at the point with the preset coordinates x_0, y_0 at altitude h and time $t = N + 1$.

The selection and construction of the best prognostic model and the procedure for extrapolating the field ξ from the point \mathbf{r}_i into the point \mathbf{r}_0 are based on the sample of spatiotemporal observations given by Eq. (13) and on the basis functions taken to be the composite difference dynamic-stochastic models of the form⁶

$$\xi_0(h, N + 1) = \sum_{\tau=1}^{N^*} A(h, \tau) \xi_i(h, N + 1 - \tau) + \sum_{j=1}^{h-1} B(h, j) \xi_i(j, N + 1) + \xi(h, N + 1) \tag{14}$$

for the MMCA algorithm. Here, N^* specifies the time delay ($N^* < [N - h - 1]/2$); $A(h, 1), \dots, A(h, N^*)$ and $B(h, 0), \dots, B(h, h - 1)$ are the unknown model parameters, and the quantity $\xi(h, N + 1)$ is the model discrepancy.

In so doing, to select the best MMCA prognostic model, we used:

1. The method of directional cluster search used to optimize the model structure with two-stage selection of models against two criteria:

a) The forecast resulting error (after Akaike¹⁰)

$$FRE = \frac{(N - N^* - 1) + s}{(N - N^* - 1) - s} RSS(s) \tag{15}$$

with

$$RSS(s) = \sum_{j=1}^{N-N^*-1} \left[\xi_{h,N-j}^{(i)} - \hat{\xi}_{h,N-j}^{(i)}(s) \right]^2, \tag{16}$$

where $RSS(s)$ is the residual square sum for the running model $\hat{\xi}_{h,N-j}^{(i)}(s)$ comprising s nonzero estimates. The parameter s specifies the complexity of the model.

In so doing, $N^* + h$ best models described by Eq. (14) are selected from the subsamples n_1 of observations performed till $t = N + 1$, and the values $\hat{\xi}_{h,N-j}^{(i)}(s)$ are estimated from the formula

$$\hat{\xi}_{h,N-j}^{(i)}(s) = X\hat{\Theta}, \tag{17}$$

$$X \in M_{(N-N^*-1) \times (N^*+h)}, \hat{\Theta} \in R^{(N^*+h)},$$

where $\hat{\Theta} = [\hat{A}_{h,1}, \dots, \hat{A}_{h,N^*}, \hat{B}_{h,0}, \dots, \hat{B}_{h,N-1}]^T$ is the minimax estimation of the parameters from the subsample n_1 calculated by special formulas⁶ (here, T denotes the transposition operation); $M_{m \times p}$ is a set of real matrices of orders $m \times p$; R^m is the m -dimensional Euclidean space.

b) The rms error of extrapolation from the subsample n_2 comprising only observations at time $t = N$, namely,

$$|\xi_{h,N}^{(i)} - \hat{\xi}_{h,N}^{(i)}(s)| \rightarrow \min. \tag{18}$$

The minimum in Eq. (18) is taken over all $N^* + h$ structures; each of them corresponds to its own model $\hat{\xi}_{h,N}^{(i)}(s)$.

2. The method of minimax estimation of the model parameters $\hat{\Theta}$ from the entire sample $n_1 + n_2$. These estimates ensure high quality of the corresponding forecast defined by the inequality

$$E |E(\xi_{h,N+1}^{(0)}) - \hat{\xi}_{h,N+1}^{(0)}|^2 \leq \delta_{h,N+1} \tag{19}$$

$$(h = \bar{h} + 1, \dots, h_k),$$

where $E(\bullet)$ is the operator of mathematical expectation; $\xi_{h,N+1}^{(0)}$ and $\delta_{h,N+1}$ are the minimax estimates, which depend on the variance of the observation error and *a priori* information about the maximum permissible prediction (extrapolation) errors.

In conclusion, it should be noted that the vector $\xi_0(h, N + 1)$ used to select and to construct the best MMCA prognostic model (it is included in the initial sample of spatiotemporal observations) must have at least one known value at height h , which in our case is calculated by the optimal extrapolation method at the altitude level with the minimum prognostic error. The assumption that the vertical structure of the field at the point r_0 is identical to that at the nearest point r_i is used to reconstruct the full altitude profile of this field at the point r_0 with the help of this model.

3. OPTIMIZATION OF THE LENGTH OF INITIAL SAMPLE AND OF THE NUMBER OF BEST STRUCTURES OF PROGNOSTIC MODELS FOR THE DEVELOPMENT OF THE OPTIMAL NUMERICAL MMCA ALGORITHM

The MMCA algorithm described above has some limitations connected with the formation of the sample of spatiotemporal observations described by Eq. (13) and the assignment of the number of the best model structures, namely:

- the number of altitude levels k must be less than 40;
- the number of the employed initial observations N must be less than 100;
- the number of the best model structures sorted by the MMCA algorithm at the first stage of selection against the criterion of the forecast resulting error specified by Eq. (15) must be no less than 5 (less number may lead to a poor model structure) and not too great to avoid much computation time.

Preliminary analysis of the available aerological data shows that the first limitation on the number of altitude levels $k = 40$ is quite reasonable for solving the problem of spatial extrapolation of the vertical structure of mesometeorological fields. At the same time, two next limitations connected with the choice of the required number of initial observations and best model structures are uncertain. Therefore, we must first seek for the optimal length of the initial sample and specify (based on real data) the number of best model structures before proceeding to practical implementation of the MMCA algorithm.

Considering these circumstances, we estimated numerically the quality of the MMCA algorithm as a function of the employed amount of initial aerological data comprising the sample of spatiotemporal observations described by Eq. (13) and the number of prognostic model structures. These estimates were based on many-year (1971–1975) radiosonde measurements of the temperature and wind velocity performed at three aerological stations in Warszawa, Minsk, and L'vov (their positions are indicated below).

Our numerical experiments showed that:

– To develop the optimal computational scheme for the spatial extrapolation of the field ξ , the number of observations should be greater by one than the number of altitude levels, that is, the number of initial realizations N can be determined from the formula

$$N = k + 1, \quad (20)$$

where k is the number of altitude levels at which the meteorological parameters are measured. In so doing, the initial data sample must comprise no less than 7 realizations of the field ξ .

– The increase of the number of altitude levels k within the examined atmospheric layer (for $k \leq 40$) significantly improves the quality of spatial extrapolation.

– The optimal number of model structures for the MMCA algorithm is 15 (for the layer between the ground and 3 km) and 10 (for the layer between 4 and 10 km), because in this case the best model structure is ensured for the MMCA together with the reliable spatial prediction of the field ξ .

In addition, we also estimated the required sampling period (temporal step) of the initial data, because in practice radiosonde observations are carried out twice a day (at 0 and 12 h, Greenwich time), that is, with a time interval of 12 h. It was established that for low altitudes (up to 3 km) the best results of spatial extrapolation were obtained with a time interval of 24 h, whereas for $h > 3$ km, the data sampling period should be 12 h. In addition, we established that the best results of spatial prediction were achieved with radiosonde observations taken at 0 (24) h, Greenwich time.

In conclusion, it should be noted that, according to Ref. 11, the highest-quality results of spatial extrapolation of mesometeorological fields in the lower layer of the troposphere are obtained with a data sampling period of 6 h and less. However, this temporal step can be achieved only in case of lidar sensing.

4. STATISTICAL ESTIMATES OF THE QUALITY OF SPATIAL EXTRAPOLATION OF THE TEMPERATURE AND WIND VELOCITY FIELDS

We used the data of many-year (1971–1975) radiosonde observations performed at five aerological

stations in Warszawa (52°11'N, 20°58'E), Kaunas (54°53'N, 23°53'E), Brest (52°07'N, 23°41'E), Minsk (53°11'N, 27°32'E), and L'vov (49°49'E, 23°57'E), which form a mesometeorological experimental site (its scheme was shown in Fig. 1 of Ref. 9), to perform the spatial extrapolation of meteorological parameters on the basis of optimal integration of two alternative prediction methods (OEM and MMCA) and to estimate the extrapolation quality. For our calculations, we selected only synchronous (for all stations) two-term (at 0 and 12 h, Greenwich time) observations from the entire initial data array. In addition all the data of radiosonde observations used in our calculations were attached to the unified geometric altitudes, which comprises, in contrast with Ref. 7, 13 standard altitude levels rather than 9. They were at altitudes 0 (the ground level), 0.2, 0.4, 0.8, 1.2, 1.6, 2.0, 2.4, 3.0, 4.0, 5.0, 6.0, and 8.0 km. They describe almost all troposphere, including the boundary layer, with high vertical resolution.

The use of these altitude levels in the problem to be solved is connected with the fact that the procedure for spatial extrapolation of mesometeorological fields (in our case, temperature and wind fields) was implemented to problems of numerical prediction of the spread of technogenic pollutants usually made for the troposphere (with the most intense spread of pollutants in the atmospheric boundary layer¹²).

To this we can add that in practical calculations of the distance of spreading of any pollutant forming a cloud we commonly used temperature and wind velocity values averaged over an atmospheric layers rather than their values measured at individual altitudes. Therefore, in analogy with Ref. 7, the procedure of layer-by-layer averaging of the temperature T and the zonal V_x and meridional V_y wind components was used to form the initial arrays of spatiotemporal observations when solving the problem of spatial extrapolation of the mesometeorological fields. This procedure was implemented with the help of equations

$$\langle T \rangle_{h_0, h} = \frac{1}{h - h_0} \int_{h_0}^h T(z) dz; \quad (21)$$

$$\langle V_x \rangle_{h_0, h} = \frac{1}{h - h_0} \int_{h_0}^h V_x(z) dz; \quad (22)$$

$$\langle V_y \rangle_{h_0, h} = \frac{1}{h - h_0} \int_{h_0}^h V_y(z) dz \quad (23)$$

where z is the altitude, the symbol $\langle \rangle$ denotes the data averaging along the vertical within the layer $h - h_0$, $h_0 = 0$ is the height of the lower boundary of the examined layer, and h is its upper boundary height. The atmospheric layers were located between 0–0.2, 0–0.4,

0–0.8, 0–1.2, 0–1.6, 0–2.0, 0–2.4, 0–3.0, 0–4.0, 0–5.0, 0–6.0, and 0–8.0 km. Here it should be noted that the layer 0–10 km (the last layer for the troposphere) was excluded from consideration, because a large error of spatial interpolation (and hence extrapolation) of the temperature and wind fields was reported for this layer in Ref. 13 due to the impact of the tropopause near 10 km (at a barometer altitude of ~250 hPa).

And finally, the quality of spatial extrapolation of the temperature $\langle T \rangle_{h_0, h}$ and the zonal $\langle V_x \rangle_{h_0, h}$ and meridional $\langle V_y \rangle_{h_0, h}$ wind components averaged over the layer was estimated with the help of the rms extrapolation error δ_ξ given by the formula

$$\delta_\xi = \left[\frac{1}{n} \sum_{i=1}^n (\Delta \xi_i)^2 \right]^{1/2} \quad (24)$$

(here, $\Delta \xi_i = \xi_i^* - \xi_i$; ξ_i^* is the deviation of the extrapolated value of the meteorological parameters ξ_i^* from its measured value for the i th atmospheric layer, and n is the number of realizations) and the probability P that the error (that is, $\Delta \xi_i$) is less than the preset value (for the average temperature, less than $\pm 1, \dots, \pm 4$ or greater than 4°C ; for the average wind components, less than $\pm 1, \dots, \pm 4$ or greater than ± 4 m/s).

Now we proceed to an analysis of the results of numerical experiments on the estimation of the quality of spatial prediction made for the temperature and wind fields by the method of optimal extrapolation (with the use of analytic dependences (11) and (12) borrowed from Ref. 9) and the integrated algorithm. To this end, we use Tables I–IV comprising the rms errors of the spatial extrapolation of the average temperature $\langle T \rangle_{h_0, h}$ and the average zonal $\langle V_x \rangle_{h_0, h}$ and meridional $\langle V_y \rangle_{h_0, h}$ wind components. They also comprise the probability (P) that the prediction errors of the same parameters are less than the preset values.

It should be immediately emphasized that in Tables I–IV we present the errors estimated for two distances (180 and 250 km) between the prediction point and the nearest station for which we have the data of radiosonde observations. In our case, we took the stations in Warszawa and Minsk as prediction points and the stations in Brest and Kaunas as points nearest to them, respectively (see Fig. 1 of Ref. 9).

We note that these distances are significantly less than the step of regular network equal to 300 km, which is commonly used in numerical weather forecast schemes.¹

Analysis of the data presented in Tables I–IV showed that:

– First, the integrated algorithm for spatial extrapolation of the mesometeorological fields based on the procedure of optimal integration of the two alternative methods (OEM and MMCA) is fairly efficient for solving the problem of numerical prediction of the average temperature and zonal and meridional wind components (especially for extrapolation of the parameters $\langle V_x \rangle_{h_0, h}$ and $\langle V_y \rangle_{h_0, h}$),

because regardless of the season and the atmospheric layer, the probabilities P that the errors are, for example, less than ± 20 and ± 2 m/s are 0.63–0.81 for the average temperature) and 0.70–0.80 (for the wind components).

– Second, the integrated algorithm for spatial prediction significantly increases (in comparison with the method of optimal extrapolation) the quality of prediction. Actually, from Tables I–IV it follows that the probabilities that the error of spatial prediction of the examined parameters made by the method of optimal extrapolation is less than ± 20 and ± 2 m/s (regardless of the season and the atmospheric layer) is greater by 0.07–0.11 for the average temperature, by 0.11–0.15 for the average zonal wind component, and by 0.14–0.29 for the average meridional wind component than the corresponding values for prediction made by the integrated algorithm.

– Third, as expected, the rms error (δ) of spatial extrapolation of the parameters $\langle T \rangle_{h_0, h}$, $\langle V_x \rangle_{h_0, h}$, and $\langle V_y \rangle_{h_0, h}$ increased in magnitude (although insignificantly) as the distance between the prediction point and the point nearest to it increased from 180 to 250 km. Only in winter, when the average zonal wind component was extrapolated, we observed the reverse dependence, that is, the rms error decreased with increasing distance between these points for the atmospheric layers whose upper boundary was at $h > 1200$ m. This salient feature was due to the fact that in winter (unlike summer) the intense western air mass transport was observed for the temperate latitudes of the Northern Hemisphere.¹⁴ And because the direction of spatial extrapolation toward Minsk coincided with the direction of air mass transport and on the contrary, the extrapolation toward Warszawa was made in the opposite direction, this very circumstance violated the direct proportionality between the rms error (δ) of the predictable average zonal wind component and the distance.

Thus, the results of numerical experiments on statistical estimation of the quality of spatial extrapolation of mesometeorological fields made for the temperature and wind field, as an example, have allowed us to draw two conclusions important for practice, namely:

– The integrated approach to the spatial extrapolation of mesometeorological fields into the territory uncovered with aerological observational data yields much better results than the method of optimal extrapolation used for numerical reconstruction of the fields.

– The integrated algorithm for spatial extrapolation of mesometeorological temperature and wind fields is quite promising (the probabilities that the errors of prediction of these meteorological parameters are less than $\pm 2^\circ\text{C}$ (± 2 m/s) mostly exceeded 0.70 even for distances as long as 250 km). It can be successively implemented for automated systems of meteorological support for local monitoring of atmospheric pollutants.

TABLE I. The rms errors δ and the probabilities p that the error of spatial prediction of the values of temperature and zonal and meridional wind components averaged over the atmospheric layers are less or greater than the preset values. The spatial prediction was made by the method of optimal extrapolation (1) and by the integrated method (2) for distances up to 180 km (winter).

Layer, m	Probability, p										δ	
	$\leq \pm 1$		$\leq \pm 2$		$\leq \pm 3$		$\leq \pm 4$		$> \pm 4$			
	1	2	1	2	1	2	1	2	1	2	1	2
a) Temperature, °q												
0-200	40	49	70	78	81	89	90	99	10	1	1.8	1.4
0-400	38	49	69	77	80	89	90	99	10	1	1.9	1.4
0-800	36	47	69	77	79	89	89	98	11	2	2.0	1.5
0-1200	35	46	68	76	78	88	88	98	12	2	2.0	1.7
0-1600	34	44	67	75	76	87	87	98	13	2	2.1	1.8
0-2000	33	43	65	74	75	86	86	98	14	2	2.2	1.8
0-2400	33	42	65	72	74	85	85	97	15	3	2.3	1.8
0-3000	32	40	64	70	73	84	84	95	16	5	2.6	2.1
0-4000	32	39	62	69	72	82	83	94	17	6	2.9	2.3
0-5000	30	37	61	69	71	91	83	92	17	8	3.1	2.5
0-6000	28	34	60	68	70	80	82	91	18	9	3.3	2.6
0-8000	27	33	59	67	69	79	81	90	19	10	3.5	2.7
b) Zonal wind, m/s												
0-200	22	42	55	80	73	83	87	97	13	3	3.2	1.8
0-400	22	42	55	79	74	83	87	97	13	3	3.3	1.9
0-800	23	41	56	78	75	82	87	97	13	3	3.3	2.0
0-1200	24	40	57	77	74	81	86	95	14	5	3.2	2.1
0-1600	25	40	57	77	76	81	86	95	14	5	3.1	2.2
0-2000	24	40	58	77	76	80	87	94	13	6	3.0	2.3
0-2400	23	40	60	76	76	80	86	94	14	6	2.8	2.3
0-3000	25	39	62	75	76	80	87	94	13	6	2.8	2.3
0-4000	26	39	63	75	76	80	86	93	14	7	2.7	2.4
0-5000	28	38	64	75	75	81	87	92	13	8	2.6	2.4
0-6000	27	37	63	75	76	80	89	92	11	8	2.6	2.4
0-8000	28	37	63	75	77	81	88	92	12	8	2.6	2.4
c) Meridional wind, m/s												
0-200	24	40	54	1	72	86	89	95	11	5	2.7	2.0
0-400	24	40	54	78	72	85	89	95	11	5	2.8	2.1
0-800	25	39	54	77	73	84	88	94	12	6	2.8	2.1
0-1200	26	38	54	78	74	83	88	94	12	6	2.9	2.1
0-1600	25	37	53	77	75	82	89	93	11	7	2.9	2.2
0-2000	25	36	53	76	76	82	87	93	13	7	3.0	2.2
0-2400	25	36	55	76	77	82	88	92	12	8	2.9	2.2
0-3000	26	37	56	75	77	82	88	92	12	8	2.8	2.2
0-4000	26	36	57	75	77	83	90	92	10	8	2.7	2.2
0-5000	27	35	58	75	77	83	91	92	9	8	2.7	2.2
0-6000	28	34	57	75	77	82	91	92	9	8	2.7	2.2
0-8000	27	33	58	74	76	81	91	92	9	8	2.7	2.2

TABLE II. The rms errors δ and the probabilities p that the errors of spatial prediction of the values of temperature and zonal and meridional wind components averaged over the atmospheric layers are less or greater than the preset values. The spatial prediction was made by the method of optimal extrapolation (1) and by the integrated method (2) for distances up to 180 km (summer).

Layer, m	Probability, p										δ	
	$\leq \pm 1$		$\leq \pm 2$		$\leq \pm 3$		$\leq \pm 4$		$> \pm 4$			
	1	2	1	2	1	2	1	2	1	2	1	2
	a) Temperature, °q											
0-200	44	51	75	81	87	94	92	100	8	0	1.5	1.2
0-400	40	50	73	81	86	94	92	100	8	0	1.5	1.3
0-800	38	49	72	80	84	94	91	100	9	0	1.6	1.3
0-1200	37	48	72	79	82	93	90	100	10	0	1.7	1.5
0-1600	3.5	46	70	79	81	92	88	98	12	2	1.9	1.6
0-2000	35	44	69	77	80	92	88	98	12	2	2.0	1.7
0-2400	34	44	69	77	78	91	87	96	13	4	2.0	1.8
0-3000	33	40	68	76	77	90	88	95	12	5	2.5	2.0
0-4000	33	38	67	75	77	89	85	95	15	5	2.7	2.2
0-5000	31	37	66	74	76	89	84	93	16	7	2.8	2.3
0-6000	30	35	65	73	75	88	83	92	17	8	3.0	2.5
0-8000	28	35	63	72	72	87	82	92	18	8	3.2	2.6
	b) Zonal wind, m/s											
0-200	26	41	60	78	76	86	86	95	14	5	2.8	1.8
0-400	26	40	60	78	77	86	86	95	14	5	2.8	2.0
0-800	25	40	59	77	76	86	87	94	13	6	2.7	2.1
0-1200	26	41	60	77	78	86	86	94	14	6	2.7	2.2
0-1600	24	40	58	76	78	85	86	94	14	6	2.7	2.2
0-2000	24	39	57	76	79	84	87	94	13	6	2.6	2.2
0-2400	24	39	59	75	80	84	88	94	12	6	2.6	2.2
0-3000	24	39	60	75	80	84	88	93	12	7	2.6	2.2
0-4000	25	39	62	74	81	83	88	93	12	7	2.5	2.2
0-5000	26	39	62	74	80	83	88	93	11	7	2.5	2.2
0-6000	26	38	62	73	80	83	88	93	12	7	2.6	2.2
0-8000	26	38	61	73	79	82	88	92	12	8	2.5	2.2
	c) Meridional wind, m/s											
0-200	21	39	49	76	70	84	85	93	15	7	3.0	2.2
0-400	22	39	50	76	71	83	85	92	15	8	3.1	2.3
0-800	22	40	50	75	72	83	86	93	14	7	3.1	2.3
0-1200	21	39	51	74	72	82	86	92	14	8	3.1	2.4
0-1600	22	38	52	74	71	81	85	91	15	9	3.0	2.4
0-2000	23	38	53	74	72	81	87	90	13	10	2.9	2.4
0-2400	22	37	54	73	73	81	87	90	13	10	2.9	2.3
0-3000	23	37	54	73	73	81	87	90	13	10	2.9	2.4
0-4000	23	36	54	72	72	80	87	90	13	10	2.8	2.4
0-5000	24	36	55	73	73	80	88	90	12	10	2.8	2.4
0-6000	24	35	56	74	73	79	87	90	13	10	2.8	2.3
0-8000	24	35	55	73	73	79	87	90	13	10	2.8	2.4

TABLE III. The rms errors δ and the probabilities p that the error of spatial prediction of the values of temperature and zonal and meridional wind components averaged over the atmospheric layers are less or greater than the preset values. The spatial prediction was made by the method of optimal extrapolation (1) and by the integrated method (2) for distances up to 250 km (winter).

Layer, m	Probability, p										δ	
	$\leq \pm 1$		$\leq \pm 2$		$\leq \pm 3$		$\leq \pm 4$		$> \pm 4$			
	1	2	1	2	1	2	1	2	1	2	1	2
a) Temperature, °q												
0-200	28	38	63	71	72	85	87	96	13	4	2.0	1.6
0-400	27	38	62	70	72	85	87	96	13	4	2.1	1.7
0-800	26	38	62	70	71	84	86	95	14	5	2.2	1.8
0-1200	26	37	61	70	70	83	85	95	15	5	2.3	1.9
0-1600	26	37	60	70	69	82	85	95	15	5	2.5	1.9
0-2000	25	36	59	70	68	80	83	94	17	6	2.7	2.0
0-2400	24	36	58	68	67	80	82	94	18	6	2.9	2.1
0-3000	24	35	56	66	66	79	82	94	18	6	3.0	2.2
0-4000	23	34	56	65	65	78	81	93	19	7	3.2	2.4
0-5000	22	33	55	65	64	76	80	92	20	8	3.3	2.6
0-6000	22	33	53	64	63	75	79	90	21	10	3.4	2.7
0-8000	20	32	53	63	62	75	78	90	22	10	3.6	2.8
b) Zonal wind, m/s												
0-200	24	38	61	75	76	84	85	93	15	7	2.9	1.9
0-400	24	38	60	74	76	84	86	93	14	7	2.9	2.0
0-800	24	38	60	74	77	84	86	92	14	8	3.0	2.0
0-1200	23	37	61	75	77	84	85	91	15	9	3.0	2.0
0-1600	24	37	62	75	76	84	85	91	15	9	2.9	2.0
0-2000	25	38	61	75	77	83	85	91	15	9	2.8	2.1
0-2400	22	37	60	75	78	82	84	91	16	9	2.8	2.1
0-3000	23	36	59	73	78	82	83	90	17	10	2.9	2.2
0-4000	24	35	59	72	78	83	83	90	17	10	3.0	2.2
0-5000	22	35	58	72	77	82	84	90	18	10	3.1	2.2
0-6000	22	34	57	72	76	82	83	90	18	10	3.2	2.2
0-8000	22	34	58	73	75	83	82	89	17	11	3.4	2.2
c) Meridional wind, m/s												
0-200	25	36	50	75	68	83	82	93	18	7	3.2	2.1
0-400	24	36	50	75	69	83	82	93	18	7	3.2	2.2
0-800	24	37	49	74	70	83	83	93	17	7	3.3	2.2
0-1200	23	37	49	73	71	82	83	92	17	8	3.4	2.3
0-1600	23	36	48	72	72	81	84	91	16	9	3.4	2.4
0-2000	24	35	48	71	74	80	84	90	16	10	3.3	2.4
0-2400	24	35	48	71	75	80	84	90	16	10	3.2	2.4
0-3000	23	35	47	71	75	80	83	90	17	10	3.1	2.4
0-4000	24	34	48	70	75	80	84	90	16	10	3.0	2.4
0-5000	25	33	49	70	75	80	85	90	15	10	3.0	2.4
0-6000	25	32	49	70	74	81	84	90	16	10	3.1	2.5
0-8000	25	32	49	70	75	81	83	89	17	11	3.0	2.5

TABLE IV. The rms errors δ and the probabilities p that the error of spatial prediction of the values of temperature and zonal and meridional wind components averaged over the atmospheric layers are less or greater than the preset values. The spatial prediction was made by the method of optimal extrapolation (1) and by the integrated method (2) for distances up to 250 km (summer).

Layer, m	Probability, p										δ	
	$\leq \pm 1$		$\leq \pm 2$		$\leq \pm 3$		$\leq \pm 4$		$> \pm 4$			
	1	2	1	2	1	2	1	2	1	2	1	2
	a) Temperature, °q											
0-200	39	52	70	79	84	92	90	100	10	0	1.8	1.3
0-400	38	51	70	79	83	92	89	100	11	0	1.8	1.3
0-800	37	48	68	78	82	91	89	100	11	0	1.9	1.4
0-1200	36	46	67	77	81	90	88	99	12	1	2.0	1.6
0-1600	35	46	66	76	79	90	88	97	12	3	2.1	1.7
0-2000	35	45	65	75	78	89	87	97	13	3	2.3	1.8
0-2400	34	43	64	75	77	88	86	95	14	5	2.3	1.9
0-3000	33	42	64	75	76	87	85	94	15	6	2.6	2.1
0-4000	32	40	63	74	75	87	84	94	16	6	2.9	2.3
0-5000	30	39	62	73	74	86	83	92	17	8	2.9	2.4
0-6000	28	37	62	72	73	85	82	91	18	9	3.1	2.6
0-8000	27	36	61	72	73	84	80	90	20	10	3.4	2.8
	b) Zonal wind, m/s											
0-200	23	42	58	75	71	84	82	92	18	8	3.0	1.9
0-400	23	41	59	75	72	84	81	92	19	8	3.1	2.1
0-800	24	42	60	74	72	84	82	92	18	8	3.1	2.2
0-1200	24	40	59	73	72	83	82	92	18	8	2.9	2.3
0-1600	24	39	60	73	72	82	81	92	19	8	2.9	2.3
0-2000	25	38	60	73	71	84	82	92	18	8	2.8	2.3
0-2400	25	37	59	73	72	83	83	93	17	7	2.8	2.3
0-3000	25	36	60	72	74	83	83	92	17	8	2.9	2.3
0-4000	24	36	60	72	74	81	84	93	16	7	2.8	2.3
0-5000	24	35	60	71	74	82	83	93	17	7	2.8	2.3
0-6000	25	35	59	70	73	82	83	92	17	8	2.8	2.3
0-8000	25	35	59	70	73	81	82	93	18	7	2.7	2.4
	c) Meridional wind, m/s											
0-200	20	41	44	73	66	82	80	91	20	9	3.5	2.3
0-400	20	42	45	73	65	83	80	91	20	9	3.5	2.4
0-800	20	40	44	73	65	82	81	92	19	8	3.4	2.4
0-1200	21	41	46	72	66	81	82	91	18	9	3.5	2.5
0-1600	20	41	47	71	67	80	82	90	18	10	3.4	2.5
0-2000	22	40	47	71	68	80	82	90	18	10	3.4	2.4
0-2400	21	38	48	71	69	80	81	90	19	10	3.4	2.4
0-3000	21	39	49	71	70	80	81	88	19	12	3.3	2.6
0-4000	20	39	50	71	71	81	82	87	18	13	3.2	2.6
0-5000	22	39	50	71	72	82	83	86	17	14	3.2	2.6
0-6000	22	37	51	70	73	81	84	86	16	14	3.1	2.7
0-8000	23	37	51	70	74	80	84	86	16	14	3.0	2.7

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