OPTICAL MEASUREMENTS OF A LARGE-APERTURE SURFACE SHAPE BY THE OVERLAP-SCANNING METHOD

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It is shown that the measuring base of an instrument testing a large-aperture surface shape by the overlap-scanning method can be considered as a filter of spatial frequencies. In this case, the complex amplitude of the filter is described by a function with parameters depending on the measuring base length and configuration of the instrument. Some results supporting appropriateness of this model in practice are presented.

The overlap-scanning method is rather widely used in optical measurements.[1–5] It consists in subdividing a tested surface (Fig. 1) of a large aperture into k parts, every of which is tested sequentially with a small-aperture instrument. The total error ΔN for a profile of the whole surface depends on the number k of instrument stops and is estimated as follows [1]:

$$\Delta N = \Delta n \sqrt{k},$$

where Δn is the error of profile estimation for one part. Degradation in measurement accuracy is caused by loss of information about a position of the measuring base (elementary aperture) in space. As a result, filtration of spatial frequencies in the shape of the tested surface occurs, and the necessity to connect results of individual elementary measurements arises. The technique and algorithms for the connecting procedure are subjects of most papers on the overlap-scanning method.[6–9] From the practical point of view, filtering properties of an elementary aperture are of no less interest as well.

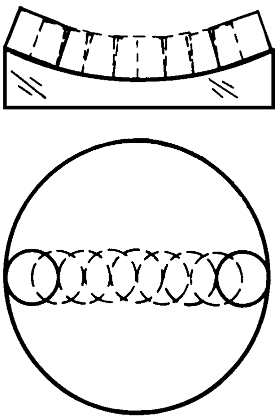

FIG. 1. Geometry of large-aperture surface testing by making use of small apertures.

Interference and shadow schemes, which are traditionally used for surface shape tests in optical instrument making, can be considered as integral ones in the sense that tests yield some information about deviation of the tested surface as a whole from a certain nominal reference surface. From this point of view, the testing scheme utilizing the overlap-scanning method should be considered as a differential technique. In this case, the concept of a surface shape (profile) becomes indefinite because one can say only about some its realization depending on random factors associated with selection of the reference surface at each particular stop of the elementary aperture. In view of special importance of the latter, below the elementary aperture will be called the measuring base.

Since the measuring base changes its spatial orientation in the overlap-scanning process, it should be expected that some information about the tested surface may be lost. It is the information about large-scale deviations of the profile; however, the information about small deviations will be revealed rather comprehensively. Therefore, we can suppose that spatial frequencies of the tested surface are filtered in the process of measurements by the overlap-scanning method.

Filtering properties of the measuring base are not always negative. With *a priori* information about the spectrum of spatial frequencies, the size of the measuring base can be selected so that all the required information about the surface quality will be preserved.

Let us consider Fig. 2. Suppose that a surface, whose profile is described by the function $y(t)$, where t is the coordinate along the surface, is studied with the measuring base of length $2T$. The ends of the measuring base are fixed at the surface points t_B and t_E. Then, as the measuring base moves, the instrument measures the modified function $Y(t)$ described by the relation

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$$Y(t) = \left[\frac{y(t_{\rm B}) + y(t_{\rm E})}{2} - y(t)\right] \cos\alpha, \qquad (1)$$

where α is the angle between the measuring base and the axis; $Y(t)$ is the sag of the function $y(t)$ within the measuring base.

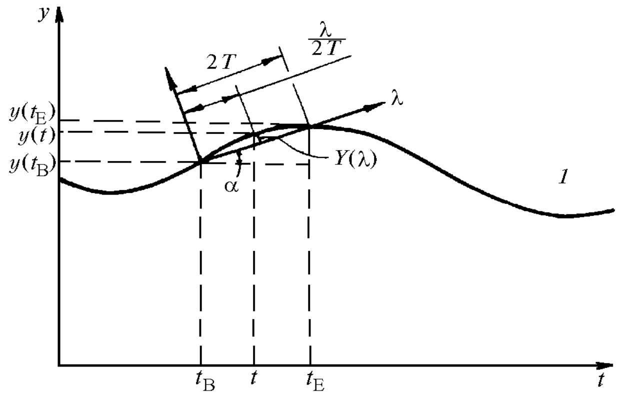

FIG. 2. On derivation of Eq. (3): the measured profile (1).

Coordinates of the points of contact of the surface and the measuring base can be written in the following form:

$$t_{\rm B} = t - 2T\lambda \cos\alpha ,$$
$$t_{\rm E} = t + 2T(1 - \lambda) \cos\alpha ,$$

where λ is the coordinate of the point t about the leading edge of the measuring base divided by its length.

Applying the Fourier transform to the function $Y(t)$ and using the well-known theorems of the Fourier analysis,[10] we obtain

$$F(Y(t)) = F(y(t))\, \varphi(\xi) , \qquad (2)$$

where $F(y(t))$ and $F(Y(t))$ are the Fourier transforms of the functions $y(t)$ and $Y(t)$, respectively; $\varphi(\xi)$ is the function describing the filter transmittance,

$$\varphi(\xi) =$$
$$= \left[\frac{\exp(2\pi i\xi\lambda 2T\cos\alpha) + \exp(-2\pi i\xi(1-\lambda)2T\cos\alpha)}{2} - 1\right] \times$$
$$\times \cos\alpha . \qquad (3)$$

This function depends on the parameters of the measuring base. They are the relative coordinate $= (y - y{\rm B})/2T$, the angle α as a characteristic of the surface curvature, and the length of the measuring base $2T$. Fulfillment of Eq. (2) proves the above-stated supposition about the filtering properties of the measuring base.

With these parameters known, the measured function $y(t)$ can be reconstructed from values of the function $Y(t)$ by making use of the well-known relation[10]

$$y(t) = Y(t) \otimes F(1/\varphi(\xi)) . \qquad (4)$$

Restrictions in applying relation (4) are evident:

− the domain of the functions $y(t)$ and $Y(t)$ must be $(-\infty; \infty)$, that is, the number k of measuring base stops must be sufficiently large. Signals with a finite domain require solution of the problem about their correct analytic continuation;

− spatial frequencies $\xi_{\rm ir}$, for which $\varphi(\xi_{\rm ir})$ vanishes, must be excluded from the domain. Since the values of $\xi_{\rm ir}$ depend on parameters of the measuring base, the latter must be selected based on *a priori* information. The signal power spectrum at these points must be zero.

To study properties of the function $\varphi(\xi)$, let us assume the following:

− the tested surface is plane, i.e., $\alpha \approx 0$ and $\cos\alpha \approx 1$;

− the sag is measured at only one point lying in the middle of the measuring base, that is, $\lambda = 1/2$.

In this case, from Eq. (3) we have the complex amplitude

$$\varphi(\xi) = \frac{\exp(2\pi i\xi T) + \exp(-2\pi i\xi T)}{2} - 1 =$$
$$= \cos 2\pi\xi T - 1 . \qquad (5)$$

The amplitude−frequency characteristic of this filter

$$|\varphi(\xi)|^2 = 1.5 + \frac{1}{2}\cos(4\pi\xi T) - 2\cos(2\pi T\xi)$$

is presented in Fig. 3.

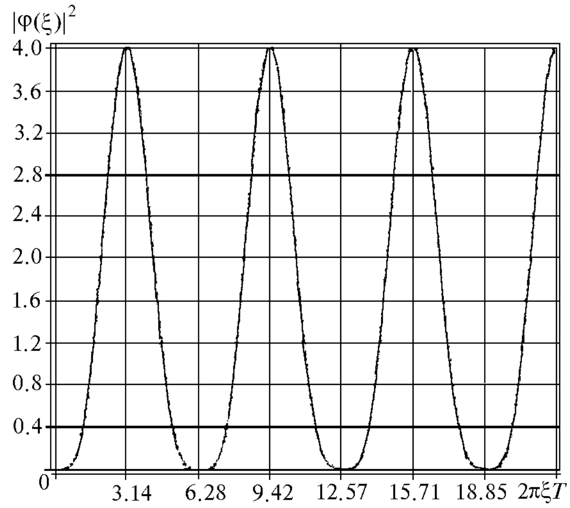

FIG. 3. The amplitude−frequency characteristic of the filter described by Eq. (5).

The filtering properties of the measuring base were simulated by the harmonic function $y(t) = \cos 2\pi\xi T$. A result of measurements of the surface profile described by the function $y(t)$ with the measuring base of length $2T$ was simulated using relations (1), (2), and (5). Simulation results are presented in Fig. 4 for different values of ξ and T. Analysis of these plots shows sufficient identity of relations (1) and (4) for $T\xi \neq n$, where $n = 0, 1, 2 \ldots$. For $T\xi \approx n$, due to error accumulation during calculation of the convolution integral, high-frequency oscillations of the function $Y(t)$ arise (Fig. 4*d*, the right-hand plot).

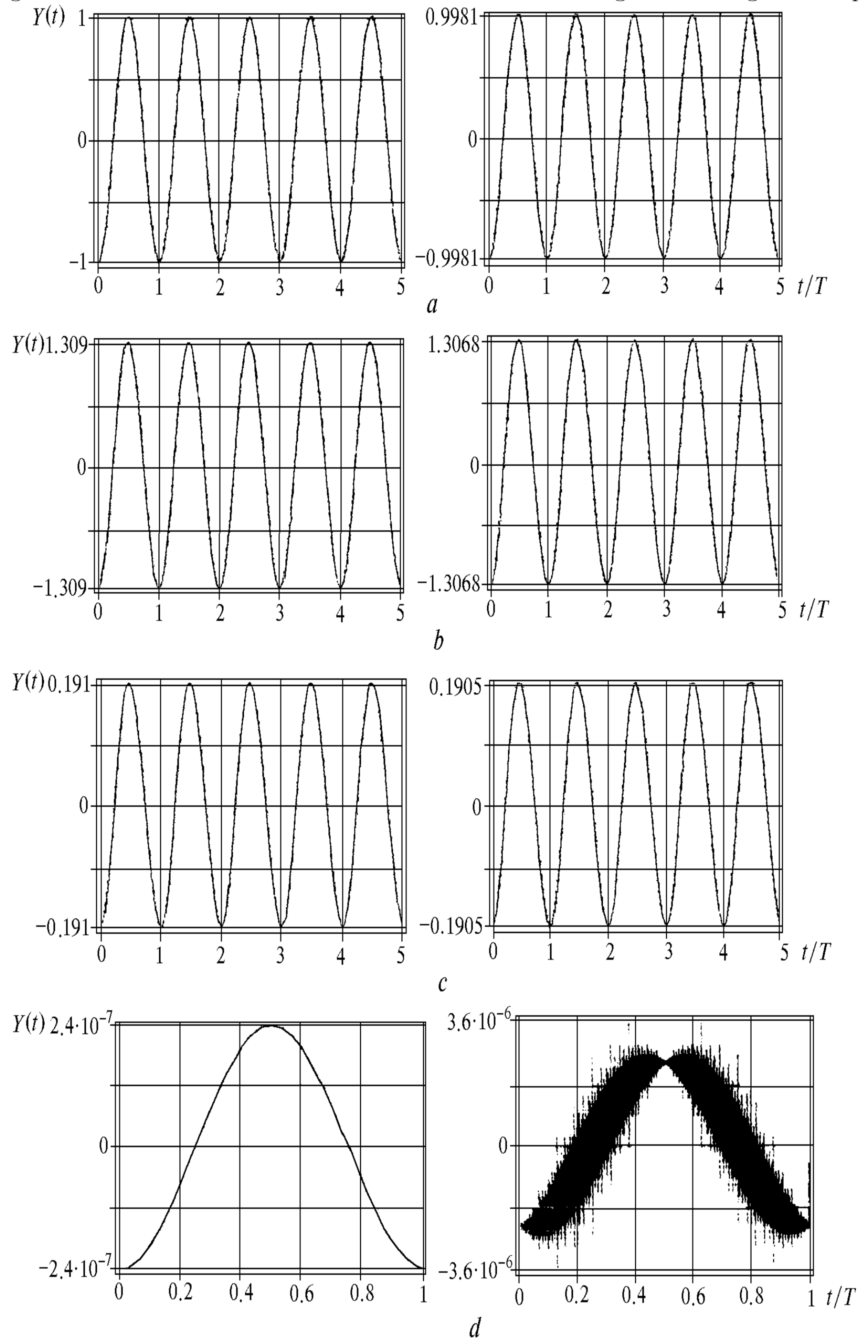

FIG. 4. Results of numerical simulation of measurements of the harmonic signal $y(x) = \cos 2\pi\xi T$ *with the measuring base of length* $2T$*:* $\xi T = 0.25$ *(a);* $\xi T = 0.3$ *(b);* $\xi T = 0.1$ *(c);* $\xi T = 1.0$ *(d). The left-hand plots are obtained from relation (1), that is, by direct simulation; the right-hand plots are obtained from convolution by Eq. (4) with the amplitude-frequency characteristic of the filter.*

The possibility to filter out undesired spatial frequencies is an interesting application of the measuring base. Such a situation arises in surface shape testing as applied to, e.g., extended objects. Extended objects differ from others by significant dependence of their shape on arrangement of supports, on which such an object rests. As supports are changed, elastic deformation of the object changes under the action of gravity. Therefore, the concept of the surface shape becomes indefinite. However, while such objects (for instance, rails) are in use, they must be mounted on supports, which can be adjusted to compensate for these random elastic deformations. Against this background, regular deformations of the profile caused by technology imperfections play a significant part. To overcome the ambiguity, a technique to measure profiles of extended objects can be proposed, in which the measuring base plays an important part. The proposed technique features the following:

– the surface profile of the extended object is tested with the measuring base;

– the length of the measuring base must be chosen so that the tested object can be considered rigid within it, that is, the sag of the extended object on this base is independent of support arrangement.

In other words, the measuring base of length $2T$ must filter out spatial frequencies ξ_{ir} of the real profile, which represent irregular deformations of the object. Complete filtration of frequencies ξ_{ir} means that the amplitude–frequency characteristic (see Fig. 3) vanishes at the points, where the condition $\xi_{ir}T = n$ is fulfilled. At the same time, the filter must pass frequencies ξ_r corresponding to regular deformations. The difference between ξ_{ir} and ξ_r determines the success of measurements. If the difference is significant, then taking $T = 1/\xi_{ir}$ we can succeed in the measurements.

As an example, we present some results obtained for a system detecting non-straightness of rails in the process of transportation on a roller conveyer. Since rollers are wore irregularly, supports are activated randomly what results in irregular deformations of the roller face of a rail. Thus, to filter out this frequency efficiently, the length of the measuring base must be chosen so that $\xi_{ir}T = 1$, therefore,

$$2T = 2/\xi_{ir} = 1.5.$$

The instrument with such measuring base allows the spatial frequency ξ_r satisfying the condition

$$\xi_r^{0.5} = \frac{0.5}{T} = \frac{0.5}{0.75} = \frac{1}{1.5}\,.$$

to be revealed most efficiently on the profile.

The transmission coefficient for this frequency is $|\varphi(\xi_r^{0.5})| = 1 - \cos(2\pi\, 0.5) = 2$.

The spectrum of defects due to technology imperfections is concentrated just near this frequency. The most interesting frequency of the regular component of deviations from straightness, from the technological point of view,

$$\xi_r^{0.23} = \frac{1}{3.2}$$

is transmitted by the measuring system with the transmission coefficient (5) $|\varphi(\xi_r^{0.23})| = 1 - \cos(2\pi\, 0.23) \approx \approx 1$. Thus, this frequency (Fig. 4*a*) is most adequately represented by the measuring instrument with the base $2T = 1.5$ m.

The described method has been implemented in a rail straightness test system in the rail-rolling mill of the Kuznetsk metallurgic plant.[12] Operation and metrological examination of the system have demonstrated its high efficiency for revealing "bottle necks" of the technology from the viewpoint of causes of rail waviness.

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