# On the contrast of images of small-size objects observed through a scattering medium with a viewing system with an active illumination 

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#### Abstract

We have obtained relationships for calculating contrast of an underwater object observed through calm sea surface with a continuous and pulse illumination. The noise due to backscatter from the medium below the object is calculated with the allowance for screening effect of the object. Image contrast is studied as a function of the object's characteristics and parameters of the observation system. At low reflectivity of the object the contrast is not a monotonic function of the object's depth.


The theory of underwater vision is a well developed branch of the ocean optics. Its main points, conclusions, and recommendations on optimal construction of observation systems (OS) have been formulated in Refs. 1 and 2. It is known that image quality, which is characterized by the image contrast and signal-to-noise ratio, significantly depends on the ratio between the power of a signal, i.e., power of light radiation reflected by an object toward the OS, and the power of backscattering noise (BN), i.e., power of radiation reflected by the scattering medium toward this same OS. The part of noise coming from medium depths above the object in the image formation is studied sufficiently well. However, this cannot be said about the noise coming from water layers that are behind the object. It is evident that the part of this BN component may exhibit a stronger effect on the detection of an object as a whole against the background noise from the medium, as compared with that in distinguishing details within the image. The difficulty of calculating the power of the "screened" BN (this term is conventional) is in the necessity of taking into account the screening effect of the object upon the water layers below it. In this paper, we propose a way to solve this problem and demonstrate it by an example of analysis for a round-shaped smallsize object in sea water. The object is observed through a smooth surface with active illumination of the object.

## 1. The formula of image transfer

Figure 1 presents the observation scheme. The observation system comprises a light source of brightness $B_{0}$ with the aperture function $D_{1}\left(\mathbf{r}_{1}, \Omega_{1}\right)$ and the imaging optical system integrated with the source and having the aperture function $D_{2}\left(\mathbf{r}_{1}^{\prime}, \boldsymbol{\Omega}_{1}^{\prime}\right)$ that are placed at a height $H$ over the water surface. The system illuminates and views a plane bounded Lambertian object which is placed in a homogeneous scattering medium at a depth $h_{0}$. The distribution of the reflection coefficient is $R_{0}\left(\mathbf{r}_{3}\right)$ (here and below $\mathbf{r}_{i}$ are coordinates of points in the planes $z_{i}, \Omega_{i}$ are
projections of unit vectors $\boldsymbol{\Omega}_{i}^{0}$ onto the plane $z=$ const).

If the OS is aligned to view the middle of the object, the power of the reflected signal received is made up of three components:

$$
\begin{equation*}
P_{i}=P_{\mathrm{ob}}+P_{\mathrm{bs}}^{0}+P_{\mathrm{bs}}^{\mathrm{e}}, \tag{1}
\end{equation*}
$$

where $P_{\mathrm{ob}}$ is power of radiation reflected by the object's surface; $P_{\mathrm{bs}}^{0}$ is the power of the noise due to the backscatter from the medium in front of the object; $P_{\mathrm{bs}}^{\mathrm{e}}$ is the BN power from the medium behind the object (with the allowance for its screening effect).


Fig. 1. Observation scheme.
If the OS is oriented to view the medium volume beyond the object, the expression for the received power takes the form

$$
\begin{equation*}
P_{\mathrm{bs}}=P_{\mathrm{bs}}^{0}+P_{\mathrm{bs}}^{\infty}, \tag{2}
\end{equation*}
$$

where $P_{\mathrm{bs}}^{\infty}$ is the power of noise due to the backscatter from the medium below the depth $h_{0}$.

The general expression for power of radiation received from the object has been derived in Ref. 3:

$$
\begin{equation*}
P_{\mathrm{ob}}=\frac{B_{0}}{\pi} \iint_{-\infty}^{+\infty} R_{0}\left(\mathbf{r}_{3}\right) E_{1}\left(\mathbf{r}_{3}\right) E_{2}\left(\mathbf{r}_{3}\right) \mathrm{d} \mathbf{r}_{3} \tag{3}
\end{equation*}
$$

Here $E_{1,2}$ are the distributions of illumination at the depth $h_{0}$, which is created by sources with the aperture functions $D_{1,2}$ :

$$
\begin{gathered}
E_{1,2}\left(\mathbf{r}_{3}\right)=\iint_{-\infty}^{+\infty} I_{1,2}\left(\mathbf{r}_{3}, \mathbf{\Omega}_{3}, h_{0}\right) \mathrm{d} \boldsymbol{\Omega}_{3} \\
I_{1}(\cdot)=\int \ldots \int_{-\infty}^{+\infty} D_{1}\left(\mathbf{r}_{1}, \Omega_{1}\right) B_{\mathrm{c}}\left(\mathbf{r}_{2} \rightarrow \mathbf{r}_{3}, \mathbf{\Omega}_{1} \rightarrow \Omega_{3}, h_{0}\right) \times \\
\times \delta\left(\mathbf{r}_{2}-\mathbf{r}_{1}-H \mathbf{\Omega}_{1}\right) \mathrm{d} \mathbf{r}_{1} \mathrm{~d} \mathbf{r}_{2} \mathrm{~d} \boldsymbol{\Omega}_{3}
\end{gathered}
$$

the scattering phase function of the medium $B_{c}$ describes the brightness at the point $\mathbf{r}_{3}$ of the scattering medium, at the distance $h_{0}$ from the source, along the direction $\Omega_{3}$ when it is irradiated by a point monodirected source being at the point $\mathbf{r}_{2}$ and emitting in the direction $\boldsymbol{\Omega}_{1}$; within the small-angle approximation, the medium function is described by the expression

$$
B_{\mathrm{c}}(\bullet)=B_{\mathrm{c}}\left(\mathbf{r}_{3}-\mathbf{r}_{2}-h_{0} \Omega_{1}, \Omega_{3}-\mathbf{\Omega}_{1}, h_{0}\right)
$$

(for brevity, refraction effects at the boundary of the air-water interface are neglected).

Let us write the spatiotemporal spectrum of the brightness $I_{1}$ as:

$$
\begin{equation*}
F_{1}^{0}\left(\mathbf{k}, \mathbf{p}, h_{0}\right)=F_{1}\left(\mathbf{k}, \mathbf{p}+\mathbf{k} L_{0}\right) F_{\mathrm{b}}\left(\mathbf{k}, \mathbf{p}, h_{0}\right) \tag{4}
\end{equation*}
$$

where $F_{1}$ and $F_{\mathrm{b}}$ are the Fourier images of the functions $D_{1}$ and $B_{\mathrm{c}} ; L_{0}=H+h_{0}$ (the spectrum of brightness $I_{2}$ at the receiver is described similarly by changing the index 1 for 2 ).

The expression (3) can be written in the "frequency" form:

$$
\begin{align*}
& P_{\mathrm{ob}}=\frac{B_{0}}{\pi}(2 \pi)^{-4} \int \ldots \int_{-\infty}^{+\infty} F_{0}\left(\mathbf{k}_{1}+\mathbf{k}_{2}\right) \times \\
& \quad \times F_{E_{1}}\left(\mathbf{k}_{1}, h_{0}\right) F_{E_{2}}\left(\mathbf{k}_{2}, h_{0}\right) \mathrm{d} \mathbf{k}_{1} \mathrm{~d} \mathbf{k}_{2} \tag{5}
\end{align*}
$$

where $F_{0}, F_{E_{1}}$, and $F_{E_{2}}$ are the Fourier images of the functions $R_{0}, E_{1}$, and $E_{2}$, and the function

$$
\begin{gathered}
F_{E_{1,2}}\left(\mathbf{k}, h_{0}\right) \equiv F_{1,2}^{0}\left(\mathbf{k}, 0, h_{0}\right)=F_{1,2}\left(\mathbf{k}, \mathbf{k} L_{0}\right) F_{\mathrm{e}}\left(\mathbf{k}, h_{0}\right) \\
F_{\mathrm{e}}\left(\mathbf{k}, h_{0}\right)=F_{\mathrm{b}}\left(\mathbf{k}, 0, h_{0}\right)
\end{gathered}
$$

describes the modulation transfer function (MTF) of a water layer with the thickness $h_{0}$.

To perform further calculations, it is necessary to specify the form of the optical transfer functions of the medium, object, light source, and OS photodetector entering into Eqs. (3)-(5).

## 2. Models of the source, receiver, medium, and the object

Let Gaussian functions
$F_{1,2}(\mathbf{k}, \mathbf{p})=\sum_{1,2} \Delta_{1,2} \exp \left[-\left(\sum_{1,2} k^{2}+\Delta_{1,2} p^{2}\right) / 4 \pi\right]$,
where $\Sigma_{1,2}$ is the aperture area of the source (receiver); $\Delta_{1,2}$ is the solid angle of radiation (reception), be used as optical transfer functions of the source and receiver.

The Fourier image of an automodel solution of the equation of radiation transfer for brightness distribution over the cross section of a narrow light beam in a medium with strongly anisotropic scattering ${ }^{4}$ is used as the optical transfer function of the scattering layer of water $F_{\mathrm{b}}$ :

$$
\begin{gather*}
F_{\mathrm{b}}\left(\mathbf{k}, \mathbf{p}, h_{0}\right)=A_{0} \exp \left(-x h_{0}\right) \times \\
\times \exp \left[-\left(G_{0} k^{2}+2 B_{0} \mathbf{k} \mathbf{p}+Q_{0} p^{2}\right) / 4 \pi\right] \tag{7}
\end{gather*}
$$

where

$$
\begin{gathered}
A_{0}=1 / \cosh \zeta_{0} ; \quad G_{0}=4 \pi\left(\zeta_{0}-\tanh \zeta_{0}\right) / x^{2} \Omega_{\infty} \\
B_{0}=2 \pi\left(1-A_{0}\right) / x ; \quad Q_{0}=\pi \Omega_{\infty} \tanh \zeta_{0} \\
\zeta_{0}=0.5 \Omega_{\infty} x h_{0} ; \quad \Omega_{\infty}=\sqrt{2 \sigma \bar{\gamma}^{2} / x}
\end{gathered}
$$

$\sigma$ is the scattering coefficient of water; $x$ is its absorption coefficient; $\bar{\gamma}^{2}$ is the dispersion of the scattering phase function.

Approximating the distribution function of the reflection coefficient at the object by the Gaussian form enables obtaining its Fourier image in the following form:

$$
\begin{equation*}
F_{0}(\mathbf{k})=R_{0} \sum_{0} \exp \left(-\Sigma_{0} k^{2} / 4 \pi\right) \tag{8}
\end{equation*}
$$

where $R_{0}$ is the reflection coefficient; $\Sigma_{0}$ is the object's area.

## 3. Power of signal coming from the object

Substituting Eqs. (6)-(8) into Eq. (5), we obtain, after some transformations, the following expression for power of the signal coming from the object:

$$
\begin{equation*}
P_{\mathrm{ob}}=P_{0} \Sigma_{2} \frac{\Delta_{2}}{\pi} R_{0} \exp \left(-2 \varkappa h_{0}\right) \frac{A_{0}^{2} \Sigma_{0}}{S_{0}\left(S_{1}^{0}+S_{2}^{0}\right)} \tag{9}
\end{equation*}
$$

where $P_{0}=B_{0} \Sigma_{1} \Delta_{1}$ is the radiation power;

$$
S_{0}=\Sigma_{0}+\frac{S_{1}^{0} S_{2}^{0}}{S_{1}^{0}+S_{2}^{0}}, \quad S_{1,2}^{0}=\Sigma_{1,2}+\Delta_{1,2} L_{0}^{2}+G_{0}
$$

## 4. The $B N$ power coming from the medium in front of the object

The expression for $P_{\text {bs }}^{0}$ can be easily obtained on the basis of formula for $P_{\mathrm{ob}}$. To do this, it is necessary to perform the substitutions $h_{0} \rightarrow h, R_{0} \rightarrow \rho_{\mathrm{b}} \mathrm{d} h$ in Eq. (9). Here $\rho_{\mathrm{b}}=\sigma_{\pi} / 4$ is the backscattering coefficient of the medium. Then one should pass to the limit $\Sigma_{0} \rightarrow \infty$ and integrate the obtained expression over the interval from 0 to $h_{0}$. As a result of these transformations, we obtain the following expression for power of noise coming from the medium in front of the object:

$$
\begin{equation*}
P_{\mathrm{bs}}^{0}=P_{0} \Sigma_{2} \frac{\Delta_{2}}{\pi} \rho_{\mathrm{b}} \int_{0}^{h_{0}} \exp (-2 x h) \frac{A^{2}}{S_{1}+S_{2}} \mathrm{~d} h \tag{10}
\end{equation*}
$$

where the parameters $A, G$, and $S_{1,2}$ are described similarly to those in Eqs. (7) and (9), but here they depend on the current value of the depth $h$.

## 5. The $B N$ power coming from the medium behind the object

Let us outline the main ways to describe the power of BN coming from the layers behind the object with the allowance for its screening effect upon the incident light field. To solve the problem, it is necessary to recalculate the power of source's radiation incident onto the object plane. Then one should multiply the obtained brightness distribution by the transmission function of the object, determine illumination from this virtual source at an arbitrary depth below the object, and integrate the illumination from the source and receiver over the entire scattering volume.

The expression for angular spatial spectrum of the brightness distribution formed by a source at the depth $h_{0}$ is defined by Eq. (4). The spatial spectrum of illumination from a virtual source $F_{1}^{0}$ at the distance $h$ from it, with the allowance for screening effect of the object, is described by the expression

$$
\begin{align*}
F_{E_{1}}(\mathbf{k}, h) & =F_{\mathrm{e}}(\mathbf{k}, h)(2 \pi)^{-2} \iint_{-\infty}^{+\infty} F_{1}\left(\boldsymbol{\omega}, \mathbf{k} h+\omega L_{0}\right) \times \\
& \times F_{\mathrm{b}}\left(\boldsymbol{\omega}, \mathbf{k} h, h_{0}\right) F_{\mathrm{ob}}(\mathbf{k}-\boldsymbol{\omega}) \mathrm{d} \boldsymbol{\omega} \tag{11}
\end{align*}
$$

where the spatial spectrum of the transmission function of the object is

$$
F_{\mathrm{ob}}(\mathbf{k})=(2 \pi)^{-2} \delta(\mathbf{k})-\Sigma_{0} \exp \left(-\Sigma_{0} k^{2} / 4 \pi\right) .
$$

The spectrum of illumination distribution formed at the receiver is described similarly by changing the indices 1 for 2 .

The expression for power of BN coming from the medium behind the object has the following form:

$$
\begin{equation*}
P_{\mathrm{bs}}^{\mathrm{e}}=\int_{0}^{\infty} p(h) \mathrm{d} h, \tag{12}
\end{equation*}
$$

where

$$
\begin{equation*}
p(h)=\frac{\rho_{\mathrm{b}}}{\pi}(2 \pi)^{-2} \iint_{-\infty}^{+\infty} F_{E_{1}}(\mathbf{k}, h) F_{E_{2}}(\mathbf{k}, h) \mathrm{d} \mathbf{k} \tag{13}
\end{equation*}
$$

Substituting Eqs. (6), (7), and (11) into Eq. (13), after cumbersome though easy transformations, we obtain the expression for the differential BN coming from the medium behind the object:
$p(h)=P_{0} \Sigma_{2} \frac{\Delta_{2}}{\pi} \rho_{\mathrm{b}} \exp \left[-2 x\left(h+h_{0}\right)\right] A^{2} A_{2}^{0} \Phi(h)$,
where

$$
\begin{gathered}
\Phi(h)=\frac{1}{\psi_{1}^{0}+\psi_{2}^{0}}+\frac{\Sigma_{0}}{S_{1}+\Sigma_{0}} \frac{\Sigma_{0}}{S_{2}+\Sigma_{0}} \frac{1}{\psi_{1}+\psi_{2}}- \\
-\frac{\Sigma_{0}}{S_{1}+\Sigma_{0}} \frac{1}{\psi_{1}+\psi_{2}^{0}}-\frac{\Sigma_{0}}{S_{2}+\Sigma_{0}} \frac{1}{\psi_{1}^{0}+\psi_{2}} ; \\
\psi_{1,2}=\Delta_{1,2} h^{2}+Q_{0} h^{2}+\Sigma_{0}- \\
-\left(\Delta_{1,2} L_{0} h+B_{0} h-\Sigma_{0}\right)^{2} /\left(S_{1,2}^{0}+\Sigma_{0}\right)+G, \\
\psi_{1,2}^{0}=\Sigma_{1,2}+\Delta_{1,2}\left(L_{0}+h\right)^{2}+G_{0}+2 B_{0} h+Q_{0} h^{2}+G .
\end{gathered}
$$

Note that the variables $A_{0}, G_{0}, B_{0}$, and $Q_{0}$ depend on the depth of the object $h_{0}$ and the variables $A$ and $G$ on the current value of the depth $h$ counted from the object.

The expression for the BN power $P_{\text {bs }}^{\infty}$ coming from the medium behind the object (when the latter is absent) can be obtained from Eqs. (11) and (13) by taking the object's area $\Sigma_{0}$ to be equal to zero.

## 6. Contrast of the object's image

The formula for determination of the object's image contrast against the medium background is written in the following form:

$$
K=\frac{P_{i}-P_{\mathrm{bs}}}{P_{i}+P_{\mathrm{bs}}}
$$

This expression can be reduced to

$$
\begin{equation*}
K=K_{i} /\left(1+P_{\mathrm{bs}}^{0} / \bar{P}\right), \tag{15}
\end{equation*}
$$

where $K_{i}=\Delta P / \bar{P} ; \quad \Delta P=0.5\left(P_{\mathrm{ob}}+P_{\mathrm{bs}}^{\mathrm{e}}-P_{\mathrm{bs}}^{\infty}\right) ; \quad \bar{P}=$ $=0.5\left(P_{\mathrm{ob}}+P_{\mathrm{bs}}^{\mathrm{e}}+P_{\mathrm{bs}}^{\infty}\right)$.
From this it follows that the value $K_{i}$ defines the contrast of the object's image without a regard for the noise $P_{\mathrm{bs}}^{0}$ coming from the medium in front of the object.

For better understanding of physical meaning of the results obtained below, we express the image contrast not in terms of the radiation power but in terms of some efficient reflection coefficients. For this purpose, we first introduce the signal power $P_{\mathrm{ob}}^{\infty}$ received from a boundless object with the reflection
coefficient equal to unity. The object is supposed to be at the depth $h_{0}$. Let all the components obtained above be normalized to it. It follows from Eq. (9) that

$$
\begin{equation*}
P_{\mathrm{ob}}^{\infty}=P_{0} \Sigma_{2} \frac{\Delta_{2}}{\pi} \exp \left(-2 x h_{0}\right) \frac{A_{0}^{2}}{S_{1}^{0}+S_{2}^{0}} \tag{16}
\end{equation*}
$$

Normalizing Eqs. (9), (10), and (14) by Eq. (16), we obtain expressions for the efficient reflection coefficients in the form

$$
\begin{gather*}
R_{\mathrm{ob}}=R_{0} \Sigma_{0} / S_{0},  \tag{17}\\
R_{\mathrm{bs}}^{0}=\rho_{\mathrm{b}} \int_{0}^{h_{0}} \exp \left[2 x\left(h_{0}-h\right)\right]\left(\frac{A}{A_{0}}\right)^{2} \frac{S_{1}^{0}+S_{2}^{0}}{S_{1}+S_{2}} \mathrm{~d} h, \\
R_{\mathrm{bs}}^{\mathrm{e}}=\rho_{\mathrm{b}}\left(S_{1}^{0}+S_{2}^{0}\right) \int_{0}^{\infty} \exp (-2 x h) A^{2} \Phi(h) \mathrm{d} h, \\
R_{\mathrm{bs}}^{\infty}=\rho_{\mathrm{b}}\left(S_{1}^{0}+S_{2}^{0}\right) \int_{0}^{\infty} \exp (-2 x h) A^{2} \Phi_{0}(h) \mathrm{d} h,
\end{gather*}
$$

where

$$
\Phi_{0}(h)=\left.\Phi(h)\right|_{\Sigma_{0} \rightarrow 0}=\left(\psi_{1}^{0}+\psi_{2}^{0}\right)^{-1} .
$$

The expression for contrast of the object's image against the medium background can be deduced from Eq. (15) as a function of efficient reflection coefficients of the object and the medium:

$$
\begin{equation*}
K=K_{i} /\left(1+R_{\mathrm{bs}}^{0} / \bar{R}\right), \tag{18}
\end{equation*}
$$

where

$$
K_{i}=\Delta R / \bar{R},
$$

$\Delta R=0.5\left(R_{\mathrm{ob}}+R_{\mathrm{bs}}^{\mathrm{e}}-R_{\mathrm{bs}}^{\infty}\right), \quad \bar{R}=0.5\left(R_{\mathrm{ob}}+R_{\mathrm{bs}}^{\mathrm{e}}+R_{\mathrm{bs}}^{\infty}\right)$.
The formulas (15) and (18) describe the general situation, i.e., an observation system with arbitrary directional patterns of the light source (SDP) and receiver (RDP). Relations for calculating the image contrast in an observation system with similar SDPs and RDPs (conventionally, $\mathrm{N}-\mathrm{N}$ system) and with a narrow SDP and wide RDP ( $\mathrm{N}-\mathrm{W}$ system) can be obtained as corollaries of these formulas.

Thus, for the $\mathrm{N}-\mathrm{N}$ observation system ( $\Sigma_{1}=\Sigma_{2}$, $\Omega_{1}=\Omega_{2}$ ) we obtain from Eq. (17) that

$$
\begin{gathered}
R_{\mathrm{ob}}=R_{0} \Sigma_{0} /\left(\Sigma_{0}+S_{1} / 2\right), \\
R_{\mathrm{bs}}^{0}=\rho_{\mathrm{b}} \int_{0}^{h_{0}} \exp \left[2 x\left(h_{0}-h\right)\right]\left(\frac{A}{A_{0}}\right)^{2} \frac{S_{1}^{0}}{S_{1}} \mathrm{~d} h, \\
R_{\mathrm{bs}}^{\mathrm{e}}=\rho_{\mathrm{b}} S_{1}^{0} \int_{0}^{\infty} \exp (-2 \varkappa h) A^{2} \Phi^{\prime}(h) \mathrm{d} h,
\end{gathered}
$$

$$
\begin{gathered}
\Phi^{\prime}(h)=\frac{1}{\psi_{1}^{0}}+\left(\frac{\Sigma_{0}}{S_{1}+\Sigma_{0}}\right)^{2} \frac{1}{\psi_{1}}-\frac{\Sigma_{0}}{S_{1}+\Sigma_{0}} \frac{2}{\psi_{1}+\psi_{1}^{0}} \\
R_{\mathrm{bs}}^{\infty}=\rho_{\mathrm{b}} S_{1}^{0} \int_{0}^{\infty} \exp (-2 x h) A^{2} / \psi_{1}^{0} \mathrm{~d} h .
\end{gathered}
$$

For the $\mathrm{N}-\mathrm{W}$ observation system $\left(\Delta_{2} \rightarrow \infty\right)$, it follows from Eq. (17) that

$$
\begin{gathered}
R_{\mathrm{ob}}=R_{0} \Sigma_{0} /\left(S_{1}+\Sigma_{0}\right), \\
R_{\mathrm{bs}}^{0}=\rho_{\mathrm{b}} \int_{0}^{h_{0}} \exp \left[2 x\left(h_{0}-h\right)\right]\left(\frac{A}{A_{0}}\right)^{2}\left(\frac{L_{0}}{H+h}\right)^{2} \mathrm{~d} h, \\
R_{\mathrm{bs}}^{\mathrm{e}}=\rho_{\mathrm{b}} \int_{0}^{\infty} \exp (-2 x h) A^{2} \Phi^{\prime \prime}(h) \mathrm{d} h ; \\
\Phi^{\prime \prime}(h)=\frac{S_{1}}{S_{1}+\Sigma_{0}}\left(\frac{L_{0}}{L_{0}+h_{0}}\right)^{2}+ \\
+\frac{\Sigma_{0}}{S_{1}+\Sigma_{0}} \frac{\Sigma_{0}}{\psi_{1}+\psi_{0}}-\frac{\Sigma_{0}}{\psi_{1}^{0}+\psi_{0}} ; \\
+\left(h / L_{0}\right)^{2}\left(\Sigma_{2}+G_{0}-2 B_{0} L_{0}+Q_{0} L_{0}^{2}\right) ; \\
R_{\mathrm{bs}}^{\infty}=\rho_{\mathrm{b}} \int_{0}^{\infty} \exp (-2 \varkappa h)\left(\frac{A L_{0}}{L_{0}+h}\right)^{2} \mathrm{~d} h .
\end{gathered}
$$

## 7. "Differential" image contrast

All the above stated refers to an observation system with continuous illumination. Nevertheless, the obtained results can be easily modified for calculating the structure of the object's image in an observation system with pulsed illumination and time gating of the photodetector signal.

Let us suppose that the OS source emits a light pulse of duration $\tau_{1}$, and the photodetector time resolution is $\tau_{2}$. The light forming the image comes to the OS from the medium layer between the depths $h_{\mathrm{s}}$, $h_{\mathrm{s}}+\Delta h$, where $h_{\mathrm{s}}$ is the current depth of gating; $\Delta h=c\left(\tau_{1}+\tau_{2}\right) / 2$ ( $c$ is the speed of light in water) .

Depending on the relation between $h_{\mathrm{s}}, \Delta h$, and $h_{0}$, one should use different formulas for calculating the image contrast:

- for $h_{0}-\Delta h \geq h_{\mathrm{s}} \geq 0$ the contrast is

$$
\begin{equation*}
K \equiv 0, \tag{21=}
\end{equation*}
$$

- for $h_{0} \geq h_{\mathrm{s}} \geq h_{0}-\Delta h$ it is
$K=\left(R_{\mathrm{ob}}+R_{\mathrm{bs}}^{\mathrm{e}}-R_{\mathrm{bs}}^{\infty}\right) /\left(R_{\mathrm{ob}}+R_{\mathrm{bs}}^{\mathrm{e}}+R_{\mathrm{bs}}^{\infty}+2 R_{\mathrm{bs}}^{0}\right)$.
Here the efficient reflection factors are described by formulas (17) with changed integration limits: for $R_{\mathrm{bs}}^{0}: h_{\mathrm{s}}, h_{0}$; for $R_{\mathrm{bs}}^{\mathrm{e}}, R_{\mathrm{bs}}^{\infty}: 0, h_{\mathrm{s}}+\Delta h-h_{0}$;
- for $\infty>h_{\mathrm{s}}>h_{0}$

$$
\begin{equation*}
K=\left(R_{\mathrm{bs}}^{\mathrm{e}}-R_{\mathrm{bs}}^{\infty}\right) /\left(R_{\mathrm{bs}}^{\mathrm{e}}+R_{\mathrm{bs}}^{\infty}\right) . \tag{21c}
\end{equation*}
$$

Here the integration limits for $R_{\mathrm{bs}}^{\mathrm{e}}$ and $R_{\mathrm{bs}}^{\infty}$ are $h_{\mathrm{s}}-h_{0}$, $h_{\mathrm{s}}+\Delta h-h_{0}$.

## 8. Results of numerical analysis

The obtained relations enable one to analyze contrast as a function of all physical factors of the problem, i.e., parameters of the object, observation system, and characteristics of the medium. Below we present the results of numerical calculations of image contrast for the following values of the OS, object's , and medium parameters: $H=300 \mathrm{~m}$, diameter of the object $d_{0}=1 \mathrm{~m}$, scattering coefficient of the medium $\sigma=0.3 \mathrm{~m}^{-1}, \quad$ absorption coefficient $\quad x=0.04 \mathrm{~m}^{-1}$, backscattering coefficient $\sigma_{\pi}=0.015 \mathrm{~m}^{-1}$, dispersion of the scattering phase function $\bar{\gamma}=0.04$.

Now let us discuss the obtained results. Calculations for the observation system of the $\mathrm{N}-\mathrm{N}$ type demonstrate that the image contrast as a function of depth for the values of the object's reflectivity $1 \geq R_{0} \geq 0.1$ monotonically decreases; this refers both to the function $K_{i}\left(h_{0}\right)$ and $K\left(h_{0}\right)$. However, for $R_{0}$ close to the brightness factor of the sea $\sigma_{\pi} / 8 \chi$, the contrast's behavior becomes more complicated. Figure 2 presents the depth functions $K_{i}$, which were calculated by Eqs. (18) and (19) for $R_{0}=0.03$. These functions demonstrate that the contrast is a non-monotonic function and changes its sign. The behavior of $K_{i}$ significantly depends on the width of the directional pattern $\Theta_{1}$ and the object's size $d_{0}$.


Fig. 2. Image contrast as a function of the object's depth ( $d_{0}=5 \mathrm{~m}, R_{0}=0.03$ ): $\Theta_{1}=0.01^{\circ}(1) ; \Theta_{1}=1^{\circ}(2)$.

To explain such a "strange" contrast's behavior, let us consider Fig. 3, which presents efficient reflection coefficients of the object and medium as functions of depth. The function $R_{\mathrm{ob}}\left(h_{0}\right)$ monotonically decreases, and functions $R_{\mathrm{bs}}^{\mathrm{e}}$ and $R_{\mathrm{bs}}^{\infty}$ of $h_{0}$ are not monotonic in the general case. The intersection points of the plots $R_{\mathrm{ob}}\left(h_{0}\right)$ and $\Delta R_{\mathrm{bs}}\left(h_{0}\right)$, where $\Delta R_{\mathrm{bs}}=R_{\mathrm{bs}}^{\infty}-R_{\mathrm{bs}}^{\mathrm{e}}$, are marked in Fig. 3 by circles and correspond to the depths where image contrast changes its sign (at these depths the object becomes invisible against the medium background). It is easy to see that the contrast can be positive, negative, and change its sign depending on the relation between the values $R_{\mathrm{ob}}, R_{\mathrm{bs}}^{\mathrm{e}}$, and $R_{\mathrm{bs}}^{\infty}$. The efficient reflection coefficient of a semiinfinite medium $R_{\mathrm{bs}}^{\infty}$ deserves special attention as a function of depth for

OS of the $\mathrm{N}-\mathrm{N}$ type (Fig. 4). It follows from the curves in Fig. 4 that the behavior of $R_{\mathrm{bs}}^{\infty}$ is significantly determined by the width of the OS directional pattern $\Theta_{1}$, but the function $R_{\mathrm{bs}}^{\infty}$ is not monotonic in the general case.


Fig. 3. Efficient reflection coefficient of the object and medium as functions of depth: $\Theta_{1}=0.01^{\circ}(a) ; \Theta_{1}=1^{\circ}(b)$; curves: $R_{\mathrm{ob}}(1) ; R_{\mathrm{bs}}^{\infty}(2) ; R_{\mathrm{bs}}^{\mathrm{e}}(3) ; \Delta R_{\mathrm{bs}}^{\mathrm{e}}$ (4).


Fig. 4. Efficient reflection coefficients of a semiinfinite medium as functions of depth: curves $\Theta_{1}=0.01^{\circ}$ (1); $\Theta_{1}=0.1^{\circ}(2) ; \Theta_{1}=1^{\circ}(3) ; \Theta_{1}=10^{\circ}(4)$.

Note that no such features are observed in the behavior for the OS of the $\mathrm{N}-\mathrm{W}$ type. Since $R_{\mathrm{bs}}^{\infty}=$ const in this case, the contrast, as a function of depth, monotonically decreases and does not change its sign. Similar results are obtained in Ref. 5 devoted to the problem of optical detection of local inhomogeneities in biological tissues. Figure 5 presents "differential" contrasts of an object as functions of the depth $h_{\mathrm{s}}$ of gating against the background of a medium in the observation system of the $\mathrm{N}-\mathrm{N}$ type with pulsed illumination (the data are calculated by the formulas (21)).


Fig. 5. "Differential" image contrast as a function of the depth of strobing $\left(\Theta_{1}=0.1^{\circ}, \quad R_{0}=0.5, \quad \Delta h=2 \mathrm{~m}, \quad D_{0}=1 \mathrm{~m}\right)$; $h_{0}=5 \mathrm{~m}$ (1), 10 m (2), 20 m (3), and 30 m (4).

Contrast is positive at the strobe depths corresponding to the object depth and negative at depths below the object. It is remarkable that the contrast value in the latter case does not depend on reflection characteristics of the object. This makes it possible to detect even objects that are "invisible" against the medium background by their "shadows". ${ }^{6}$ Some of the general trends of contrast's behavior are as follows: both in the positive and negative part, contrast grows with the increase in the object's dimensions; it decreases with the object's depth and
deteriorates with the increase in strobe duration $\Delta h$. Similar trends are seen also for the observation system of the $\mathrm{N}-\mathrm{W}$ type.

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